J. Plasma Fusion Res. SERIES, Vol. 4 (2001) 271–275

Nonlinear Destabilization of Double Tearing Modes in Reversed Magnetic Shear Plasmas

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(Received: 5 December 2000 / Accepted: 18 August 2001)

Abstract

A new nonlinear destabilization of double tearing modes (DTM) is observed for nonmonotonic q-profiles and the mechanism is investigated in the single helicity reduced MHD simulation in cylindrical geometry. This nonlinear destabilization of DTM is caused by the generation of the higher harmonics from the main harmonics (3/1 in the present case) by the nonlinear mode coupling. Hence, there appears no anomalous resistivity caused by the turblence. In this nonlinear destabilization mechanism, the triangular deformation of the islands causes the rapid growth of DTM after going through the Rutherford-type regime by forming the steep current around the outer X-points, that is the structure driven mode.

Keywords:

reversed shear, double tearing mode, nonlinear destabilization, structure driven mode

1. Introduction

The formation of the non-monotonic safety factor (q-) profile, and/or the reversed shear profile, is considered to be one of the attractive method to attain high performance steady state operation of a tokamak and the MHD stability for this profile is one of important issues to be theoretically clarified for the development of a steady state tokamak. The plasma with this q profile can be linearly unstable against resistive modes even in a low-beta state. Especially, double tearing modes (DTM) show the large growth rate of the resistive internal mode and can drive the plasma to termination almost exponentially in time with the linear growth rate. The nonlinear behaviors of the double tearing mode have been intensively studied by some authors through MHD simulations [1,2]. Recently, we found the new phenomena of the double tearing mode in the nonlinear phase; that is, when two resonance surfaces are apart from each other, the mode gently grows magnetic islands at each resonance surface like in Rutherford regime of the conventional tearing mode [3] but it suddenly shows the rapid growth after both magnetic islands grow enough to interact with each other [4]. It must be noted that this process is observed in a plasma with helical symmetry, where all harmonics have the resonance surfaces at the same radius, so that the newly observed phenomena seems to be very different from any theories proposed so far like the nonlinear coupling among different helicities and also the destabilization through the renormalized turbulence transport process, which have been observed in MHD simulations of the major disruption [5,6]. This process is very important because, even if the plasma safely passes the regime unstable against the conventional double tearing mode with assistance of the magnetic well or the detail current profile control, the slowly growing tearing-like modes can be suddenly destabilized by the nonlinear process and lead to the reconstruction of the current profile to the monotonic q profile. The purpose of this paper is to show the details of this nonlinear destabilization of double tearing mode.

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2. Model and Simulation Results

We employ the reduced set of resistive MHD equations in cylindrical plasma with helical symmetry and solve them by the finite difference in the radial direction and Fourier expansion in the toroidal direction [7].

$$\frac{\partial u}{\partial t} = \frac{1}{r}[u, \phi] + \frac{1}{r}[\psi, j] + \frac{B_0}{R_0}\frac{\partial j}{\partial \varphi} + \nu \nabla_\perp^2 u \quad (1)$$

$$\frac{\partial \psi}{\partial t} = \frac{1}{r}[\psi, \phi] + \frac{B_0}{R_0}\frac{\partial \phi}{\partial \varphi} + \eta j - E \quad (2)$$

$$j = \frac{\partial^2}{\partial r^2}\psi + \frac{1}{r}\frac{\partial}{\partial r}\psi + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\psi \quad (3)$$

$$u = \frac{\partial^2}{\partial r^2}\phi + \frac{1}{r}\frac{\partial}{\partial r}\phi + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\phi \quad (4)$$

$$[a, b] = \frac{\partial a}{\partial r}\frac{\partial b}{\partial \theta} - \frac{\partial b}{\partial r}\frac{\partial a}{\partial \theta}. \quad (5)$$

