Neoclassical Toroidal Viscosity Calculations in Tokamaks using a δf Monte Carlo Simulation

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Outlines (1)

- Neoclassical toroidal viscosity (NTV) arises from very small asymmetric magnetic perturbations of $10^{-2} \sim 10^{-4} \times B_0$ by the error field, MHD activities, or external perturbation coils applied to mitigate ELMs **(RMPs)**.

- NTV is considered to damps plasma toroidal rotation, which may affect the stability of other MHD modes (RWM, locked modes).

- Observed toroidal rotation damping rates in NSTX, JET, etc., have been studied with analytic theories of NTV which are derived from bounce-average drift-kinetic equation **[ex. Shaing et al, Nucl. Fusion 2010].**
- Analytic formulae are usually given in an asymptotic limit which is valid only in a certain range of collisionality and $\mathbf{E} \times \mathbf{B}$ rotation speed, or connection formula of such approximated solutions. They also rely on many approximations that are used in conventional neoclassical transport theories.
- It has not been well examined how quantitatively accurate those analytic formulae are.

Outlines (2)

- To develop a simulation scheme for precise and quantitative reliable evaluation of NTV, **a drift-kinetic neoclassical transport code** for helical plasmas, **FORTEC-3D,** is applied to direct simulation of NTV by the δf Monte Carlo method. (Satake et al., PPCF 2011, PRL 2011)

- Benchmark tests for $\mathbf{E}\times\mathbf{B}\to 0$ case has revealed that the asymptotic analytic formulae overestimates the NTV in low-collisionality regime, while the combined analytic formula by Park et al. [PRL 2009] agrees with FORTEC-3D direct simulation in wide range of collision frequency.

- We further benchmarked the NTV calculations in the finite- $\mathbf{E}\times\mathbf{B}$ rotation cases, in low-collisionality tokamak with a single-helicity magnetic perturbation.
- It is found that the radial profile and the dependence of NTV on E_r show qualitative difference between the δf simulation and the analytic formula when $|E_r|$ becomes large.
- This difference is related to parallel flow shear which develops around the resonant surface in FORTEC-3D simulation.

Basic relations for NTV calculations

● Momentum balance equation

$$\frac{\partial}{\partial t}(mn\mathbf{u}) = -\nabla\cdot\overleftrightarrow{\mathbf{P}} + en\mathbf{u}\times\mathbf{B} + en\mathbf{E} + \mathbf{F} + \mathbf{S}_m \quad \begin{cases} \mathbf{F} = \int d^3v\, m\mathbf{v}C(\delta f) & \text{: Friction force} \\ \mathbf{S}_m = \int d^3v\, m\mathbf{v}S & \text{: Momentum input} \end{cases}$$

● Radial particle flux [$\langle \nabla\psi\cdot\left(\frac{\mathbf{b}}{B}\right)\times \text{above eq.}\rangle$]

$$\Gamma_\psi = \left\langle \frac{\mathbf{F}_\perp\cdot\mathbf{B}\times\nabla\psi}{eB^2}\right\rangle + \langle n\mathbf{v}_{\mathbf{E}\times\mathbf{B}}\cdot\nabla\psi\rangle - \left\langle \frac{\mathbf{B}\times\nabla\psi\cdot\nabla\cdot\overleftrightarrow{\mathbf{P}}}{eB^2}\right\rangle + \left\langle \frac{\mathbf{S}_{m\perp}\cdot\mathbf{B}\times\nabla\psi}{eB^2}\right\rangle + \left\langle \frac{\partial}{\partial t}\left(\frac{n}{\Omega}\mathbf{b}\times\mathbf{u}_\perp\right)\cdot\nabla\psi\right\rangle$$

Γ_{cl} Classical flux; $\Gamma_{E_\wedge}$ Radial $\mathbf{E}\times\mathbf{B}$ flux (neglected in FORTEC-3D); Γ_{NC}; $\Gamma_{s_\perp}$ Momentum-input driven flux; Γ_{pl} Polarization flux

● Neoclassical flux

$$\Gamma_{NC} = -\left\langle \frac{\mathbf{B}\times\nabla\psi\cdot\nabla\cdot\overleftrightarrow{\mathbf{P}}}{eB^2}\right\rangle = -\frac{G}{e\iota}\left\langle \frac{\mathbf{B}\cdot\nabla\cdot\overleftrightarrow{\mathbf{P}}}{B^2}\right\rangle + \frac{1}{e\iota}\left\langle \mathbf{e}_\zeta\cdot\nabla\cdot\overleftrightarrow{\mathbf{P}}\right\rangle = \Gamma_{p_\parallel} + \Gamma_{TV}$$

$\Gamma_{p_\parallel}$: (Banana-plateau & Pfirsch-Schluter fluxes)

Γ_{TV}: **Flux driven by toroidal viscosity (non-ambipolar)**

● Time evolution of toroidal angular momentum

($\langle \mathbf{e}_\zeta\cdot\ \rangle$ product of the momentum balance eq.)

$$\sum_a \left\langle \frac{\partial}{\partial t}\mathcal{L}^a_\zeta\right\rangle = \sum_a \left(-e_a\iota\Gamma^a_{TV} + T^a_\zeta\right) + \langle \mathbf{J}\times\mathbf{B}\cdot\mathbf{e}_\zeta\rangle\,.$$

$\langle\mathcal{L}_\zeta\rangle \equiv \langle mn\mathbf{u}\cdot\mathbf{e}_\zeta\rangle$

a: particle species

Toroidal rotation is accelerated by the toroidal viscosity $e\iota\Gamma_{TV} = \left\langle \mathbf{e}_\zeta\cdot\nabla\cdot\overleftrightarrow{\mathbf{P}}\right\rangle$, external torque input T_ζ , and the torque of $\mathbf{J}\times\mathbf{B}$ force.

