J. Plasma Fusion Res. SERIES, Vol. 8 (2009)

NUMERICAL MODELLING OF HIGH ENERGY ION TRANSPORT IN TOKAMAK PLASMAS

Kiyotaka HAMAMATSU

Japan Atomic Energy Agency, Naka Fusion Research Institute, 801-1 Mukohyama, Naka, Ibaraki, 311-0193, JAPAN

(Received: 12 September 2008 / Accepted: 22 January 2009)

To evaluate an anomalous radial transport of high energy ions, i.e., alpha particles and/or energetic ions due to NBI, we formulate a simple numerical model. By assuming that the anomalous transport is described by diffusion and flow terms, the transport term is averaged over bounce and circulating motion of particles and is added to the bounce averaged Fokker-Planck equation. As a numerical result of the presented model, time evolution of the velocity distribution function of alpha particles is presented.

Keywords: bounce averaged Fokker-Planck equation, radial transport, high energy ion, numerical analysis

1. Introduction

The achievement of the high performance plasma with controlled fusion reaction is the main subject in the fusion reactor. As a numerical approach, we are developing the TOPICS-IB (TOPICS extended to Integrated simulation for Burning plasma) code[1] to simulate the burning tokamak plasmas, where many basic processes, e.g., the radial transport of core plasma, MHD equilibrium and instabilities, alpha heating, current drive, plasma flow in the scrap-off layer, etc., are complexly related. The numerical performance of TOPIC-IB code depends on numerical modelling of each basic process. One of the most important physical issues is the analysis of transport properties of high energy ions, which are alpha particles and/or energetic ions injected by neutral beam. The slowing down processes of energetic ions can be analyzed by using the OFMC (Orbit Following Monte-Carlo) code [2] by using many test particles . Furthermore, many numerical studies are also actively pushed forward understanding of anomalous transport, for example, which is caused by TAE-mode instabilities[3]. Numerical costs of these numerical simulations are very expensive. To calculate the radial transport of high energy ions under the control of TOPICS-IB code, we propose a simple numerical model based on the bounce averaged Fokker-Planck equation. A numerical code is developed to solve the time evolution of the velocity distribution function and an obtained numerical result is presented.

2. Basic Equation

The drift-kinetic equation[4] is written as

$$\frac{\partial f}{\partial t} + v\cos\eta \boldsymbol{b}\cdot\boldsymbol{\nabla} f + \dot{\eta}\frac{\partial f}{\partial \eta} = \sum_j C_j(f) + S_{\mathrm{p}} - L_{\mathrm{th}}(f) - \boldsymbol{\nabla}\cdot\boldsymbol{\Gamma}^{\mathrm{AN}} \quad (1)$$

author's e-mail: hamamatsu.kiyotaka@jaea.go.jp

where v is the particle speed, η is the pitch angle of the particle velocity, and $\boldsymbol{b}$ is a unit vector along the magnetic field $\boldsymbol{B}$. The time derivative of the pitch angle, $\dot{\eta}$, shows bounce or circulating motion of particles in a tokamak configuration. $C_j(f)$ is the Coulomb collision term to denote collisions with bulk plasma species j. The bulk plasma is assumed to be Maxwellian, whose density and temperature are calculated by a transport code of the bulk plasma. S_{p} is a particle source caused by the fusion reaction or ionization of NBI. The loss term

$$L_{\mathrm{th}}(f) = \frac{f}{\tau_{\mathrm{th}}}\left(\frac{m}{2\pi w_{\mathrm{L}} T_{\mathrm{ash}}}\right)^{3/2} \exp\left[-\frac{mv^2}{w_{\mathrm{L}} T_{\mathrm{ash}}}\right] \quad (2)$$

removes slowed down alpha particle or injected ion as helium ash or bulk plasma, where τ_{th} is the thermal collision time. The velocity range of removed particles is expressed by the constant value w_{L}. In this article, the electric field and the drift motion across the magnetic field line are ignored. And a radial transport is assumed to be caused by a divergence of an anomalous particle flux $\boldsymbol{\Gamma}^{\mathrm{AN}}$.