The safety factor profile used in the following is

$$q(r) = q_c\{1 + (\frac{r}{r_0})^{2\lambda}\}^{\frac{1}{\lambda}}[1 + A\exp\{-(\frac{r - r_\delta}{\delta})^2\}], \quad (6)$$

$$\psi(r) = -\frac{B_0}{R_0}\int_0^r \frac{rdr}{q(r)}.$$

We fix the parameters $\lambda = 1$, $r_0 = 0.412$, $\delta = 0.273$, $r_\delta = 0$ and $A = 3$ through this paper and change only q_c which changes the distance of two resonance surfaces, Δr. Also, we fix the poloidal/toroidal mode numbers (m/n) to 3/1, respectively. Hence, the all harmonics are expressed as $(m/n) = (3l, l)$, $l = 0, 1, ..., l_{max}$, where l_{max} is the maximum number of Fourier mode. The maximum number of the Fourie components of the mode is taken to be 100 and the maximum number of equally spaced radial grid is 401. The linear stability analysis against the resistive mode in this q-profile shows that, increasing Δr, the exponent factor α of resistivity η with respect to the growth rate $\gamma(\eta^\alpha)$ changes from $\alpha = 1/3$ of the resistive internal mode in the limit of $\Delta r = 0$ to $\alpha = 3/5$ of the conventional tearing mode in the limit of $\Delta r = \infty$ [8]. Corresponding to this change of the linear stability, the nonlinear behavior of the mode also changes from the exponential growth with the linear growth rate to the quasi-linear saturation of the magnetic islands around both resonance surfaces. The new type of the nonlinear behavior of the mode is found in the midway of these two well known behaviors of the double tearing mode. The typical example of the temporal evolution of magnetic and kinetic energies of this new phenomena is shown in Figs. 1(a) and (b) for $l_{max} = 40$. After the exponential growth in the linear regime, the mode reduces its spontaneous growth rate and tends to enter the Rutherford-type regime. In this phase, the kinetic energy almost saturates, while the magnetic energy continues to increase with reduced temporal rate and magnetic islands grow in time. Then, after magnetic islands growing around each resonance surface to contact with each other, the mode shows the abrupt growth. We note that the linear stability of this mode follows the tearing mode scaling at the initial equilibrium. In order to study of the origin of this abrupt growth of the double tearing mode during the nonlinear phase, we have performed several simulations. The possible candidate of this destabilization is the quasi-linear modification of the q profile and the acceleration of the linear instability. The simulation, where the perturbations set to zero just before the start of the abrupt growth, shows that the mode returns back to the linear growth phase and again enters the Rutherford-type regime. The modified q-profile also does not destabilize any other higher harmonics in the linear stability sense. In this way, the quasi-linear modification of the q-profile cannot reproduce the abrupt growth of the mode. This was also confirmed by the comparison of simulations with reducing the maximum number l_{max} of Fourier mode. In Figs. 1, time traces for different numbers of l_{max} are also plotted. Figures 1 show the temporal evolution of the mode before the abrupt growth is not sensitive so much on l_{max}, while the behavior of the abrupt growth strongly depends on l_{max}; that is, reducing l_{max} from some critical number, the growth becomes more gentle and finally the mode tends to the saturation without relaxing the excess of the magnetic energy. On the other hand, the simulations with l_{max} greater than the critical number give almost the same result. The critical number of l_{max} depends on the distance between resonance surfaces, Δr, and it is $l_{max} = 20$ for the typical example in Figs. 1. These simulations clearly show that the abrupt growth of the double tearing mode after the Rutherford-type phase is induced by the nonlinear coupling among the higher harmonics, although the harmonics higher than some critical number do not play an essential role in this process. We will return this problem later. Now, let's study whether the magnetic perturbation $\psi_{l>0}$ or the kinetic

