Analytic Representation for Parallel Flow of Hot Ions Produced by Tangential Neutral Beam Injection

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(Received 7 June 2018 / Accepted 12 September 2018)

Tangential neutral beam injection deposits momentum in a fusion plasma, which produces macroscopic flow of bulk particles. Concurrently, equilibrium distribution of hot ions produced by the injection is determined by collisions with the bulk particles. Through these processes, macroscopic flow of hot ions can be produced. Effects of the flow of hot ions have not been focused on since their inertial force is thought to be much smaller than the bulk one due to smallness of density of hot ions. In this study, we derive an analytic representation for parallel flow of hot ions. From the analytic representation, we find that amplitude of the parallel flow is determined by three parameters, injection speed, injection pitch angle, and electron temperature. Also we find that the inertial force of hot ions is not negligible even if the density of hot ions is much smaller than that of bulk ions.

Keywords: hot ion, flow, neutral beam injection, slowing-down distribution, inertial force of hot ions

DOI: 10.1585/pfr.13.1403119

1. Introduction

A neutral beam injection (NBI) system is key equipment for a fusion device. One of the most important roles for the system is to heat plasma to a high temperature enough for fusion. Another important role is to deposit momentum in the plasma, which produces macroscopic plasma flow. The resultant plasma flow improves plasma performance. The flow can stabilize some magnetohydrodynamic (MHD) modes, such as resistive wall mode (RWM) that limits the achievable β value. Also, sheared flow can suppress microscopic turbulence through sheared radial electric field, which reduces anomalous transport. Therefore, control of the plasma flow is essential to realize confinement of a stable high-β plasma. For the control of flow profile, the NBI system can be an "actuator."

In this paper, we consider a situation such that the NB is injected in a plasma steadily. The NB injection produces hot ions through collisions with bulk particles. This gives the source term for dynamics of hot ions. These hot ions achieve the steady state through collisions with the bulk particles. Through these processes, the bulk plasma flow is produced, and we assume that it is prescribed in this paper. The distribution function of hot ions for the steady state has been discussed in [1], which solves the steady state Fokker-Planck equation with the source term. The source term is given by delta functions of injection speed (or energy) and injection pitch angle. The solution represents the slowing-down distribution in the energy (speed) and scattering in the pitch angle. In this paper, we investigate the macroscopic flow of "hot" ions (not "bulk" ions). The flow of hot ions has not been focused on because it is not expected to affect the overall plasma dynamics. The inertial force of hot ions is thought to be small due to the smallness of the number density. Based on this assumption, the hybrid kinetic-MHD model, which can analyze the interaction between MHD modes and hot particles, neglects the inertial force of hot ions [2–4]. However, recently, we show that the flow of hot ions can affect the MHD stability (such as RWM) through energy exchange term between bulk plasma and hot ions [5]. In this paper, we derive an analytic representation for parallel flow of hot ions. The present representation can be applied to the situation that the NB is tangentially injected in a tokamak.

The remainder of this paper is organized as follows. In Sec. 2, we derive an analytic solution of the equilibrium distribution function for hot ions which collide with bulk particles with flow. In Sec. 3, based on the analytic solution of the distribution function described in Sec. 2, we derive an analytic representation for parallel flow of hot ions. In Sec. 4, we investigate the parallel flow based on the analytic solution derived in Sec. 3. Also we study the importance of the inertial force of hot ions. Section 5 summarizes the results.