We discuss the equation in the pseudo toroidal coordinate system, where ρ, ζ ,and θ are minor radius, toroidal angle, and poloidal angle, respectively. The relations between the pseudo toroidal coordinates and the cylindrical coordinates are $R = R_0 + \rho\cos\theta$, $\zeta = \zeta$, and $z = \rho\sin\theta$. The axisymmetric magnetic field $\boldsymbol{B}$ is expressed by

$$\boldsymbol{B} = \psi'_{\mathrm{P}}(\rho)\boldsymbol{\nabla}\rho\times\boldsymbol{\nabla}\zeta + I\boldsymbol{\nabla}\zeta, \quad (3)$$

where ψ_{P} is the poloidal magnetic flux and its prime denotes $\mathrm{d}/\mathrm{d}\rho$ and I means $B_{\mathrm{t0}}R_0$ on the magnetic axis. To average Eq.(1) over the particle motion in this magnetic field, we follow the standard procedure[5]. The second term of the left-hand side of Eq.(1) becomes

$$v\cos\eta\boldsymbol{b}\cdot\boldsymbol{\nabla}f = \frac{v\cos\eta}{qR}\frac{\partial f}{\partial\theta}. \tag{4}$$

The safety factor q can be expressed by $q(\rho) = \psi'_{\mathrm{T}}(\rho)/\psi'_{\mathrm{P}}(\rho)$, where ψ_{T} is the toroidal magnetic flux. The third term is also rewritten by

$$\dot{\eta}\frac{\partial f}{\partial\eta} = \frac{\mu}{mv\sin\eta}\frac{I}{qR^3}\frac{\partial R}{\partial\theta}\frac{\partial f}{\partial\eta}, \tag{5}$$

where μ is magnetic moment and the relation $\boldsymbol{\nabla}\cdot\boldsymbol{B} = 0$ is used.

When the particle velocity and pitch angle on the equatorial plane in the low field side are shown by v_0 and η_0, by using the energy conservation and the adiabaticity of magnetic moment, the parallel velocity or pitch angle at the poloidal position θ is obtained as:

$$\begin{aligned} v_{\parallel}(\rho,\theta) &= v_0\cos\eta(\rho,\theta) \\ &= \pm v_0\sqrt{1-\psi_{\mathrm{B}}(\rho,\theta)\sin^2\eta_0}. \end{aligned} \tag{6}$$

The definition of function ψ_{B} is $\psi_{\mathrm{B}}(\rho,\theta) = B(\rho,\theta)/B(\rho,\theta=0)$. This equation shows a bounce or circulating motion along the magnetic field line. We define the period of bounce motion: $\tau_{\mathrm{B}} = \oint qR\mathrm{d}\theta/|v_{\parallel}|$. The orbit average of "$X$" during the period τ_{B} can be expressed by

$$\langle X\rangle_{\mathrm{B}} = \frac{1}{\tau_{\mathrm{B}}}\oint\frac{qR\mathrm{d}\theta}{|v_{\parallel}|}X. \tag{7}$$

When it is considered that the bounce time τ_{B} is considerably shorter than the collision relaxation time τ_{C} and the anomalous transport time, the velocity distribution function can be expanded by the power of τ_{B}, i.e. $f = \mathcal{F}_0 + (\tau_{\mathrm{B}}/\tau_{\mathrm{C}})\mathcal{F}_1 + (\tau_{\mathrm{B}}/\tau_{\mathrm{C}})^2\mathcal{F}_2 + \cdots$, where $\mathcal{F}_{\mathrm{i}}$ is not dependent on the bounce phase. By the bounce orbit averaging, Eq.(4) and (5) are annihilated. As a result of bounce averaging, Eq.(1) becomes

$$\begin{aligned} \frac{\partial\lambda\mathcal{F}_0}{\partial t} &= \sum_j\langle C_j\rangle_{\mathrm{B}} + \lambda\langle S_{\mathrm{p}}\rangle_{\mathrm{B}} - \lambda L_{\mathrm{th}}(\mathcal{F}) \\ &\quad -\lambda\Big\langle\boldsymbol{\nabla}\cdot\boldsymbol{\Gamma}^{\mathrm{AN}}\Big\rangle_{\mathrm{B}}, \end{aligned} \tag{8}$$

where $\lambda = v_0\cos\eta_0\tau_{\mathrm{B}}/R_0$ is proportional to length of bounce orbit. We note that Eq.(8) is described in the (v_0,η_0,ρ)-space.