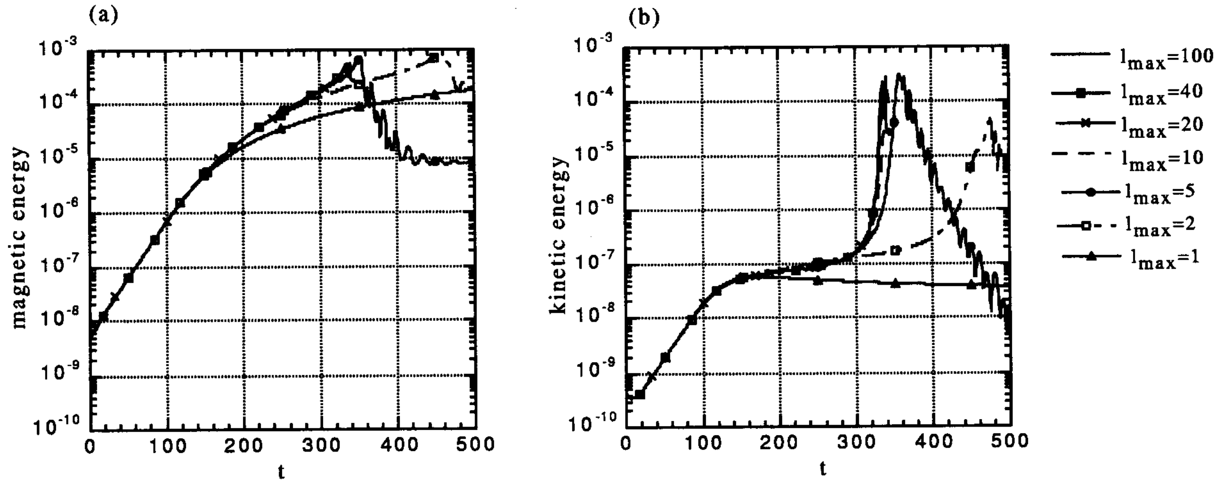

Fig. 1 Time evolutions of the magnetic and kinetic energies of 3/1-harmonics, with the different mode numbers, l_{max}.

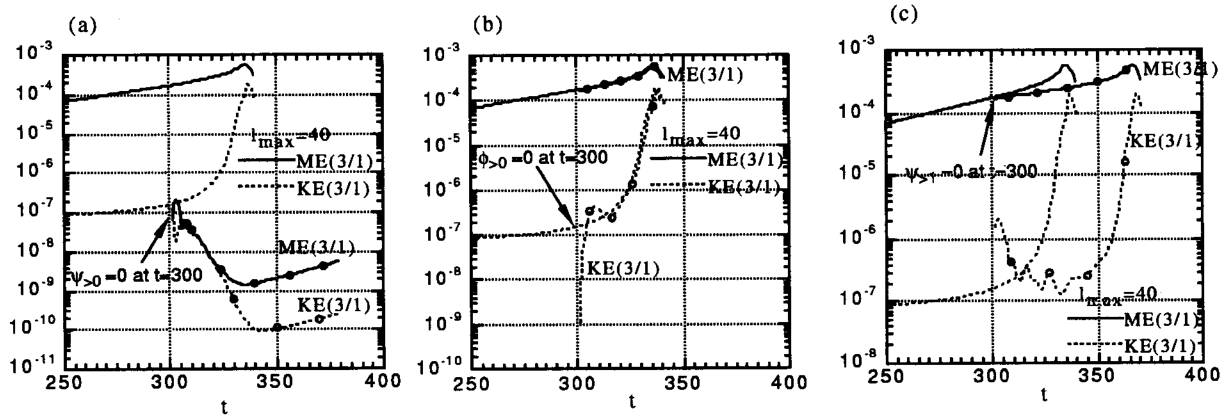

Fig. 2 Time evolutions of the magnetic and kinetic energies of 3/1-harmonics, which are restarted at $t = 300$ by setting, (a) $\psi_{l>0} = 0$, (b) $\phi_{l>0} = 0$ and (c) $\psi_{l>1} = 0$.