2. Analytic Solution of Equilibrium Distribution Function for Hot Ions Interacting with Flowing Bulk Plasma

In this section, we derive an analytic solution of an equilibrium distribution function for hot ions which collide

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with bulk particles with flow. We start from the steady-state Fokker-Planck equation that governs the dynamics of hot ions,

$$\vec{v}_h \cdot \nabla f_h + \frac{Q_h}{M_h}\left(\vec{E} + \vec{v}_h \times \vec{B}\right) \cdot \nabla_{\vec{v}_h} f_h = \sum_{j=i,e} \mathcal{F}(f_h, f_j), \quad (1)$$

where f is the equilibrium distribution function (the subscripts h, i, and e represent for hot ions, bulk ions, and bulk electrons, respectively), Q is the charge, M is the mass, $\vec{E}$ is the equilibrium electric field, $\vec{B}$ is the equilibrium magnetic field, and $\mathcal{F}$ is the collision operator. We note that $\vec{v}$ is the particle velocity in the laboratory frame. We assume that collisions between hot ions can be neglected. The collision operator of the Landau form reads

$$\mathcal{F}(f_h, f_j) = \frac{Q_h^2 Q_j^2}{8\pi\varepsilon_0^2 M_h} \ln\Lambda \times \frac{\partial}{\partial \vec{v}_h} \cdot \int dv_j^3\, \omega_{hj} \cdot \left(\frac{1}{M_h}\frac{\partial}{\partial \vec{v}_h} - \frac{1}{M_j}\frac{\partial}{\partial \vec{v}_j}\right) f_h f_j, \quad (2)$$

where ε_0 is the vacuum permittivity, $\ln\Lambda$ is the Coulomb cut-off factor, and $\partial/\partial\vec{v}_h$ and $\partial/\partial\vec{v}_j$ operate on f_h and f_j, respectively. We note that the tensor

$$\omega_{hj} = \frac{\partial^2 g_{hj}}{\partial\vec{v}_h \partial\vec{v}_h} = \frac{1}{g_{hj}^3}\left(g_{hj}^2 \mathcal{I} - \vec{g}_{hj}\vec{g}_{hj}\right), \quad (3)$$

depends on the relative velocity $\vec{g}_{hj} = \vec{v}_h - \vec{v}_j$, where $\mathcal{I}$ is a unit tensor. In what follows, we assume that equilibrium flow of bulk ions is prescribed by $\vec{V}_i$. In addition, we assume that the equilibrium electric field is determined by the ideal Ohm's law as $\vec{E} + \vec{V}_i \times \vec{B} = 0$. After variable transformation $\vec{u}_h = \vec{v}_h - \vec{V}_i$ and $\vec{u}_j = \vec{v}_j - \vec{V}_i$, we observe $\vec{g}_{hj}$ is invariant, which indicates that Eqs. (2) and (3) are also invariant for putting $\vec{v}_h \to \vec{u}_h$ and $\vec{v}_j \to \vec{u}_j$. This fact is due to the Galilei invariance of the collision operator [6]. Then, assuming f_h is spatially uniform, Eq. (1) reduces to

$$\frac{Q_h}{M_h}\left(\vec{u}_h \times \vec{B}\right) \cdot \nabla_{\vec{u}_h} f_h = \sum_{j=i,e} \mathcal{F}(f_h, f_j), \quad (4)$$

which is formally equivalent to the equation in [1]. Then following the derivation in [1] with assuming that the equilibrium distribution function is axisymmetric about the magnetic field, we can obtain the analytic form of equilibrium distribution function for hot ions as

$$f_h(u_h, \xi) = \frac{S^0 \tau_s}{u_h^3 + u_c^3} \sum_{l=0}^{\infty} \frac{2l+1}{2} P_l(\xi_0) P_l(\xi) \times \left(\frac{u_h^3}{u_0^3}\frac{u_0^3 + u_c^3}{u_h^3 + u_c^3}\right)^{\frac{1}{6}l(l+1)Z_2} U(u_0 - u), \quad (5)$$

where $\xi = (\vec{u}_h/u_h) \cdot \hat{b} = \cos\theta$ is the pitch ($\hat{b} = \vec{B}/B$ is the unit vector along the equilibrium magnetic field and θ is the pitch angle), ξ_0 is the injection pitch of the hot ions, and S^0 is the source term defined by $N_{0h} = (2\pi S^0 \tau_s/3)\ln[(u_0^3 + u_c^3)/u_c^3]$. Here N_{0h} is the uniform number density of hot ions, $\tau_s = u_c^3/(2\Gamma_{he} Z_1)$ is the Spitzer slowing down time, u_0 is the injection speed of hot ions by neutral beam injection, and u_c is the crossover speed. The crossover speed is expressed as $u_c = [3\sqrt{\pi} M_e Z_1/(4M_h)]^{1/3} v_{the}$, where