Evaluation of pressure tensor and NTV in the δf method

The guiding-center distribution function : $f = f_M(\psi, v) + \delta f(\psi, \theta, \zeta, v_{||}, v_\perp)$

$$\Rightarrow \quad \overleftrightarrow{\mathbf{P}} \simeq \overleftrightarrow{\mathbf{P}}_{CGL} = p_0(\psi)\overleftrightarrow{\mathbf{I}} + \delta P_{||}\mathbf{bb} + \delta P_\perp(\overleftrightarrow{\mathbf{I}} - \mathbf{bb}).$$

DKE for δf is solved in FORTEC-3D

Taking an flux surface average ⇒ $\left\langle \mathbf{e}_\zeta \cdot \nabla \cdot \overleftrightarrow{\mathbf{P}} \right\rangle = \frac{1}{2}\left\langle \frac{\partial}{\partial \zeta}\delta P \right\rangle$,

where $\delta P = \delta P_{||} + \delta P_\perp = m \int d^3v (v_{||}^2/2 + v_\perp^2)\delta f.$

Making use of the magnetic field expression in Fourier series in Boozer coordinates, NTV is calculated as follows:

$$B(\psi, \theta, \zeta) = B_0 \left[1 - \sum_{m \geq 1} \epsilon_m(\psi)\cos(m\theta) + \sum_{m \geq 0, n \neq 0} \delta_{m,n}(\psi)\cos(m\theta - n\zeta) \right],$$

$$\frac{1}{2}\left\langle \frac{\partial}{\partial \zeta}\delta P \right\rangle = \frac{G + \iota I}{2V'} \oint d\theta d\zeta \frac{1}{B^2}\frac{\partial \delta P}{\partial \zeta} = \frac{G + \iota I}{V'} B_0 \oint d\theta d\zeta \frac{\delta P}{B^3} \sum_{m,n} n\delta_{m,n}\sin(m\theta - n\zeta).$$

Then, the toroidal viscosity is evaluated by decomposing into the following form:

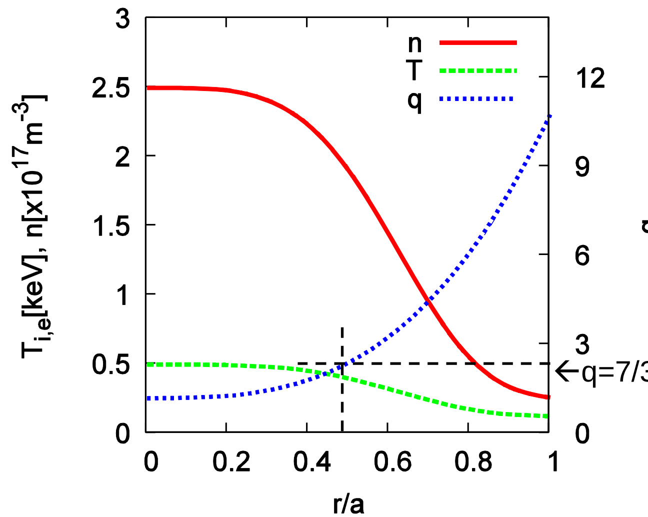
$$\left\langle \mathbf{e}_\zeta \cdot \nabla \cdot \overleftrightarrow{\mathbf{P}} \right\rangle = \sum_{m,n} \left\langle \mathbf{e}_\zeta \cdot \nabla \cdot \overleftrightarrow{\mathbf{P}} \right\rangle_{m,n} = B_0 \sum_{m,n} n\delta_{m,n} Q_{m,n} \; .$$

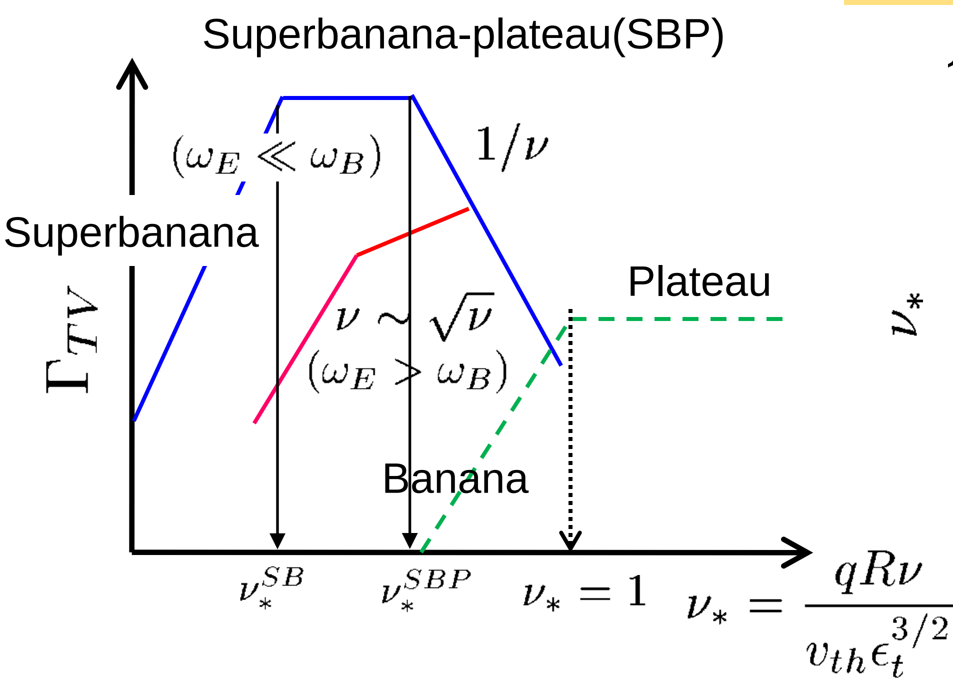
$$Q_{m,n} \equiv \left\langle \frac{\delta P}{B}\sin(m\theta - n\zeta) \right\rangle .$$