perturbation $\phi_{l>0}$ is the key factor of this nonlinear coupling. For this sake, we have done the simulations, where the magnetic perturbations or the kinetic perturbations are set to zero just before the abrupt growth and the reproducibility of the abrupt growth is checked. The result is that the simulation with $\psi_{l>0}(t = 300) = 0$ can not reproduce the nonlinear growth, while, in the case of $\phi_{l>0}(t = 300) = 0$, the kinetic perturbations are easily recovered to the same level as the referenced case in a very short time and shows the abrupt growth. This comparison confirms that the nonlinear destabilization is originated to the coupling of magnetic perturbations through $\boldsymbol{J} \times \boldsymbol{B}$. Another interesting result of simulation is that, when we retain the fundamental perturbations with $\psi_{l=0}$ and $\psi_{l=1}$, setting all other $\psi_{l>1}$ to zero at $t = 300$, and resume the full mode simulation, the mode starts the abrupt growth after once decreasing its amplitude. All these simulation results are summarized in Figs. 2(a), (b) and (c), where temporal evolution of the magnetic and kinetic energies of the fundamental mode with $l = 1$ (or, the $m/n = 3/1$ mode) are shown. Now, we study of the temporal evolution of the energy spectrum. We have performed the simulation with harmonics number up to 100. Figures 3(a) and (b) show the magnetic and kinetic energy spectrums at $t = 320$ (halfway of the nonlinear destabilization) for simulations with different harmonics number. Except the small reversed spectrum at the maximum harmonics number, the simulations with harmonics number higher than some critical number show the same spectrum. This means that the mode energy transferred to higher harmonics through mode coupling among the low and

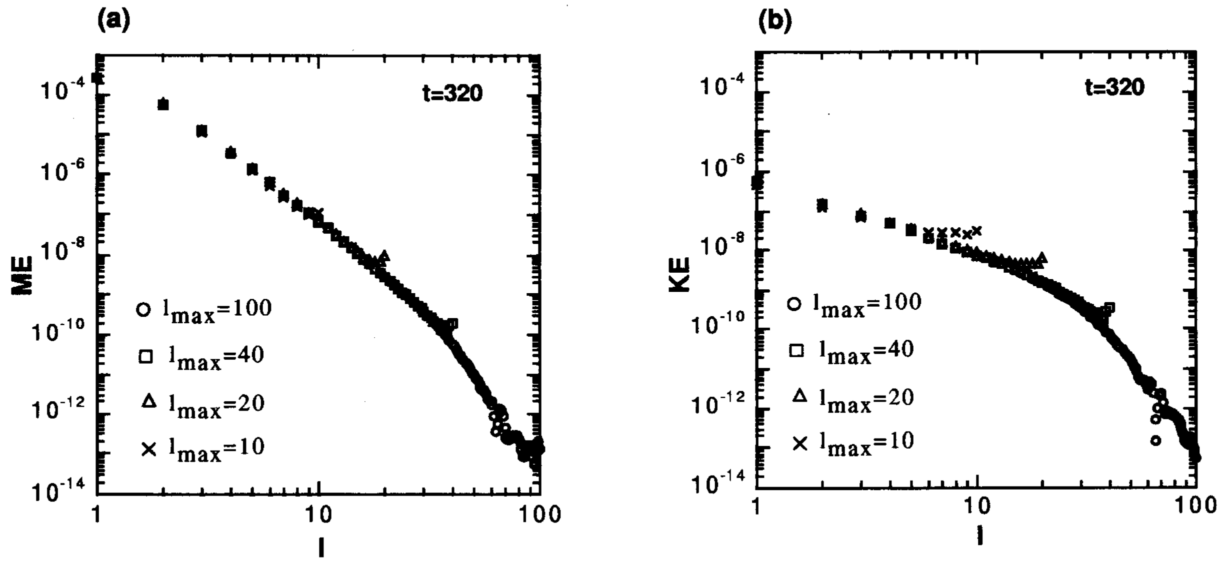

Fig. 3 Energy spectra of simulations with different Fourie mode numbers at $t = 320$

