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Saturation of Zonal Flow in Gyrofluid Simulations of Electron temperature Gradient Driven Turbulence

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Abstract

An enhanced zonal flow is observed in gyrofluid simulations of electron temperature gradient (ETG) driven turbulence with weak magnetic shears. The Kelvin-Helmholtz (KH) instability is proposed as a primary damping mechanism of such flow. Some considerable evidences for the KH mode excitation are presented. Results seem to suggest a possibility of turbulence transition from the ETG-dominated one to the KH-dominated one due to the zonal flow dynamics in weak shear plasmas.

Keywords:

KH instability, zonal flow saturation, spectral analysis, gyrofliud simulation, ETG turbulence

1. Introduction

In last decades, it is widely recognized that the anomalous transport plausibly originates from the drift wave fluctuations such as ion/electron temperature gradient (ITG/ETG) driven turbulence in fusion plasmas [1]. The importance of zonal flows in suppressing turbulent ion and electron heat transport has been intensively emphasized, especially in the ITG turbulence [2]. The saturation level of zonal flow has also received much attention [3]. This is because it is one of key factors to influence the flow-shearing rate, except for the spectral structure of zonal flow [4]. Recently, an enhanced zonal flow generated by the slab ETG turbulence has been observed in a regime with both weak magnetic shear and steep electron temperature gradient by performing a long time gyrofluid simulation, in which the electron heat transport is reduced much [5,6]. Similar phenomena were also observed very recently in the drift-Alfven turbulence in the edge plasmas of stellarator and tokamak configurations [7]. Hence, a natural question arises, which refers to the zonal flow saturation in such simulations.

It is generally known that zonal flows can be damped by collisional effects [8]. In low collisional plasmas, there exist some other mechanisms. For example, first of all, the zonal flow may be saturated by the spectral modulation of turbulence due to the flow, while it is also generated through a modulational instability [9]. On the other hand, a strongly sheared flow may excite a secondary Kelvin-Helmholtz (KH) instability, which can absorb the flow energy and, then, saturate the growing flow [10-12]. In addition, the coupling of zonal flows through the geodesic curvature to pressure fluctuations may also influence the level of zonal flows in a toroidal plasma. In this work, the KH instability is proposed as a primary mechanism to limit the level of enhanced zonal flow in gyrofluid slab ETG turbulence. Some considerable evidences for the KH mode excitation are presented based on numerical analyses of the stability of zonal flow with regard to KH instability, the spatial Fourier spectra and the time-frequency wavelet spectra of turbulent ETG fluctuations.

2. Basic model and simulation results

A three-field (normalized potential, ϕ , parallel electron velocity, $v_\parallel$, and electron pressure, p) gyrofluid model is employed to describe the electrostatic slab ETG turbulence in tokamak core plasmas, i.e., [9]

$$(1-\nabla_\perp^2)\partial_t\phi-(1+K\nabla_\perp^2)\partial_y\phi-\nabla_\parallel v_\parallel=\lfloor\phi,\nabla_\perp^2\phi\rfloor+\mu_\perp\nabla_\perp^4\phi, \tag{1}$$

$$\partial_t v_\parallel-\nabla_\parallel(\phi+p)=-[\phi,v_\parallel]-\eta_\perp\nabla_\perp^2 v_\parallel, \tag{2}$$

$$\partial_t p+K\partial_y\phi+\Gamma\nabla_\parallel v_\parallel+(\Gamma-1)\sqrt{8/\pi}\,|k_\parallel|(p+\phi)=-[\phi,p]+\chi_\perp\nabla_\perp^2 p \tag{3}$$

Where $\Gamma = 5/3$ and $K = 1 + \eta_e$ with $\eta_e = \mathrm{dln}T_e/\mathrm{dln}n$. A 3-dimensional (3D) initial value code is used to solve Eqs. (1-3) [9]. Lots of simulations are performed for different mag-

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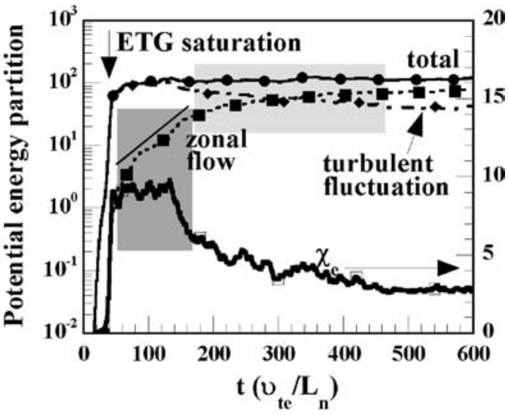