When a bulk plasma species is isotropic, the bounce averaged Coulomb collision operator has been calculated as the following[5]:

$$\begin{aligned} \langle C_j\rangle_{\mathrm{B}} &= -\frac{1}{v_0^2}\frac{\partial}{\partial v_0}\Big(v_0^2 S_{0v}\Big) \\ &\quad -\frac{1}{v_0\sin\eta_0}\frac{\partial}{\partial\eta_0}\Big(\sin\eta_0 S_{0\eta}\Big), \end{aligned} \tag{9}$$

$$\begin{aligned} S_{0v} &= -D_{0vv}\frac{\partial\mathcal{F}_0}{\partial v_0} + F_{0v}\mathcal{F}_0, \\ S_{0\eta} &= -D_{0\eta\eta}\frac{1}{v_0}\frac{\partial\mathcal{F}_0}{\partial\eta_0}, \\ D_{0vv} &= \lambda\langle D_{vv}\rangle_{\mathrm{B}}, \quad F_{0v} = \lambda\langle F_v\rangle_{\mathrm{B}}, \\ D_{0\eta\eta} &= \lambda\Big\langle\frac{\tan^2\eta_0}{\tan^2\eta}D_{\eta\eta}\Big\rangle_{\mathrm{B}}, \end{aligned}$$

where D_{vv}, F_v, and $D_{\eta\eta}$ are the conventional Coulomb collision coefficients[6]. The bounce averaging of particle source term and thermal loss term is absolute.

3. Modelling of Anomalous Transport

We consider the anomalous transport in the direction of minor radius ρ. The last term of Eq.(8) is described as

$$\lambda\Big\langle\boldsymbol{\nabla}\cdot\boldsymbol{\Gamma}^{\mathrm{AN}}\Big\rangle_{\mathrm{B}} = \frac{\lambda}{\tau_{\mathrm{B}}}\oint\frac{qR\mathrm{d}\theta}{|v_{\parallel}|}\frac{1}{\rho R}\frac{\partial}{\partial\rho}\Big(\rho R\Gamma_{\rho}^{\mathrm{AN}}\Big),$$

where the bounce time τ_{B} is enough shorter than the time-scale of anomalous transport. Since R is canceled out in front of the differential operator of ρ, the integration of θ and the differential operator of ρ are commutative. Therefore,

$$\lambda\Big\langle\boldsymbol{\nabla}\cdot\boldsymbol{\Gamma}^{\mathrm{AN}}\Big\rangle_{\mathrm{B}} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\lambda\langle\Gamma_{\rho}^{\mathrm{AN}}\rangle_{\mathrm{B}}\right). \tag{10}$$

is obtained.

The anomalous transport is assumed to be describe by diffusion and flow, i.e., the bounce averaged particle flux is expressed as

$$\lambda\langle\Gamma_{\rho}^{\mathrm{AN}}\rangle_{\mathrm{B}} = -D_0^{\mathrm{AN}}\frac{\partial\lambda\mathcal{F}_0}{\partial\rho} + F_0^{\mathrm{AN}}\lambda\mathcal{F}_0, \tag{11}$$

where

$$\begin{aligned} D_0^{\mathrm{AN}}(v_0,\eta_0,\rho) &= \big\langle D^{\mathrm{AN}}\big\rangle_{\mathrm{B}}, \\ F_0^{\mathrm{AN}}(v_0,\eta_0,\rho) &= \big\langle F^{\mathrm{AN}}\big\rangle_{\mathrm{B}}. \end{aligned} \tag{12}$$

In this article, we show an anomalous transport caused by a disturbance of the TAE-mode type, as an example. The anomalous transport coefficients are assumed by

$$\begin{aligned} D^{\mathrm{AN}}(v,\eta,\rho,\theta) &= \hat{D}^{\mathrm{AN}}(\rho)\mathcal{W}(v,\eta,\rho,\theta), \\ F^{\mathrm{AN}}(v,\eta,\rho,\theta) &= \hat{F}^{\mathrm{AN}}(\rho)\mathcal{W}(v,\eta,\rho,\theta). \end{aligned} \tag{13}$$