intermediate ones just flows to higher ones and do not flow back to the lower ones. In this way, the higher harmonics play as the sink term of the excess energy of the system, and does not play any role on the behavior of the main mode, as the same as shown in comparison of temporal evolution. On the other hand, the intermediate harmonics have the close coupling with each other, especially with the fundamental mode of m/n = 3/1. The difference of the role of harmonics with low/intermediate number and higher number can be seen from the comparison of the slopes of the energy spectrum both in magnetic and kinetic energy. That is, the slope of kinetic energy spectrum clearly shows the two stage structure and, during the nonlinear instability, its shape does not change so much. Also, we can see that the slope of magnetic energy is grater than that of the kinetic one in the low/intermediate harmonics regime, while those are almost the same in the higher harmonics regime. The latter fact shows that the higher harmonics is just the driven modes without any singular MHD characters of themselves. This is confirmed by analyzing the dependence of the radial structure on the harmonic number. We note that all harmonics have the resonance at the same radius because of the helical symmetry. In the low and intermediate harmonic number, the radial structures of ψ and ϕ are different from each other and show the resonance natures, while those are almost same and lost the resonance character in the higher harmonic number regime. Also, the harmonics tend to localize at the outer resonance

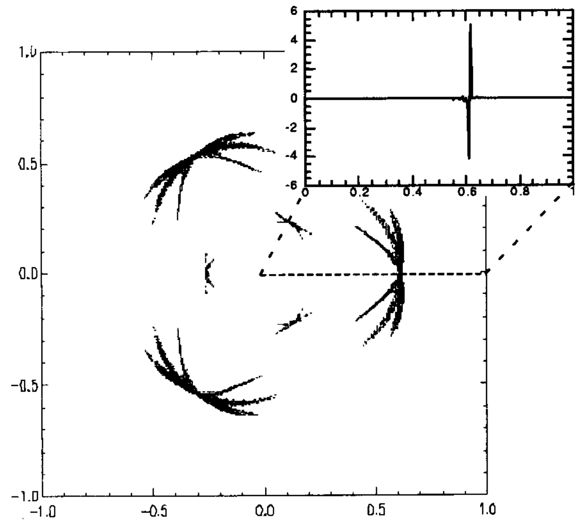

Fig. 4 Typical profile of the rotation of the $\boldsymbol{J} \times \boldsymbol{B}$ force at $t = 320$.

surface, as increasing the harmonic number. This is the reflection of the fact that the inner magnetic island increases the degree of the triangular deformation and it has the sharp edge at the X-point of the outer magnetic island. The growth of the inner magnetic island is prevented by the outer magnetic island and the resultant compression of the magnetic surface induces the skin current along the separatrix surface. The further growth of the magnetic island increases the deformation of the island shape and the skin current becomes to be concentrated to the X-points of the outer magnetic islands. The formation of this localized skin current and the accelerated dissipation seems to give the weak dependence of the spontaneous growth rate of the nonlinear instability. Figure 4 shows the typical profile of the rotation of the $\boldsymbol{J} \times \boldsymbol{B}$ force. In this way, the increase of the triangular deformation of magnetic islands and the resultant localization of the skin current to the X-point is the key factor of this new process. In this sense, this is a kind of the structure driven mode.

3. Summary and Discussions

We have studied the detail process of the nonlinear instability caused in the reversed shear profile in a tokamak by using the reduced MHD model with helical symmetry. This instability was shown to be originated not from any type of the quasi-linear destabilization but from the nonlinear coupling among higher harmonics. But it is also not the turbulence driven instability, where the increase of the transport coefficients driven by the higher harmonics accelerates the growth of the mode. In fact, only the higher harmonics of limited number contribute the instability and the critical number depends on the distance of the two resonance surfaces. This is also seen from the change of the shape of the magnetic islands. The increase of the triangular deformation of islands concentrates the skin current profile to the X-point and relaxes the excess magnetic energy through dissipation of this pin-point skin current at the X-point. All these results indicate that the destabilization phenomena newly observed in the double tearing mode simulation is caused by the new mechanism of the nonlinear destabilization, which may be said the structure driven mode.

In this study, the mode coupling effect between the different helicity is ignored. If there exist the perturbations with finite amplitude, which are resonating with rational surfaces near the most unstable one, this effect may transfer the energy of the main harmonics not only to the higher harmonics but also to those with the different helicity and may delay the growth of the higher harmonics with the same helicity. Hence, the term of the Rutherford type regime before the structure driven instability may become longer than that in the single helicity simulation.

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