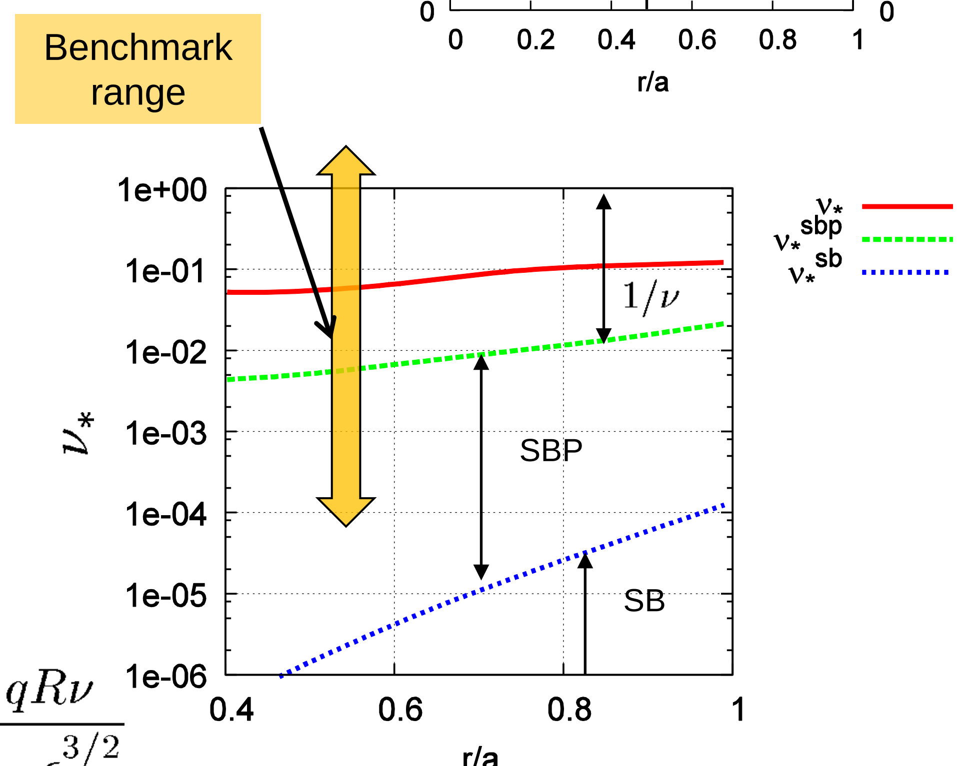
In this expression, one needs to evaluate **only the $Q_{m,n}$ components which has corresponding non-zero $\delta_{m,n} \; (n \neq 0)$ perturbations applied.**

Basic properties of neoclassical toroidal viscosity
Definition of asymptotic collisionality regimes

- Collisionality dependence is checked for $E_r = 0$ cases.
- Parameter survey is carried out by magnifying the collisionality in the range SBP to plateau regimes.
- Single-helicity perturbation, $\delta_{7,3} = 0.02\,(r/a)^2 \cos(7\theta - 3\zeta)$ is superimposed on cylindrical tokamak field, which has a resonant surface at r≒0.49a where q=7/3.







Combined NTV theory (J. K. Park)

- Including the missing components in conventional bounce-average theories:
  - Resonance between bounce motions and electric precession
  - Resonance between magnetic and electric precession (SBP and SB)
- Do not use assumptions that limits the range of collisionality (such as $1/\nu$, superbanana, etc.)
- Combined formula for NTV torque has been derived with effective Krook collision operator

$$\langle \hat{\phi} \cdot \vec{\nabla} \cdot \overleftrightarrow{\Pi}_a \rangle_\ell = \frac{\epsilon^{-1/2} p_a}{\sqrt{2}\pi^{3/2} R_0} \int_0^1 d\kappa^2 \delta_{w,\ell}^2 \int_0^\infty dx \mathcal{R}_{a1\ell} \left[u^\varphi + 2.0\sigma \left| \frac{1}{e} \frac{dT_a}{d\chi} \right| \right] \quad (11)$$

Torque (Transport) $\propto \sum_{m,m'} \delta_{m,n}\delta_{m',n}$ Resonance Rotation with offset

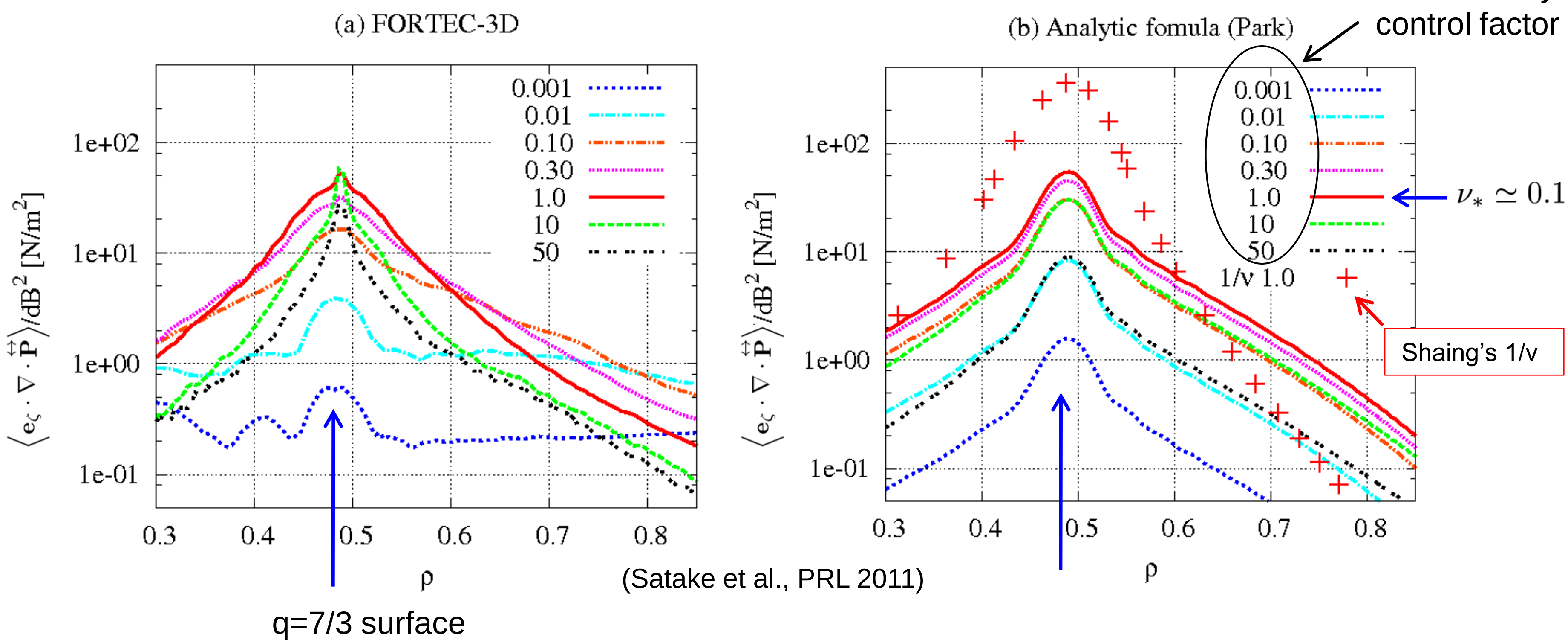
$$\mathcal{R}_{ay\ell} = \frac{1}{2} \frac{n^2 (1 + (\frac{\ell}{2})^2) \frac{\nu_a}{2\epsilon} x (x - \frac{5}{2})^y e^{-x}}{\left[\ell \frac{\pi\sqrt{\epsilon}}{4\sqrt{2}} \omega_{ta} \sqrt{x} - n\omega_E - n\sigma \frac{q^3}{4\epsilon} (\omega_{ta}^2 / \omega_{ga}) x \right]^2 + \left[(1 + (\frac{\ell}{2})^2) \frac{\nu_a}{2\epsilon} \right]^2 x^{-3}} . \quad (12)$$