The weight function, $\mathcal{W}$, is given by

$$\mathcal{W}(v,\eta,\rho,\theta) = \exp\left[-\left(\frac{v_{\parallel}-V_{\mathrm{A}}(\rho,\theta)}{\Delta V_{\mathrm{A}}(\rho,\theta)}\right)^2\right], \tag{14}$$

where V_{A} is the Alfven velocity and Δ shows a resonance width. The absolute values of $\hat{D}^{\mathrm{AN}}(\rho)$ and $\hat{F}^{\mathrm{AN}}(\rho)$ are determined to assure that the maximum values of $D_0^{\mathrm{AN}}(v_0,\eta_0,\rho)$ and $F_0^{\mathrm{AN}}(v_0,\eta_0,\rho)$ with fixed ρ are proportional to the absolute value of pressure gradient of high energy particles, which is calculated by the second moment of $\mathcal{F}_0$.

4. Numerical Results

First the numerical method is briefly explained. We wish to solve Eq.(8) in the domain

$$0 < v_0 < v_{0\max}, \quad 0 < \eta_0 < \pi, \quad 0 < \rho < a, \tag{15}$$

where a is the minor radius and $v_{0\max}$ is numerical maximum of the speed. We do this by converting the differential equation to an algebraic equation using the finite difference method. In this procedure, we weave the algebraic matrix so that a symmetrical property of $\mathcal{F}$ is kept, more specifically, $\mathcal{F}_0(v_0, \eta_0, \rho) = \mathcal{F}_o(v_0, \pi - \eta_0, \rho)$ in the pitch angle of trapped particle range, which broadens with increasing of ρ. The algebraic matrix, which contains five bands, is inverted by using the conjugate gradient method. As for the numerical method for time advancing of Eq.(8), we employ the Crank-Nicholson scheme.

We show a numerical result of velocity distribution function of alpha particles produced by the D-T fusion reaction. The anomalous transport is started from an initial steady state balanced by the thermal loss term without radial transport. The main parameters are the following ITER-like parameters: major radius is $R_0 = 6.2$m, minor radius is $a = 2$m, toroidal magnetic field is $B_{t0} = 5.3$T at the magnetic axis, profile of safety factor is $q(\rho) = 0.8 + 2.2(\rho/a)^2$. It is assumed that the bulk plasma is stationary and composed by the deuterium, tritium and electrons, whose radial profiles are the followings: $T_D(\rho) = T_T(\rho) = T_e(\rho) = 20\times(1-\rho^2)^{1.5}$keV, $n_D(\rho) = n_T(\rho) = n_e(\rho)/2 = 5\times10^{19}(1-\rho^2)^{0.3}/\mathrm{m}^3$. Here, the helium ash is not considered and T_{ash} in Eq.(2) is substituted by $T_D(\rho)$. It is assumed that $w_L = 3$, since the value of w_L in this range has little effect on the transport of high energy particles. The velocity distribution function is calculated with an initial condition of $\mathcal{F}_0 = 0$ and reaches almost a steady state after one second. The radial profiles of pressure of active alphas are shown by the dotted curve in Fig.1. We consider this profile is the initial state of the anomalous transport process.

In this article, we show a radial transport caused only by an anomalous diffusion. Figure 2 (a), (b), and (c) describe the intensity of diffusion coefficient $D_0^{AN}(v_0, \eta_0, \rho)$ on the (v_0, η_0)-plane at $\rho/a = 0.05$, 0.35, and 0.85, respectively, where Δ in Eq.(14) is 5% and $m_\alpha v_{0\max}^2/2$ =5MeV. The resonance region in velocity space expands with increasing of aspect ratio. The white lines show the boundary between passing particles and trapped particles. The diffusion coefficient in trapped particle region becomes symmetrical against $v_\parallel = 0$. The profile of peak value of diffusion coefficient $D_0^{AN}(v_0, \eta_0, \rho)$ at every magnetic surface is shown in Fig.3 as a function of ρ, where the absolute value is given as a numerical parameter. In this numerical model alpha particles are not lost from the

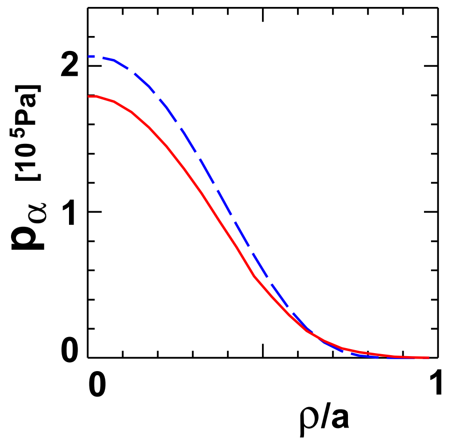

Fig. 1 Pressure profile of α particles. The dotted curve shows the initial state($t = 0$) and the solid curve shows the result of radial diffusion at $t = 0.5$s.