Fig. 1 Time evolution of the turbulent and zonal potential energy as well as electron heat conductivity. $\hat{s}$ = 0.1, η_e = 6. The dark (light) shaded part marks the approximately exponentially fast (slow) growing phase of zonal flows. Short straight line is for reference only.

netic shear and η_e. The artificial viscosities are nominated as $\mu_\perp = \eta_\perp = \chi_\perp = 0.5$; Space domain is set as $L_x = 200\rho_e$, $L_y = 20\pi\rho_e$ and $L_z = 2\pi L_n$. As a result, a high electron confinement state, which is characterized by weak magnetic shear, steep electron temperature gradient, enhanced zonal flow and reduced heat transport, is observed as typically shown in Fig. 1. It can be seen that the zonal flow component undergoes two different growing phases after the initial saturation of ETG fluctuations. The first fast growing phase clearly shows a zonal flow instability on average, in which the heat conductivity is higher. A slow growing phase follows accompanying with decreasing turbulent fluctuations and heat transport. Finally, the zonal flow is limited at some level, and turbulent fluctuations, which are dominated by a few coherent structures with k_y = 0.2 ~ 0.3, approach a quasi-steady state with a low transport level.

In the simulations above, the weak magnetic shear is found to be a crucial factor to control the zonal flow dynamics and turbulent transport property. Analyses based on the well-known Hasegawa-Mima turbulence, which is a simplified version of Eqs. (1-3), show that the weak shear favors the zonal flow instability in ETG fluctuations, which is well in agreement with the observations in ETG simulations. It is noticed that the level of zonal flow determines the electron transport property in weak shear ETG turbulence. Hence, the saturation mechanism of zonal flows is an important issue.

3. Saturation of enhanced zonal flow

Lots of simulations similar to the above one have been also performed with different viscosities and higher spatial resolution. A common feature is generally observed: although initially saturated ETG fluctuations are highly turbulent, they slowly condense to some coherent structures as the zonal flows grow up and saturate at some level. It seems to show that enhanced zonal flows, suppressed ETG fluctuations and coherent vortices coexist in the quasi-steady state. In what follows, numerical experiments are designed to test a working hypothesis that a KH (namely, flow-driven) instability may limit the zonal flow level. Two questions are addressed: whether is the observed zonal flow unstable with regard to the KH mode? If yes, how is the KH fluctuation identified in the spatio-temporal spectra of the mixed fluctuations?

3.1 Stability of KH mode

For a non-monotonously varied flow, it is not tractable to get an analytical derivation of the KH instability. Alternatively, a numerical check can be performed to directly examine the stability of KH mode in the enhanced zonal flow observed in Fig. 1. This can be done by removing the nonlinear coupling, electron temperature and density gradients, and all turbulent fluctuations from the simulation. Note that as shown in Fig. 1, the growth of zonal flow becomes slow at around t = 200, which may presage the excitation of KH instability. Hence, the zonal flow at the normalized time t = 200 are sampled as a constant energy source of KH fluctuations. Small random noise is chosen as the initial perturbation. With these settings, only the flow-driven KH instability may occur if the zonal flow is unstable with regard to it. Fig. 2 displays the time evolution of fluctuating potential, which shows that the KH mode is marginally unstable. Further numerical experiments by artificially increasing (or decreasing) the flow amplitude by 10% show that the KH fluctuations become unstable (or stable) as plotted by the dashed-dot curves. These observations indicate that when the zonal flow grows up to some high level, which approaches the amplitude threshold of a KH instability, the flow itself may be damped through exciting the KH mode. The growth of zonal flow is then slowed down. It shows that the occurrence of KH instability is possible after around t = 200. To further confirm this point, the flow at t = 500, at which the zonal flow approaches the quasi-steady state, is taken to analyze the KH stability. It is found that the KH mode, which propagates along y direction, is weakly unstable in the spectral region of k_y = 0.2 ~ 0.3. The unstable modes correspond well to the dominant turbulent fluctuations in the simulation of Fig. 1. Note that the weak magnetic shear also favors the KH instability. Hence,

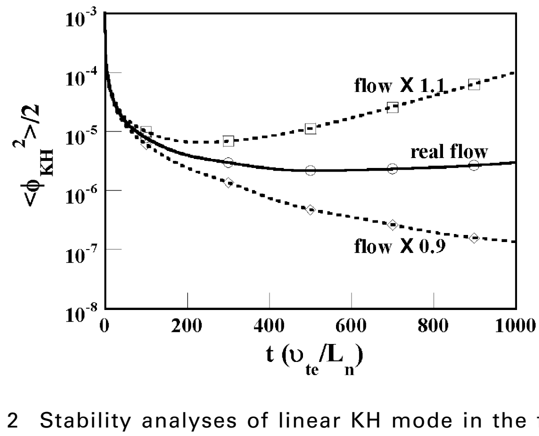

Fig. 2 Stability analyses of linear KH mode in the flow observed at t = 200 in Fig. 1. It shows the KH mode is marginally unstable.