Bounce Frequency — Electric Precession — Magnetic Precession — Collisionality

[Park et al, PRL102, 065002 (2009)]

Benchmark test : (m,n)=(7,3) single-mode perturbation, $E_r = 0$ limit

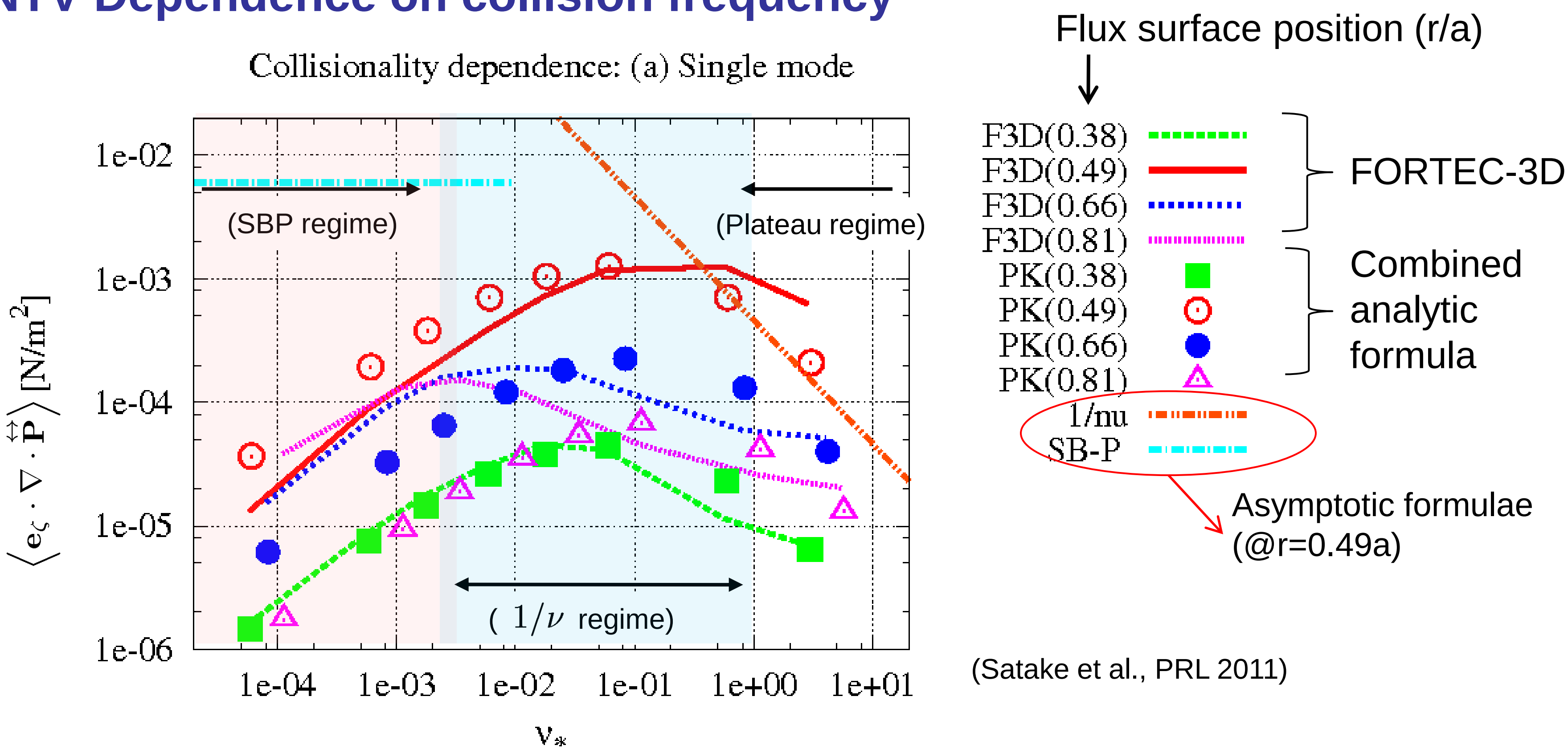
Radial profile of neoclassical toroidal viscosity



- **The overall tendency of radial profile is similar. Both results have a strong peak of NTV at the resonant flux surface (q=7/3).**
- The peak values are close (within factor 2~3) between FORTEC-3D and the combined formula, but FORTEC-3D has larger NTV tail values at the off-resonant tails.
- Shaing's $1/\nu$ formula for $\nu_* \simeq 0.1$ is also compared here. The peak value is much larger, while it drops rapidly towards the off-resonant surfaces.