$$Z_1 = \sum_i \frac{N_{0i} Z_i^2 M_h}{N_{0e} M_i}, \quad (6)$$

is a parameter depending on the mass ratio [$N_{0i(e)}$ is the number density of ions (electrons) and Z is the charge number] and $v_{the} = \sqrt{2T_e/M_e}$ is the electron thermal speed (T_e is the electron temperature). The coefficient Γ_{he} is defined by $\Gamma_{he} = Z_h^2 e^4 N_{0e} \ln\Lambda/(8\pi\varepsilon_0^2 M_h^2)$, where e is the elementary charge. In Eq. (5), P_l is the Legendre polynomials,

$$Z_2 = \frac{\sum_i Z_i^2 N_{0i}}{N_{0e}} \frac{1}{Z_1}, \quad (7)$$

is a parameter depending on the mass ratio, and U is the step function.

3. Analytic Representation for Parallel Flow of Hot Ions Produced by Tangential Neutral Beam Injection

Using the analytic solution of the distribution function for hot ions interacting with bulk particles with flow [Eq. (5)], we compute the parallel flow of hot ions. Let us start from the definition of the macroscopic flow of hot ions,

$$N_{0h}\vec{V}_h = \int \vec{v}_h f_h dv_h^3 = \int \vec{u}_h f_h du_h^3 + N_{0h}\vec{V}_i. \quad (8)$$

When we substitute Eq. (5) into Eq. (8), it is difficult to compute a general form of flow. Hence in this study, we focus on the "parallel" flow, which would be observed when neutral beam is injected tangentially into a tokamak plasma. From Eq. (8), we obtain

$$N_{0h} V_{h\parallel} = \int u_h \xi f_h du_h^3 + N_{0h} V_{i\parallel}. \quad (9)$$

The second term in the right hand side of Eq. (9) represents the flow of bulk ions and is assumed to be prescribed. Substituting Eq. (5) into the first term of the right hand side of Eq. (9), we obtain

$$\int u_h \xi f_h du_h^3 = 2\pi S^0 \tau_s \sum_{l=0}^{\infty} \frac{2l+1}{2} P_l(\xi_0) \left(\frac{u_0^3 + u_c^3}{u_0^3}\right)^{\frac{1}{6}l(l+1)Z_2} \times \int_{-1}^{1} d\xi\, \xi P_l(\xi) \int_0^{u_0} du_h \left(\frac{u_h^3}{u_h^3 + u_c^3}\right)^{\frac{1}{6}l(l+1)Z_2 + 1}. \quad (10)$$

In Eq. (10), we observe that the integration by ξ becomes rather simple because $\int_{-1}^{1} \xi P_l(\xi) d\xi = 0$ for $l > 1$ [7]. Since

$\int_{-1}^{1} \xi P_0(\xi)d\xi = 0$ for $l = 0$, only $l = 1$ component survives, i.e., $\int_{-1}^{1} \xi P_1(\xi)d\xi = 2/3$. Then Eq. (10) reduces to

$$\int u_h \xi f_h du_h^3 = 2\pi S^0 \tau_s \xi_0 \left(\frac{u_0^3 + u_c^3}{u_0^3}\right)^{\frac{1}{3}Z_2} \times \int_0^{u_0} du_h \left(\frac{u_h^3}{u_h^3 + u_c^3}\right)^{\frac{1}{3}Z_2+1}.$$

