

## Generating Carrier-Envelope-Phase Stabilized Few-Cycle Pulses from a Free-Electron Laser Oscillator

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We propose a scheme to generate carrier-envelope-phase (CEP) stabilized few-cycle optical pulses from a free-electron laser oscillator. The CEP stabilization is realized by the continuous injection of CEP-stabilized seed pulses from an external laser to the free-electron laser oscillator whose cavity length is perfectly synchronized to the electron bunch repetition. Operated at a midinfrared wavelength, the proposed method is able to drive a photon source based on high harmonic generation (HHG) to explore the generation of isolated attosecond pulses at photon energies above 1 keV with a repetition of  $> 10$  MHz. The HHG photon source will open a door to full-scale experiments of attosecond x-ray pulses and push ultrafast laser science to the zeptosecond regime.

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Modern laser technologies have realized the generation of ultraintense optical pulses comprising only a few field oscillation cycles to open new avenues for strong field and ultrafast science [1,2]. One of the remarkable applications of few-cycle lasers is the generation of isolated attosecond pulses with full-spatial and temporal coherence via high harmonic generation (HHG) for studying electron dynamics in matter with an attosecond time scale [3]. In such experiments, the stabilization of the carrier-envelope phase (CEP), which is the timing of the field oscillations with respect to the pulse peak, is essential not only for triggering the dynamics of a quantum system with attosecond temporal resolution [4] but also for maximizing HHG photon yield [5].

A maximum photon energy obtained from HHG, the so-called single-atom cutoff energy, can be increased by enhancing the laser intensity. Most HHG photon sources, however, have been limited to photon energies of  $\leq 150$  eV. This is because the coherent addition of HHG-generated photons in the target gas under the condition of phase matching between injected laser and HHG photons becomes difficult as the HHG photon energy increases [6].

Theoretical and experimental studies recently revealed that the HHG cutoff energy under the phase-matched condition depends on the drive laser wavelength as  $h\nu_{\text{cutoff}} \propto \lambda^{1.7}$ , and the efficient generation of HHG photons of soft and hard x-ray energies becomes possible by replacing conventional HHG drivers operated at  $0.8\text{--}1\ \mu\text{m}$  by midinfrared (mid-IR) lasers [6,7]. The generation of 1.6 keV photons was demonstrated at HHG driven by  $3.9\text{-}\mu\text{m}$  laser pulses of six cycles, 10 mJ, and 20 Hz [6]. For exploring attosecond science in soft and hard x-ray regions, it is highly desirable to develop CEP-stabilized few-cycle mid-IR lasers with a high-repetition rate and a high-average power as demonstrated at a near-IR wavelength [8].

In the present Letter, we propose a method to generate CEP-stabilized few-cycle pulses from a free-electron laser (FEL) oscillator. Operated at a mid-IR wavelength, the method is applicable for generating isolated attosecond pulses, via HHG, at photon energies above 1 keV and a repetition rate of  $> 10$  MHz [9].

The duration of optical pulses generated in a FEL oscillator is governed by lasing dynamics through the single-pass gain, round-trip loss of the cavity, electron bunch length, slippage distance, and cavity length detuning [10,11]. The slippage distance is defined as  $L_s = \lambda N_u$ , the product of the FEL wavelength  $\lambda$ , and the number of undulator periods  $N_u$ . Cavity length detuning is introduced in FEL oscillators to compensate for the effect of laser lethargy, i.e., a group velocity slower than the vacuum speed of light. In a FEL oscillator in the strong-slippage regime, in which the electron bunch is shorter than the slippage distance, an electron bunch superradiantly emits a few-cycle optical pulse in the limit of small cavity length detuning [11–14].

FEL lasing in a perfectly synchronized optical cavity (or zero-detuning length) was demonstrated at the Japan Atomic Energy Research Institute (JAERI); the FEL pulse was characterized to be 2.32 optical cycles at a wavelength of  $23.3\ \mu\text{m}$  [15,16]. In experimental and numerical studies, it was found that the lasing in a perfectly synchronized optical cavity only occurs in the high-gain and small-loss FEL oscillators in the strong slippage regime and requires a relatively long rising time to reach saturation [15,17,18], which is supported by a superconducting linac.