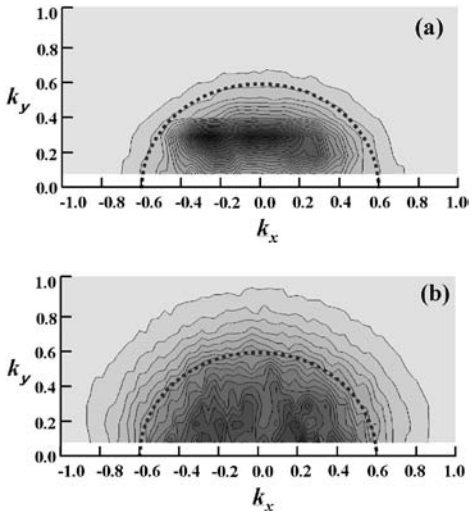

Fig. 3 Spatial spectral density distributions of the turbulent fluctuations $\langle S\{\phi^2\}\rangle_{z,t}(k_x, k_y)$ in $k_x - k_y$ space in the simulations with (a) and without (b) zonal flows. The dashed curves are for reference only.

these numerical experiments indicate that the role of magnetic shear in ETG turbulence seems to involve a complex interaction between zonal flows and the secondary KH mode, which may significantly influence final saturation level of ETG turbulence.

3.2 Spatial fourier spectral analysis

Generally, any perturbation or fluctuation can be described by a spatio-temporal spectrum under the *ansatz* $\tilde{f} \propto \exp[i(\vec{k}\cdot\vec{x} - \omega t)]$. The flow driven KH mode is characterized by longer wavelength and lower frequency. To diagnose the KH fluctuation in ETG simulations, it is necessary to analyze the turbulence spectra. Spatial Fourier spectral analyses show that turbulent fluctuations are characterized by a shrinking k_y spectral distribution in the longer wavelength region with a peak at $k_y \approx 0.3$ and an isotropic distribution at shorter wavelengths in the $k_x - k_y$ space, as shown in Fig. 3(a). The spectral distribution of dominant turbulent fluctuations at around $k_y = 0.3$ corresponds well to that of unstable KH modes. For comparative purpose, a *reference simulation* is performed with the same parameters as in Fig. 1, but the zonal flow component is set to zero in time. An isotropic spectral distribution in both longer and shorter wavelength regions is observed as shown in Fig. 3(b), which is of typical characteristics of the pure ETG turbulence. It may result from the nonlinear energy cascade or inverse energy cascade. Furthermore, analyses on two-dimensional autocorrelation functions of the turbulent fluctuations show that in the simulation with zonal flow, the turbulent fluctuation is characterized by a strongly inhomogeneous correlation in the x and y directions. The ETG turbulence is strongly modulated by a wave-like fluctuation propagating along the y direction so that it seems to obviously possess a periodicity. The wave number is estimated as $k_y \approx 0.262$, which also approaches the peak spectrum of the KH mode.

3.3 Time-frequency wavelet analysis

To visually see the excitation of KH mode in time evolution, a new spectral method, termed wavelet transform, is used to analyze time series of the turbulent fluctuation. The basic idea for the use of this technique is that: if the enhanced zonal flow can drive a KH instability, which may back-react to saturate the exponentially growing flow, the KH fluctuation should be diagnosed in the time-frequency sequence of wavelet energy spectra. A software program of wavelet transform is employed and the Morlet wavelet is used in the implementation [13]. Fig. 4 plots contours of turbulent potential energy [i.e., $\phi(k_y)^2/2$] in the time-frequency plane in the simulation of Fig. 1. It clearly reveals the excitation of a