The last term, which represents the integration by u_h, can be solved analytically as [8]

$$\int_0^{u_0} du_h \left(\frac{u_h^3}{u_h^3 + u_c^3}\right)^{\frac{1}{3}Z_2+1} = \frac{u_c}{3}\frac{\upsilon^\mu}{\mu} {}_2F_1(\mu - 1/3, \mu; 1 + \mu; -\upsilon), \tag{11}$$

where $\mu = (Z_2 + 4)/3$, $\upsilon = u_0^3/u_c^3$, and ${}_2F_1$ is the Gauss's hypergeometric function. Using these expressions, we obtain an analytic form of the parallel flow of hot ions as

$$N_{0h}V_{h\parallel} = \frac{2\pi}{3} S^0 \tau_s u_c \xi_0 \left(\frac{\upsilon + 1}{\upsilon}\right)^{\frac{1}{3}Z_2} \frac{\upsilon^\mu}{\mu} {}_2F_1 + N_{0h}V_{i\parallel}, \tag{12}$$

where the arguments of ${}_2F_1$ are same with ones in Eq. (11). Equation (12) is the analytic representation for the parallel flow of hot ions. Here, we compare the present analysis with the previous ones on NB current drive [9,10]. The previous studies tried to obtain the current density of hot ions with tangential neutral beam injection by assuming that the distribution function is characterized by only $l = 1$ Legendre polynomial. In these literatures, the current density (or flow multiplied by the density) is expressed in an integral form. Therefore, the difference between the present and previous analyses is only the use of the hypergeometric function instead of integral in Eq. (11). The present analysis clearly shows the validity of the assumption and that the expression has an analytic solution.

In what follows, we make some assumptions to reduce Eq. (12). We consider a single bulk ion specie, and the bulk and hot ions have the same charge number $Z_i = Z_h = 1$. In addition, we assume that the bulk ions and electrons have the same density $N_{0i} = N_{0e}$. Then from Eqs. (6) and (7), Eq. (12) reduces to

$$V_{h\parallel} = \frac{u_c \xi_0}{\ln(1 + \nu)} \left(\frac{\nu + 1}{\nu}\right)^{\mu - \frac{4}{3}} \frac{\nu^\mu}{\mu} {}_2F_1 + V_{i\parallel}. \tag{13}$$

From Eq. (13), we observe that the parallel flow of hot ions is characterized by the crossover velocity u_c, injection pitch ξ_0, mass ratio $\mu = (M_i/M_h + 4)/3$, and a parameter $\nu = u_0^3/u_c^3$ that depends on injection speed, mass ratio, and electron temperature.

4. Analysis of Inertial Force of Hot Ions

In the previous section, we have succeeded to obtain an analytic representation for the the parallel flow of hot ions produced by tangential neutral beam injection as shown in Eq. (13). First we observe that the first term in the right hand side of Eq. (13) is spatially uniform, hence the spatial profile is characterized by the parallel flow of bulk ions. This is because we assume that f_h is uniform. Since the profile of $V_{h\parallel}$ is totally determined by the bulk ion flow profile, in this paper we assume a uniform bulk ion flow to exclude the complexity arising from non-uniformity. Equation (13) indicates that the amplitude of parallel flow of hot ions is characterized by the injection parameters (speed $\upsilon = u_0^3/u_c^3$ and pitch ξ_0), the electron temperature (through the crossover speed u_c), and the mass ratio μ. In what follows, we investigate how these parameters affect the amplitude of parallel flow of hot ions, $V_{h\parallel}$.