FEL oscillators have been operated at a wide range of wavelengths including mid-IR [19,20], but CEP stabilization has never been demonstrated because the evolution of FEL pulses is initiated by the shot noise, microscopic fluctuation of the longitudinal density of the electrons. In our proposal, CEP-stabilized few-cycle pulses are realized

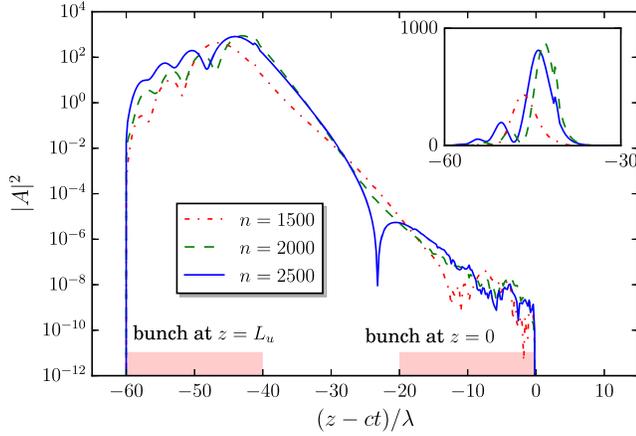


FIG. 1. Temporal shapes of FEL pulses in a perfectly synchronized optical cavity. The profile of the electron bunch at the entrance,  $z = 0$ , and the exit,  $z = L_u$ , of the undulator is also plotted. The inset is the same FEL pulses plotted with a linear scale.

by combining FEL lasing in a perfectly synchronized optical cavity and an external seed laser with CEP stabilization. In the following text, the generation of CEP-stabilized few-cycle FEL pulses is discussed based on the results of time-dependent one-dimensional FEL simulations that employ a FEL code similar to the analysis of the JAERI FEL [15,16].

Our simulation code stems from the well-established FEL simulation model: macroparticle tracking in a ponderomotive potential formed by laser and undulator fields [21,22]. A minor modification, the addition of a convective source term  $\partial_\tau S$  in Ref. [16], has been applied for dealing with few-cycle pulses. Our previous study revealed that the shot noise of an electron beam plays a critical role not only in initiating the FEL lasing but also in sustaining the lasing after saturation in a perfectly synchronized cavity [23]. In our simulation, the shot noise is implemented according to a model proposed by Penman and McNeil [24] and coherent spontaneous emission [25] is not included.

We note the validity of one-dimensional simulations in our study. Since the transverse profile and phase front of the FEL pulse are primarily determined by the eigenmodes of the oscillator, FEL gain reduction due to the beam divergence can be evaluated with a geometric filling factor representing the transverse overlap between the well-defined optical and electron beam profiles. One-dimensional simulations, thus, yield a reasonable approximation to reproduce lasing behavior in FEL oscillators [11,26,27].

We assume a design similar to the JAERI FEL but change the wavelength to  $6 \mu\text{m}$  considering potential applications of the FEL to HHG as listed in Table I.

Figure 1 shows the temporal profiles of FEL pulses after 1500, 2000, and 2500 round trips in a perfectly synchronized FEL oscillator. In this plot, the longitudinal coordinate is defined such that the leading edge of the electron bunch

TABLE I. Parameters of the FEL oscillators.

	JAERI FEL	This work
Electron beam		
Energy (MeV)	16.5	50
Bunch charge (pC)	510	100
Normalized emittance ( $x/y$ ) (mm mrad)	40/22	12/12
Bunch length <sup>a</sup> (ps)	5	0.4
Peak current (A)	200	250
Bunch repetition (MHz)	10	10
Undulator		
Undulator parameter (rms)	0.7	1.25
Pitch (cm)	3.3	4.5
Number of periods	52	40
FEL		
Wavelength ( $\mu\text{m}$ )	22.3	6
Rayleigh length (m)	1.0	0.52
FEL parameter, $\rho$	0.0044	0.0052
Cavity loss	6%	4%

<sup>a</sup>The bunch length is the FWHM of a triangular bunch for the JAERI FEL and the full width of a rectangular bunch for the simulations in this work.

is located at the undulator entrance,  $z = 0$ , at the reference time,  $t = 0$ . The pulse intensity is expressed as a dimensionless value normalized by the high-gain FEL parameter  $\rho$  [28]. The position and profile of the electron bunch at the entrance and exit of the undulator are also depicted to demonstrate that the FEL lasing is in the strong-slippage regime.

Figure 1 illustrates characteristics of FEL pulses evolved in a high-gain and strong-slippage FEL oscillator with a perfectly synchronized optical cavity. The optical pulse consists of an exponential lobe of the leading edge and a main peak followed by ringing. The duration of the main peak, 4.4 cycles (FWHM) after 2500 round trips, is much shorter than that of the electron bunch. A main peak followed by ringing is common to superradiance observed in two-level systems [29] and identical to previous results for the analysis of a high-gain FEL amplifier [28] and a perfectly synchronized FEL oscillator in the transient regime [30], both of which indicated the FEL lasing to be superradiance.