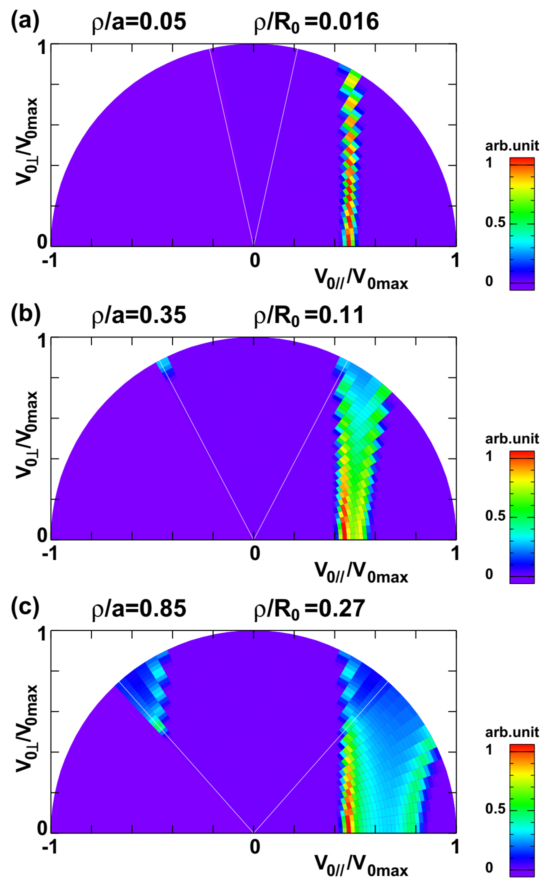

Fig. 2 Intensity of diffusion coefficient $D_0^{AN}(v_0, \eta_0, \rho)$. (a), (b), and (c) are located at $\rho/a = 0.05$, =0.35 and =0.85, respectively.

plasma surface $\rho/a = 1$, since the diffusion coefficient fades away near the plasma surface.

Figure 4 shows the time evolution of the stored energy ,W_α, and total particle number, N_α, by $t = 0.5$s The radial profile of α particle pressure at $t = 0.5$s is described by the solid curve in Fig.1. The veloc-

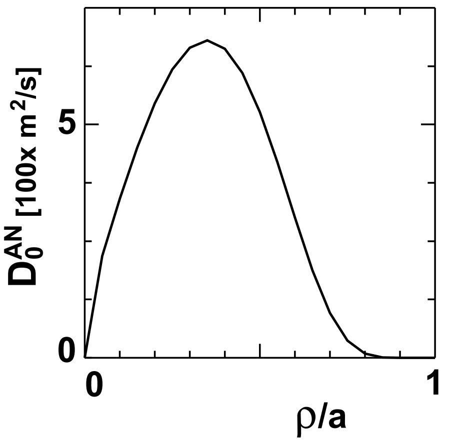

Fig. 3 Profile of maximum value of diffusion coefficient $D_0^{\mathrm{AN}}(v_0, \eta_0, \rho)$ at every magnetic surface.

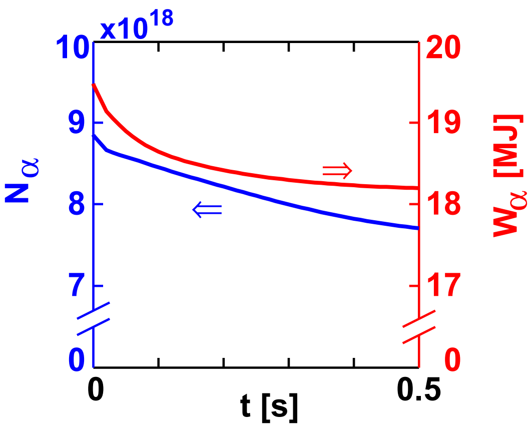

Fig. 4 Time evolution of stored energy ,W_α, and total particle number, N_α.