4.1 Dependence of parallel flow of hot ions on injection parameters

We first investigate how the amplitude of parallel flow of hot ions, $V_{h\parallel}$, depends on the injection parameters (injection speed u_0 and pitch ξ_0). To this end, we fix the following parameters as $M_h/M_i = 1$, $N_h/N_{0i} = 2 \times 10^{-2}$, $N_{0i} = N_{0e} = 0.5 \times 10^{20}\mathrm{m}^{-3}$, $T_{0e} = 3\mathrm{keV}$, and $\ln\Lambda = 17$. These are JT-60SA-like parameters. When these parameters are fixed, the first term of Eq. (13) reads $V_{h\parallel} = (3u_c\xi_0/5)(1 + 1/\upsilon)^{1/3}\upsilon^{5/3}/\ln(1 + \upsilon){}_2F_1(4/3, 5/3; 8/3; -\upsilon)$ where $\upsilon = u_0^3/u_c^3$. This shows that $|V_{h\parallel}|$ is large when $|\xi_0|$ is large and that $V_{h\parallel}$ is symmetric with respect to ξ_0. Naturally, large injection pitch, which is almost tangential to the magnetic field, can drive large parallel flow and the direction depends on its sign. These properties are clearly shown in the analytic representation. The factor related to υ can be divided in two parts, an elementary function part $(1 + 1/\upsilon)^{1/3}\upsilon^{5/3}/\ln(1 + \upsilon)$ and a hypergeometric function part ${}_2F_1(4/3, 5/3; 8/3; -\upsilon)$. Figure 1 shows the behavior of two parts when we varied the injection speed from 0 to $1.6V_A$ (V_A is the Alfvén speed), where $u_0 = 1.6V_A$ corresponds to the energy $E_0 = M_h u_0^2/2 \sim 500\mathrm{keV}$ similar to the JT-60SA N-NB injection. Clearly shown in Fig. 1 (a) , for fixed electron temperature and mass ratio, the dependence of $V_{h\parallel}$ on u_0 is almost linear. The near-linear dependence for small u_0 can be shown as follows. For small $\nu = u_0^3/u_c^3$, the elementary part is approximated as $(1+1/\nu)^{1/3}\nu^{5/3}/\ln(1+\nu) = \nu^{1/3}[1+O(\nu)]$, while the hypergeometric part is approximated as ${}_2F_1(4/3, 5/3; 8/3; -\nu) = 1 + O(\nu)$. These are clearly shown in Fig. 1 (b). Therefore, the "product" in Fig. 1 (a) in small u_0/V_A becomes linear. From Fig. 1, we obtain $V_{h\parallel} \sim 1.2(u_c/V_A)\xi_0 u_0 \sim 0.57\xi_0 u_0$ for the present parameters. Therefore, when $u_0 = 1.6V_A$ and $\xi_0 = 1$, we obtain $V_{h\parallel}/V_A \sim 0.9 = O(1)$, which indicates the flow can be Alfvénic.

4.2 Dependence of $V_{h\parallel}$ on electron temperature

The other important parameter to determine the amplitude of $V_{h\parallel}$ is the electron temperature, T_e, i.e., elec-

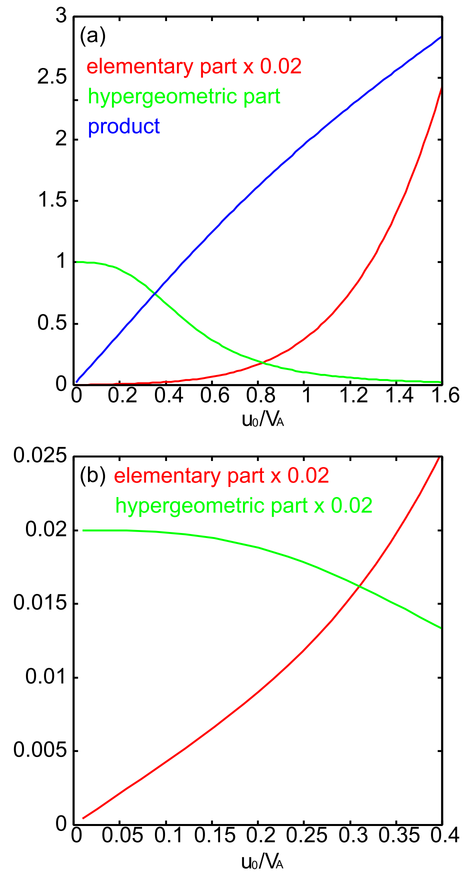

Fig. 1 (a) Plots of the elementary part $(1 + 1/\upsilon)^{1/3}\upsilon^{5/3}/\ln(1 + \upsilon)$, the hypergeometric part ${}_2F_1(4/3, 5/3; 8/3; -\upsilon)$, and their product as functions of u_0/V_A. The first one is multiplied by 0.02. (b) Plots of elementary and hypergeometric parts multiplied by 0.02 in the range $0 < u_0/V_A \leq 0.4$.