Benchmark test : (m,n)=(7,3) single-mode perturbation, $E_r = 0$ limit

NTV Dependence on collision frequency



(Satake et al., PRL 2011)

- No clear $1/\nu$, or Superbanana-plateau type dependence are found in the FORTEC-3D simulation.
- Asymptotic limit formula is valid only in a narrow range $\nu_* \simeq 1$.
- **Good agreement with Park's combined formula is found over the wide range of ν_*, in the $E_r = 0$ limit.**

Benchmark of NTV in perturbed tokamaks : Finite-E_r case

How is the force balance relation used in NC transport theory?

Example: in Park's formula ($\chi =$ poloidal flux)

$$\langle \mathbf{e}_\zeta \cdot \nabla \cdot \mathbf{\Pi} \rangle \simeq -\frac{\sqrt{\epsilon} pT}{\sqrt{2} e \pi^{3/2} R_0} \sum_{nmm'} \int_0^1 d\kappa^2 \delta^2_{l;mm'n}(\kappa) \int_0^\infty dx \left[A_1 + (x - 5/2) A_2 \right] \mathcal{R}_{1l}(x),$$

$$A_1 = \frac{d \ln p}{d\chi} + \frac{e}{T} \frac{d\Phi}{d\chi}, \quad A_2 = \frac{d \ln T}{d\chi}$$

← "Driving forces" of NTV

Radial Force balance relation in torus plasmas : $A_1 = \frac{e}{T}(qu^\theta - u^\zeta).$

Impose incompressibility on the flow tangent to a flux surface: $\nabla \cdot (n\mathbf{u}_{||} + n\mathbf{u}_\perp) = 0$

$$\Rightarrow nu_{||} = -\frac{I}{eB}\left(\frac{dp}{d\chi} + en\frac{d\Phi}{d\chi}\right) + K(\chi)B$$

From neoclassical theory: $K(\chi) = k\frac{Ip}{e\langle B^2 \rangle} A_2, \; nu^\theta = K\mathbf{B}\cdot\nabla\theta$

Note: $\partial/\partial\zeta = 0$ is used here!

$$\Rightarrow u^\zeta = -T(A_1 - kA_2)/e$$ **: Neoclassical offset rotation**

NTV can finally be rewritten in terms of u_l^ζ :

$$\langle \mathbf{e}_\zeta \cdot \nabla \cdot \mathbf{\Pi} \rangle \simeq -\frac{\sqrt{\epsilon} p u_l^\zeta}{\sqrt{2} e \pi^{3/2} R_0} \sum_{nmm'} \int_0^1 d\kappa^2 \delta^2_{l;mm'n}(\kappa) \int_0^\infty dx \mathcal{R}_{1l}(x),$$

where $u_l^\zeta \equiv u^\zeta - \frac{T}{e}\left[k + \frac{\int dx \mathcal{R}_{2l}(x)}{\int dx \mathcal{R}_{1l}(x)}\right] A_2, \quad R_{2l}(x) \equiv (x - 5/2) R_{1l}(x).$

← u^ζ is treated as a driving force here.

Benchmark of NTV in perturbed tokamaks : Finite-E_r case

How are simulations with $E \times B$ rotation compared with theory?

$$u_l^\zeta \equiv u^\zeta - \frac{T}{e}\left[k + \frac{\int dx \mathcal{R}_{2l}(x)}{\int dx \mathcal{R}_{1l}(x)}\right] A_2.$$

- In order to avoid ambiguity coming from approximating $k(\epsilon_t, \nu_*) \approx 1$ used in the most analytic formulae ($k \simeq 1.17$ in the $\epsilon_t, \nu_* \to 0$ limit), benchmarks are carried out by setting $A_2 \propto dT/dr = 0$.

- In FORTEC-3D, an E_r profile is given from the force balance relation with assuming $u_\parallel \simeq 0$, e.g., $E_r = \frac{T_i}{e}\frac{d}{d\chi}\ln n_i$. **This E_r profile is called "the E_{r0} case".**

- **To see the NTV dependence on E_r, the E_{r0} case profile is multiplied by ±0.5, 1.0, etc.**

- The collisionality is $\nu_* \sim 0.06(@r/a = 0.49)$, around which the NTV becomes maximum in the $E_r \to 0$ limit.