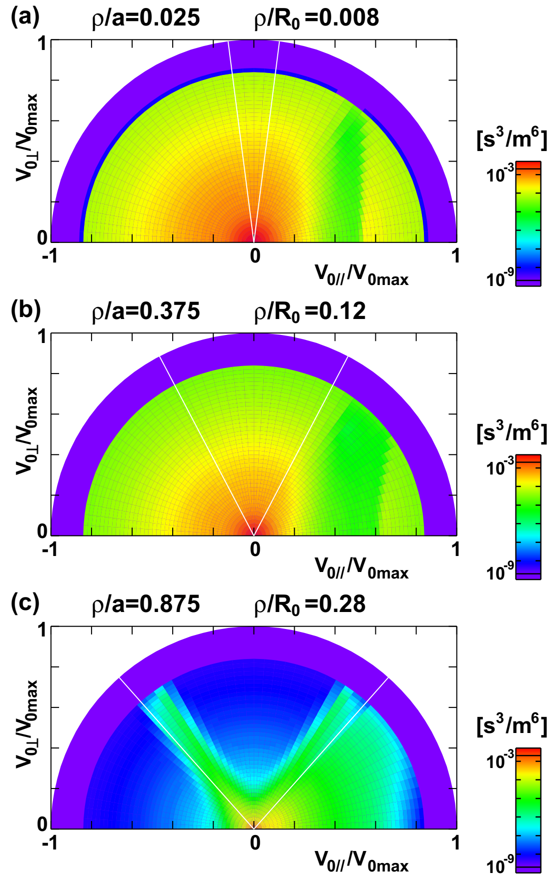

Fig. 5 Intensity of velocity distribution function, $\mathcal{F}_0(v_0, \eta_0, \rho)$. (a), (b), and (c) are located at $\rho/a = 0.025$, 0.375 and 0.875, respectively.

ity distribution functions, $\mathcal{F}_0(v_0, \eta_0, \rho)$, at $t = 0.5$s are shown in Fig.5, where (a), (b), and (c) correspond to $\rho/a = 0.025$, 0.375, and 0.875, respectively. The velocity regions of lost particles in Fig.5(a) and (b) are broader than the resonance region shown by Fig.2(a) and (b). And also the velocity region of existing particles in Fig.5(c) is broader than the resonance region shown by Fig.2(c). These are caused by the diffusion and slowing down due to the Coulomb collisions.

5. Summary

To evaluate the radial transport of high energy ions, we have formulated the bounce averaged Fokker-Planck equation with the radial transport term, which contains diffusion and flow terms. And a sample of numerical analysis of the formulated equation is presented. The CPU time required to run the presented sample on a personal computer with frequency of 2GHz is about 30 minutes, where (51×51) velocity grid points, 21 radial grid points, and time step of 0.1ms are used. We think that the presented numerical model can be brought into the TOPICS-IB code. In this article, we do not treat the neoclassical transport process based on disturbed drift orbits due to the Coulomb collision. Although we assume that the anomalous transport is caused by the TAE mode, a consistency of transport coefficient with the MHD theory is not considered. These problems are left to future studies.

6. Acknowledgement

It is a pleasure to acknowledge useful discussions with Tomonori Takizuka. This work was supported in part by the Grant-in-Aid for Scientific Research from Japan Society for the Promotion of Science (JSPS).

[1] N. Hayashi, et.al., Nucl. Fusion **47**, 682 (2007).
[2] K. Tani, et.al., J. Phys. Soc. Jpn. **50**, 172 (1981).
[3] Y. Todo, et.al., Phys. Plasmas **2**, 2711 (1995).
[4] R.D. Hazeltine, et.al., *Plasma Confinement* (Dover Publications, INC., New York, 1992) §4.
[5] J. Killeen, et.al., *Computational methods for kinetic models of magnetically confined plasmas* (Springer-Verlag, New York, 1986) §3.
[6] C.F.F. Karney, Computational Phys. Reports **4**, 195 (1986).