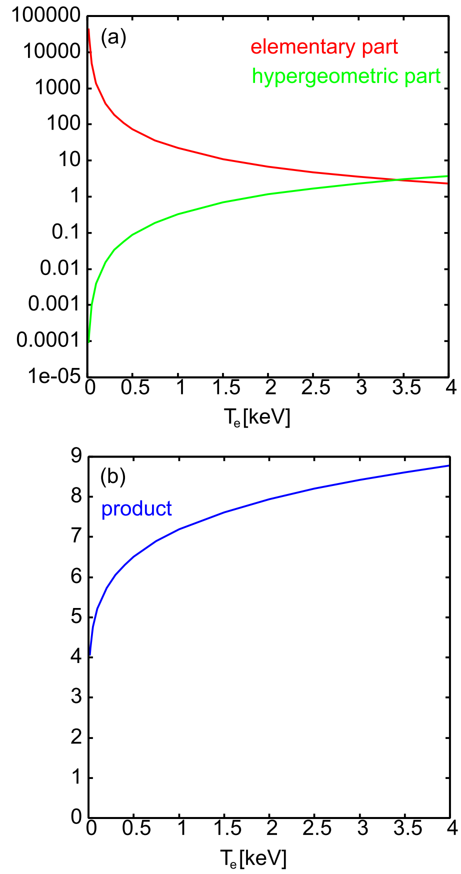

Fig. 2 (a) Plots of the elementary part $(1 + 1/\upsilon)^{1/3}\upsilon^{5/3}/\ln(1 + \upsilon)$, the hypergeometric part ${}_2F_1(4/3, 5/3; 8/3; -\upsilon)$ as functions of T_e. The first and second lines multiplied by 10^{-8} and 10^2 respectively. (b) Plot of the product as a function of T_e.

tron thermal speed. The electron thermal speed affects the crossover speed u_c, which affects the amplitude of $V_{h\|}$. We fix the injection parameters $\xi_0 = 0.8$ and $u_0/V_A = 1.6$, which corresponds to the tangential neutral beam injection in JT-60SA. Other parameters are fixed as same with the previous subsection except for the electron temperature. In this case, the analytic representation (13) reads $V_{h\|} = 0.8u_c(1 + 1/\upsilon)^{1/3}\upsilon^{5/3}/\ln(1 + \upsilon)\,{}_2F_1(4/3, 5/3; 8/3; -\upsilon)$ where $\upsilon = u_0^3 u_c^{-3}$. The factor related to υ can be divided into two parts as in the previous subsection. Figure 2 (a) shows the behavior of elementary and hypergeometric parts when we varied the electron temperature, and Fig. 2 (b) indicates their product. Figure 2 (b) indicates that the product is approximately scales by $T_e^{1/6} \propto u_c^{1/3}$. Therefore, the flow amplitude depends on the crossover speed as $V_{h\|} \propto u_c^{4/3}$ in total.

4.3 Dependence of $V_{h\|}$ on mass ratio

The last parameter to determine the amplitude of $V_{h\|}$ is the mass ratio. Here we define the mass ratio parameter M_h/M_i and study the mass ratio effect on $V_{h\|}$. To this end, we fix the following parameters as follows, $N_h/N_{0i} = 2 \times 10^{-2}$, $N_{0i} = N_{0e} = 0.5 \times 10^{20}\mathrm{m}^{-3}$, $T_{0e} = 3\mathrm{keV}$, $\xi_0 = 0.8$, and $u_0/V_A = 1.6$. We consider a situation such that the bulk and hot ions can be hydrogen or deuterium. These can be realized in the "initial research phase" in JT-60SA. In this case, the mass ratio can be 0.5, 1, and 2. When the mass ratio is changed, we obtain $V_{h\|}/V_A = 0.614$, 0.646, and 0.664 for $M_h/M_i = 0.5$, 1, and 2, respectively. The dependence of $V_{h\|}$ on the mass ratio is weak. Since u_0 is fixed, increasing M_h/M_i means the injection energy is increased.