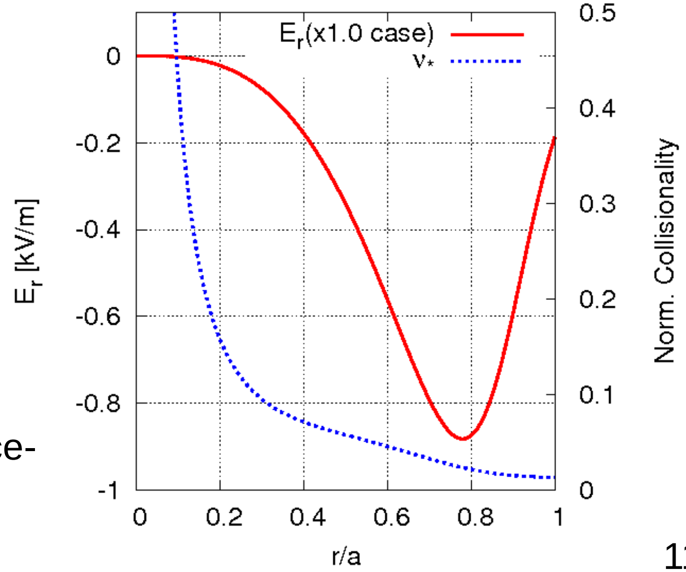
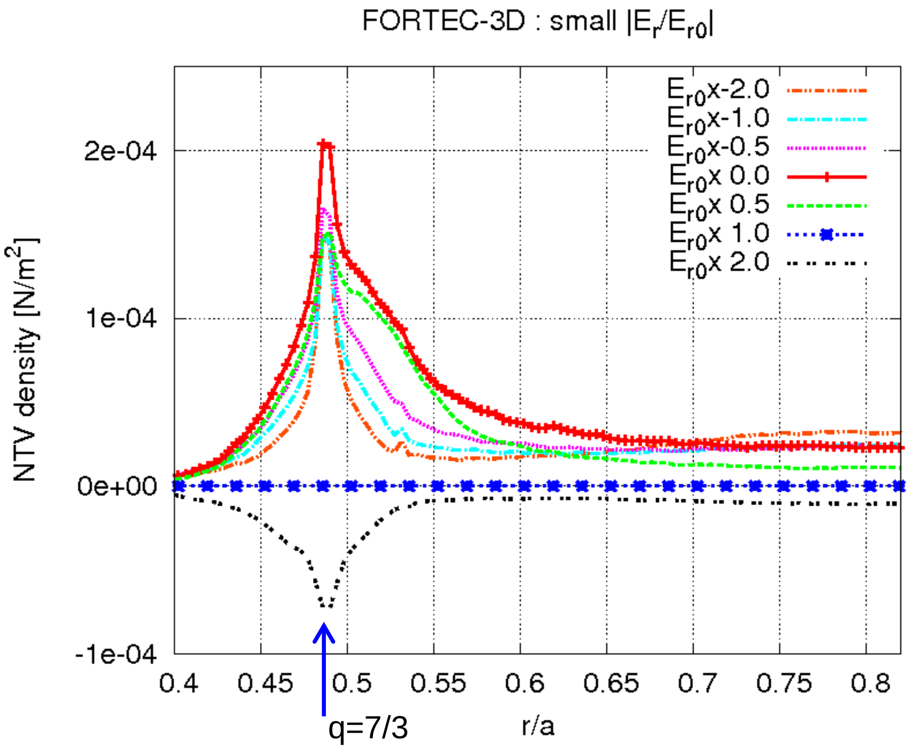
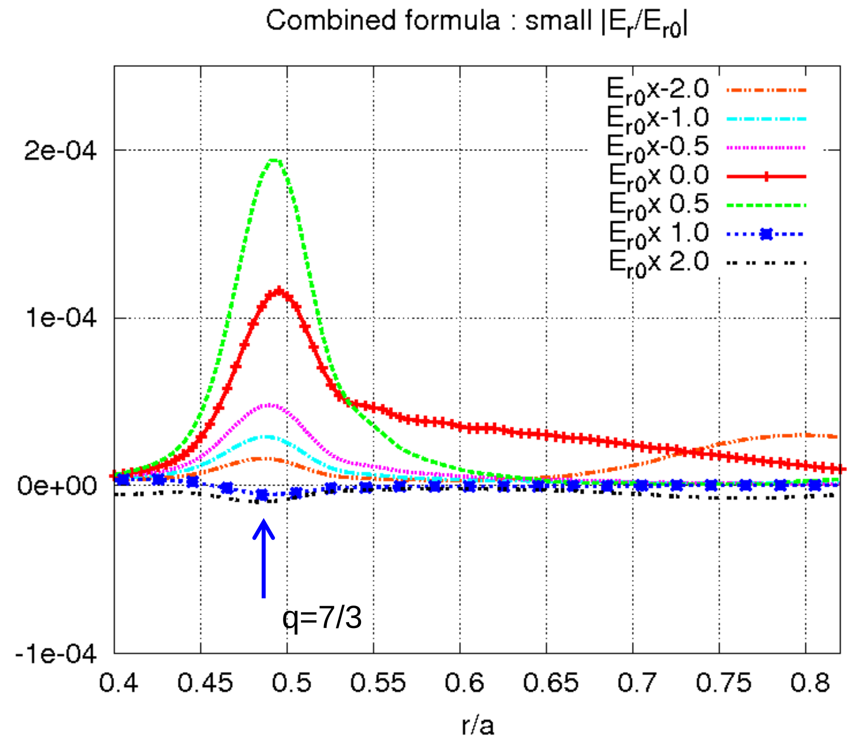


Figure: The radial profiles of ν_* and E_{r0} (the force-balance profile of E_r).