The logarithmic plot of the FEL pulse in Fig. 1 shows that the dynamic range of the laser pulse intensity from the leading edge to the peak is greater than  $10^{11}$ . The leading part of the optical pulse contains incoherent shot noise with random amplitude and phase. The amplitude and phase of the field in the exponential envelope, the main peak, and the ringing are all governed by the interaction between the electrons and the radiation initiated by the shot noise in the leading part. Consequently, the carrier frequency and phase of the FEL pulses are not stabilized.

The simulation result shown in Fig. 1 suggests the possible stabilization of the optical pulse frequency and

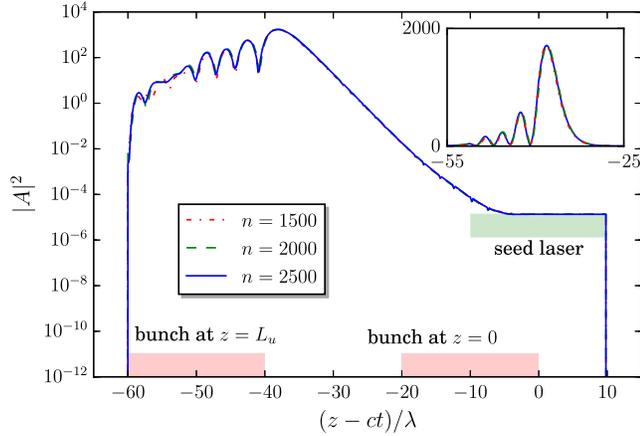


FIG. 2. Temporal shapes of FEL pulses in a perfectly synchronized optical cavity with an external seed laser after 1500, 2000, and 2500 round trips.

phase by fixing the amplitude and phase of the shot noise in the leading part of the FEL pulse. This stabilization is realized by overlapping the pulse head with an external seed laser pulse whose frequency and phase are stabilized. We conducted a simulation to confirm the scheme of CEP stabilization. Figure 2 shows FEL pulses obtained in a simulated FEL oscillator with injection seeding, where all the parameters are the same as in Fig. 1. The seed laser pulse is assumed to have the resonant wavelength, an intracavity intensity of  $|A_{\text{seed}}|^2 = 1.3 \times 10^{-5}$ , and a temporal duration of  $20\lambda$  with CEP stabilization. The seed pulse timing is chosen such that half of the seed pulse overlaps with the FEL pulse and the rest is out of the FEL pulse to indicate the seed laser intensity not affected by the FEL interaction. In Fig. 2, we can see that the seed laser efficiently stabilizes the FEL oscillator with a perfectly synchronized cavity. The FEL pulse after the saturation retains an almost identical temporal shape: the main pulse of 3.8 cycles (FWHM) followed by periodic ringing.

The effects of the CEP stabilization can be clearly seen in Fig. 3, which shows the instantaneous intensity and phase of FEL pulses evolving in a perfectly synchronized cavity for the two cases without and with a seed laser. The instantaneous phase  $\phi_L$  is defined such that the complex field is expressed as  $|A| \{ \exp i[\omega_r(z/c - t) + \phi_L] \}$ , where  $\omega_r$  is the FEL resonance frequency. The simulation parameters are the same as Figs. 1 and 2, respectively. A few-cycle FEL pulse is established after a start-up period,  $\sim 500$  round trips. In the few-cycle FEL pulse, the carrier phase is continuous across the pulse except for the leading edge. Since the pulse leading edge is a free boundary governed by shot noise, the pulse always suffers from the fluctuation introduced by the shot noise. As a result, the pulse has a chance to lose memory of the original carrier phase or to be replaced by another pulse. These variations of the FEL pulse occur in the time scale of the FEL pulse evolution from the shot noise to the saturation,  $\sim 500$  round trips in