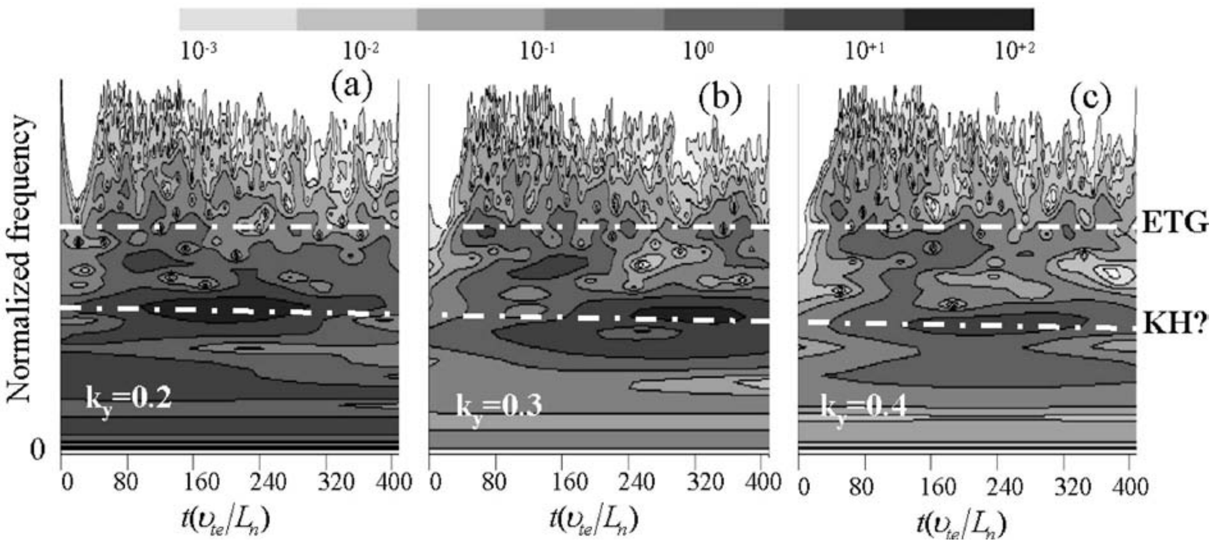

Fig. 4 Time-frequency wavelet energy spectra of the turbulent fluctuations in zonal flow dominated ETG turbulence. It shows the excitation of a low frequency fluctuation (KH mode?) at the end of the fast growing phase of zonal flows in Fig. 1.

lower frequency fluctuation from $t \approx 160$ besides the ETG fluctuation with higher frequency. It shows a visible frequency gap between these two fluctuations. Such phenomenon is not observed in the *reference simulation* without zonal flows. The latter displays a rather wider frequency spectral structure after the saturation of ETG turbulence, which may be subject to the nonlinear energy cascade or inverse cascade processes like the spatial spectrum. Note that the time $t \approx 160$ to excite the low-frequency fluctuation is approximately in agreement with the transition point $t \approx 200$ of the evolution of zonal flows in Fig. 1. Furthermore, the wavelet analysis of linear KH instability in Fig. 2 shows that the frequency of KH mode well corresponds to that of the low-frequency fluctuation observed in Fig. 4. Hence, from the correspondence of the spatio-temporal spectra of KH mode with the spectral distribution of the turbulent fluctuations in simulations, the low-frequency fluctuation may be reasonably inferred as the secondary KH mode. Although the wavelet technique itself is still suffered from some errors and noise problem, the observations from Fig. 4 may provide a significant evidence for the excitation of KH mode in ETG turbulence due to the enhanced zonal flows.

4. Summary and discussion

In summary, based on 3D electrostatic slab ETG turbulence simulations, the saturation of zonal flow with regard to a KH instability is analyzed. All numerical analyses for the excitation of KH mode in ETG simulations consistently support the working hypothesis that enhanced zonal flow may be dominantly limited by a flow-driven KH instability. It may suggest a possibility of turbulence transition from the ETG-dominated one to the KH-dominated one due to the enhanced zonal flows in weak shear plasmas.

Although the dynamics of zonal flow involves complicated nonlinear interactions, some quasi-linear processes, such as the modulation instability for flow generation and the KH instability for flow saturation, may be the most primary mechanisms in ETG simulations above. It depends on the time scales and nonlinear coupling of different fluctuations. In slab ETG simulations with weak shears, the generation of zonal flow and the excitation of KH fluctuation are much slower than that of ETG turbulence. Additional calculation, in which the zonal flow at $t = 500$ in Fig. 1 is artificially imposed in the *reference simulation* in Sec. 3.2, shows that direct nonlinear coupling between ETG fluctuation and the KH mode does not seem to be strong [6]. Hence, the KH instability in this work can be identified through the comparison of the spatio-temporal spectra of both linear KH mode and pure ETG fluctuation with the spectral distribution of mixed fluctuations in ETG simulations. Other nonlinear mechanisms, such as the spectral modulation of turbulent fluctuations by the flow [9], energy cascade or inverse energy cascade, may also occur in ETG turbulence, but they are not dominant ones. Furthermore, the zonal flow is also damped by electron collision in ETG turbulence [14]. Although the viscosity damping has been applied to the zonal flow component in the simulations above, it is worthwhile to perform a simulation, which includes collisional damping effects, to compare the contributions of different damping mechanisms to the saturated level of zonal flow. The work is in progress.

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