Benchmark test : (m,n)=(7,3) single-mode perturbation, Finite-E_r

Comparison of the NTV radial profile (1) Small-$|E_r/E_{r0}|$



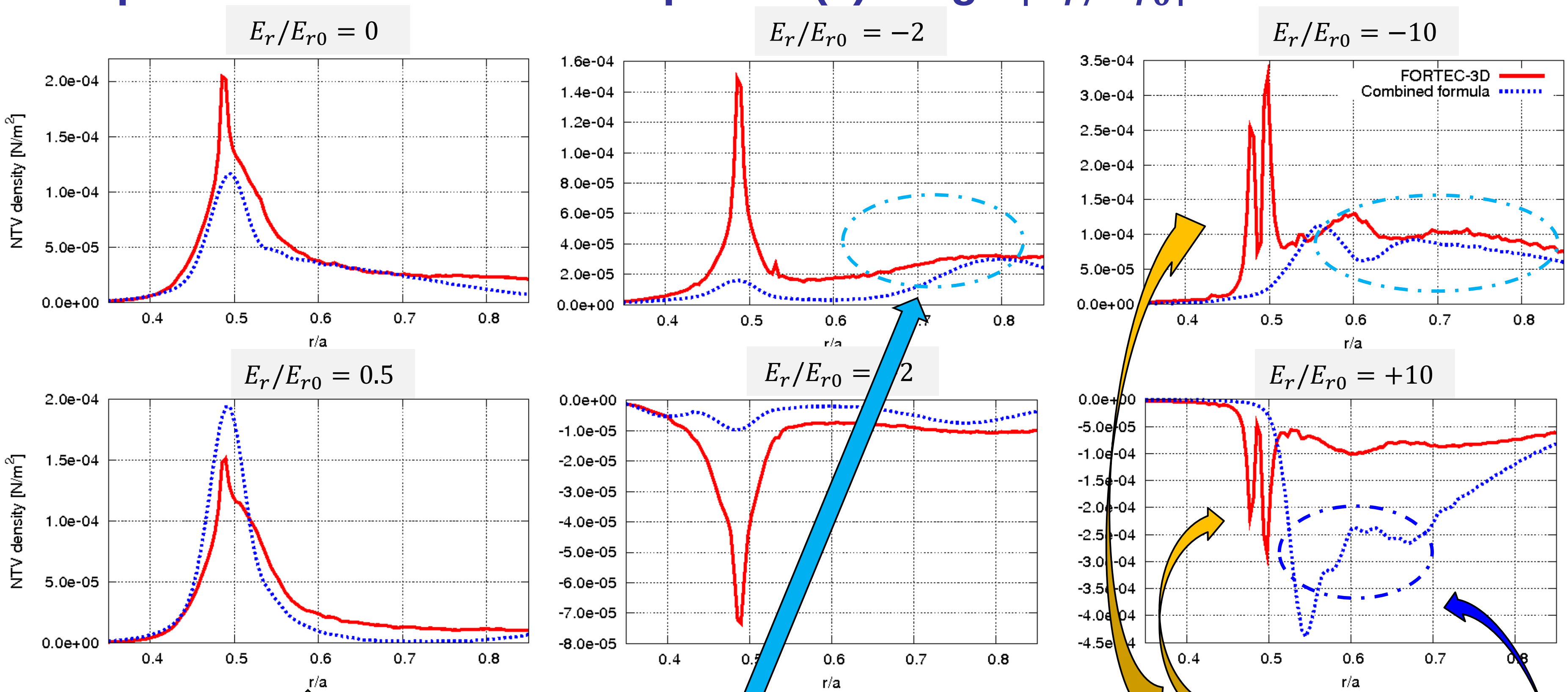


- When E_r/E_{r0} is small, single peak appears at the q=7/3 surface as in $E_r = 0$ case. In the force-balance case ($E_r = E_{r0}$), NTV vanishes everywhere.

- The discrepancy b/w FORTEC-3D and the analytic formula at the resonant flux surface becomes larger as $|E_r/E_{r0}|$ increases.

Benchmark test : (m,n)=(7,3) single-mode perturbation, Finite-E_r

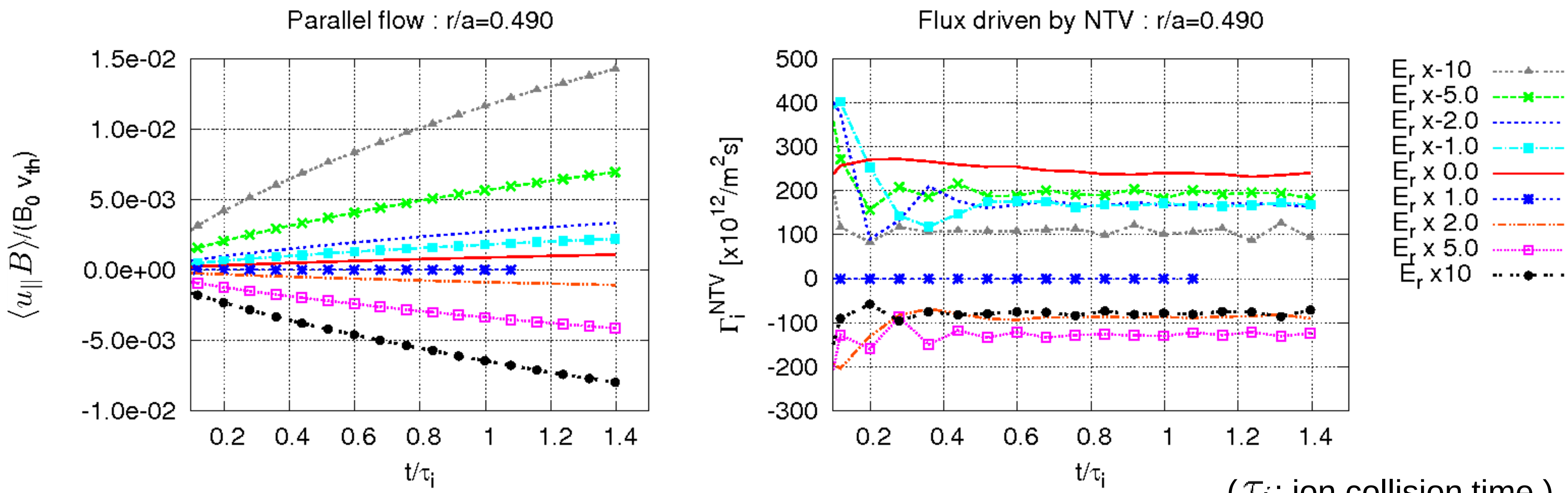
Comparison of the NTV radial profile (2) Large-$|E_r/E_{r0}|$



- Good agreement when $|E_r/E_{r0}| < 1$.
- At the outside region (r/a>0.6) two calculations agrees within a factor O (1) when E_r/E_{r0} <0.
- Twin peaks appears only in FORTEC-3D. Peaks at the q=7/3 resonant disappears in the analytic formula.
- Large viscosity at r/a>0.5 is estimated only in the analytic formula, when E_r/E_{r0} is large positive (negative-E_r).