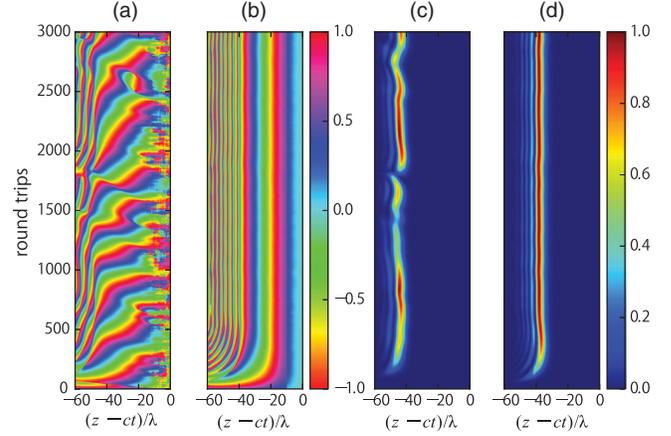


FIG. 3. Contour plots of the instantaneous phase of FEL pulses in units of  $\pi$ rad (a) without injection seeding and (b) with injection seeding. Contour plots of the instantaneous intensity of FEL pulses normalized to the maximum intensity (c) without injection seeding and (d) with injection seeding.

the case. The FEL pulse evolution with an external seed laser exhibits a quite different aspect, in which the pulse shape and CEP after the onset of saturation are stabilized.

For the CEP stabilization, the seed laser intensity must be sufficiently large compared to the shot noise intensity. A set of simulations was performed to determine the amount of CEP fluctuations and pulse energy fluctuation as a function of the seed laser intensity. We plot the simulation result, i.e., the variation of the CEP and pulse energy after saturation against the intracavity seed laser intensity in Fig. 4. The average intensity of the shot noise at the leading edge of the pulse,  $-20 < (z - ct)/\lambda < 0$ , for the simulation parameters is found to be  $|A_{\text{noise}}|^2 = 1.83 \times 10^{-8}$ , as indicated by the broken line. The CEP is uniformly random when the seed laser intensity is less than the shot noise. However, the CEP is stabilized for a seed intensity exceeding the shot noise level and the rms error of the CEP,  $\Delta\phi$ , monotonically decreases with the scale of  $\Delta\phi \propto (|A_{\text{seed}}|^2)^{-0.56}$ , which is

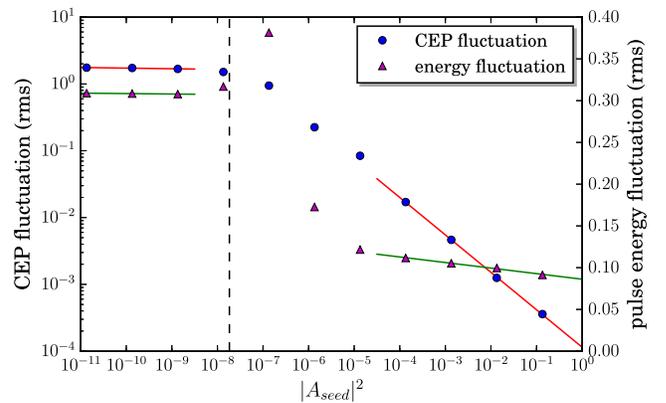


FIG. 4. Fluctuation of the carrier-envelope phase and pulse energy as a function of intracavity seed laser intensity. The broken line is the shot noise intensity.

almost consistent with a phase error equal to the vector sum of the seed laser and the random shot noise:  $\Delta\phi_{\text{sum}} \propto (|A_{\text{seed}}|^2)^{-0.5}$ . The injection seeding also stabilizes the FEL pulse energy. The FEL conversion efficiency with injection seeding was found to be 10% for the simulation parameters, which corresponds to an extracted pulse energy of 0.5 mJ neglecting diffraction and absorption losses in the optical cavity. The extracted pulse can be reasonably stacked in an external cavity to enhance the pulse energy up to tens of times [31].

Next, we discuss the practical implementation and performance of the proposed method in terms of a required seed laser, the synchronization of the electron, FEL, and seed pulses, and finally possible CEP errors due to nonideal electron beams.

A seed laser must generate a train of CEP-stabilized optical pulses at a repetition rate matching that of a train of electron bunches. The role of the seed laser is to induce an electron energy modulation with a stabilized wavelength, amplitude, and phase. The interaction of the electron bunch and seed pulse can be conducted with various configurations. The simplest case involves overlapping a seed laser and an electron bunch in an early section of the FEL undulator as we assumed in the above simulations. Alternatively, a dedicated short-period undulator installed upstream of the FEL undulator can be used for the seeding, in which the seed pulse may have a different polarization from the FEL pulse to share a single dielectric mirror for input and output coupling. We confirmed that seeding the entire electron beam provides the CEP stabilization similar to Figs. 2 and 3 as far as the seed laser keeps the same peak intensity. Seeding the entire electron beam, however, requires a seed laser of a high-average power and seems less attractive for a practical implementation. The intensity of the seed pulse in Fig. 2 corresponds to an intracavity pulse energy of 0.34 nJ for a 400-fs pulse. Such laser pulses are realized with an optical parametric amplifier followed by difference frequency generation [32].