4.4 Estimate of inertial force of hot ions

In the previous subsections, we observe that the amplitude of parallel flow of hot ions is determined mainly by the injection parameters, and that the electron temperature can affect the amplitude while the mass ratio has little effects. The inertial force of hot ions can be approximated by $I_h = M_h N_h V_{h\|}^2/L_{V_{h\|}}$ where $L_{V_{h\|}}$ is the scale length for $V_{h\|}$. Here, we assume $L_{V_{h\|}} \sim L_{V_{i\|}} \sim R$ where R is the major radius. The ratio of inertial force of hot and bulk ions reads $I_h/I_i = (M_h/M_i)(N_h/N_i)(V_{h\|}/V_{i\|})^2$. In previous calculations, N_h/N_i and $V_{i\|}$ are arbitrary. As shown in the previous subsection, $V_{h\parallel}/V_A \sim 1$. Hence, even for fast bulk plasma rotation $V_{i\parallel}/V_A = 10^{-2}$, the term $(V_{h\parallel}/V_{i\parallel})^2$ can be large. Then even if the density of hot ions is small, e.g., $N_h/N_i = 10^{-3}$, the inertial force of hot ions can be compatible with that of bulk ions.

Next we consider the inertial force of hot ions in the momentum equation of the hybrid kinetic-MHD model [2], $M_h N_h \vec{V}_h \cdot \nabla \vec{V}_h = \vec{J} \times \vec{B} - \nabla p - \nabla \cdot \mathsf{P}_\mathsf{h}$, where $\vec{J}$, p, and P_h are equilibrium current density, bulk pressure, and pressure tensor of hot ions. The right hand side can be approximated as $L = |\vec{J} \times \vec{B}| \sim |\nabla p| \sim [B_0^2/(\mu_0 a)]\beta$, where a is the minor radius and β is the total pressure normalized by B_0^2/μ_0. Then we observe

$$\frac{I_h}{L} = \frac{a}{L_{V_{h\parallel}}} \frac{M_h}{M_i} \frac{N_h}{N_i} \left(\frac{V_{h\parallel}}{V_A}\right)^2 \beta^{-1}.$$

When we approximate $a/L_{V_{h\parallel}} \sim 0.3$, $M_h/M_i \sim 1$, $N_h/N_i = 10^{-2}$, $V_{h\parallel}/V_A = 1$, and $\beta = 10^{-2}$, we obtain $I_h/L \sim 0.3$. Therefore, the inertial force of hot ions can be prominent in the momentum equation. This tendency is enhanced when we increase the parameters u_0, ξ_0, and T_e since $V_{h\parallel}$ gets large.

5. Summary

In this paper, we focus on the inertial force of hot ions produced by the neutral beam injection. We point out that hot ions interacting with bulk ions and electrons with flow can be described by the Fokker-Planck equation that is formally same with the static case. Hence, if we assume uniform equilibrium distribution function, we can obtain an analytic form of slowing-down distribution function. From this analytic distribution function, we have succeeded to compute the parallel flow of hot ions analytically. This representation is derived for the first time. From this representation, we observe that the parallel flow of hot ions is characterized by the injection parameters, electron temperature and mass ratio. Further analysis shows that injection parameters determine the parallel flow, and that electron temperature can affect the amplitude while the mass ratio has a minor effect on the parallel flow. Finally, we discuss the inertial force of hot ions in the framework of hybrid kinetic-MHD theory. The order estimate indicates that the inertial force of hot ions can be compatible with the pressure gradient term, hence in these cases, the framework of hybrid kinetic-MHD theory should be extended. This extension will be studied in future.

Acknowledgments

The authors are grateful to Dr. S. Ide and Dr. S. Higashijima for their support. JS is indebted to an anonymous referee for providing insightful comments to significantly improve the manuscript.

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