Few-cycle FEL pulses are obtained in a FEL oscillator in the high-gain and large-slippage regime, when the FEL cavity length is tuned close to the perfect synchronization. The tuning width was  $1 \mu\text{m}$  at the JAERI-FEL experiment and  $0.25 \mu\text{m}$  for the simulation in the present study. Such cavity length tuning can be achieved with the help of a mode-locked laser synchronized to an accelerator as demonstrated in the JAERI FEL [15]. We can use the seed laser for this purpose as well. Timing jitter in a train of electron bunches can destroy the condition of “perfect synchronization.” In our previous papers, we discussed the jitter issue and found that timing jitter must be less than 100 fs (rms) to reproduce the experimental result at the JAERI FEL [15,23]. A modern synchronization method [33] has enough accuracy for our proposal even with the cavity tuning width smaller than that of the JAERI FEL.

Inhomogeneous slice properties along the electron bunch longitudinal profile degrade FEL performance. Our proposal, however, is not sensitive to the slice properties, because a single narrow FEL pulse, slipping forward from the bunch tail to the head, interacts with an entire electron bunch. An analytical study suggested that the lasing behavior of short-pulse FEL oscillators is governed by an integrated gain parameter, which is defined as a gain parameter integrated over the slippage distance [11]. All the possible bunch-to-bunch fluctuations in the electron beam, therefore, can be reduced to fluctuations in the integrated gain parameter and can be modeled as jitter in the peak current in one-dimensional simulations.

The variations in the CEP caused by bunch-to-bunch FEL gain variations were numerically evaluated for  $|A_{\text{seed}}|^2 = 1.3 \times 10^{-5}$ . In the simulations, we introduced random jitter into the electron peak current to vary the integrated gain parameter over a bunch train and evaluated the variations in the CEP. Figure 5 plots the calculated CEP errors as a function of the amount of jitter in the peak current; the results indicate that the rms error of the CEP becomes 0.15 rad for a peak current jitter of 20%. In a practical design of FELs, accelerator parameters can be optimized to minimize the jitter in the energy spread and emittance at a FEL undulator [34].

In conclusion, we have proposed a scheme to generate CEP-stabilized few-cycle laser pulses from a FEL oscillator with a CEP-stabilized external seed laser. Operated at mid-IR wavelengths, the proposed method realizes a unique driver for HHG: CEP-stabilized few-cycle pulses of sub-millijoules at a high repetition rate,  $> 10 \text{ MHz}$ , which can be reasonably stacked in an external cavity to enhance the pulse energy up to tens of times. The method breaks the fundamental limitation of the wavelength and repetition rate in solid-state-laser-based few-cycle pulse generation and enables one to generate, via HHG, isolated attosecond

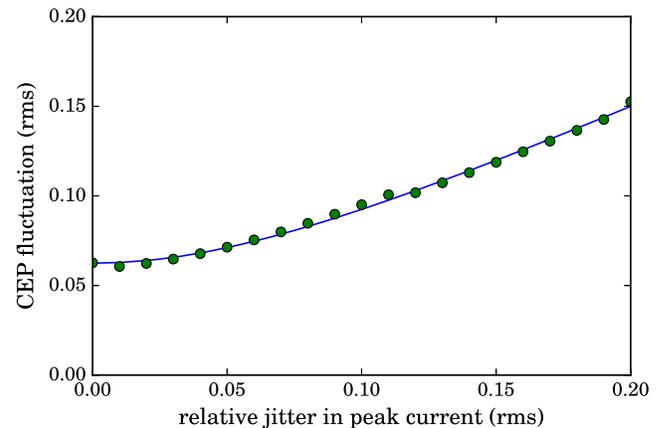


FIG. 5. Fluctuation of the carrier-envelope phase,  $\Delta\phi$ , as a function of the rms jitter in the peak current,  $\Delta I$ . The solid curve is the best-fit curve employing a function form  $\Delta\phi = \sqrt{a^2 + b^2(\Delta I)^2}$ .

pulses at photon energies above 1 keV with CEP stabilization. The method will open a door to full-scale experiments of attosecond x-ray pulses and push ultrafast laser science to the zeptosecond regime.

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