Benchmark of NTV in perturbed tokamaks : Finite-E_r case

Effect of parallel flow in the Monte Carlo simulation

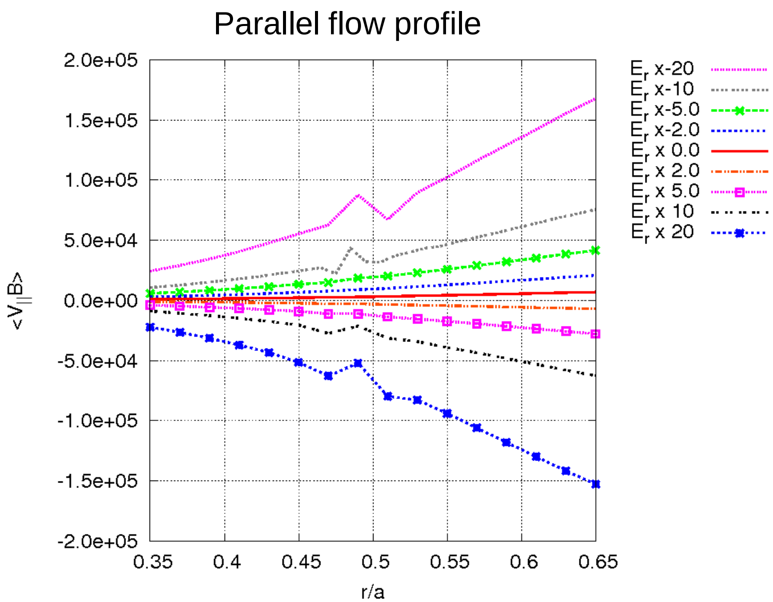


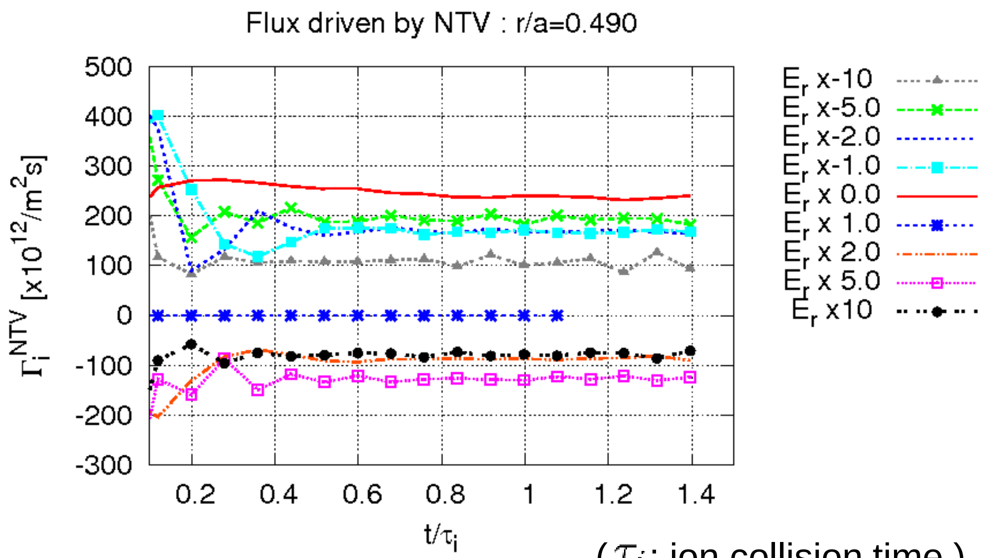
(τ_i: ion collision time)

- Though $u_{\|}$ evolves slowly in simulation, it remains small, $u_{\|}/v_{th} \ll 0.1$, in the present benchmark cases.
- The evolution of $u_{\|}$ did not seem to affect the evaluation of NTV in the simulations even for large E_r cases, however …

Benchmark of NTV in perturbed tokamaks : Finite-E_r case

Effect of parallel flow in the Monte Carlo simulation





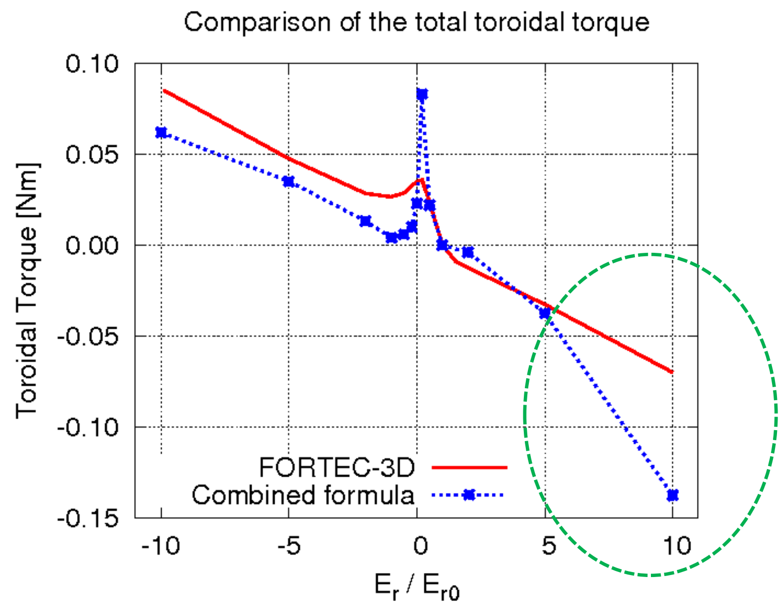
(τ_i: ion collision time)

- Though $u_{||}$ evolves slowly in simulation, it remains small, $u_{||}/v_{th} \ll 0.1$, in the present benchmark cases.
- The evolution of $u_{||}$ did not seem to affect the evaluation of NTV in the simulations even for large E_r cases, however …
- We found that the $u_{||}$ profiles tend to have shear around the resonant rational flux surface when $E \times B$ rotation is large.
- **Parallel flow shear viscosity, $\nabla \cdot (mn\mathbf{u}_{||}\mathbf{u}_{||})$ is a candidate to explain the twin-peak profile of NTV.**

Benchmark test : (m,n)=(7,3) single-mode perturbation, Finite-E_r

NTV Dependence on the E_r amplitude (2) Total toroidal torque

- **The total toroidal torque (NTV integrated on the whole volume) is also compared.**
- Since the analytic formula predicts larger NTV on the outer region of plasma when $E_r/E_{r0} \gg 1$, large discrepancy appears when integrated in the plasma volume.



- **Why combined analytic formula result is asymmetric in E_r ?**

Too much approximation for precession drift (frequency and the direction) ?

$$\langle \hat{\phi} \cdot \vec{\nabla} \cdot \overleftrightarrow{\Pi}_a \rangle_\ell = \frac{\epsilon^{-1/2} p_a}{\sqrt{2}\pi^{3/2} R_0} \int_0^1 d\kappa^2 \delta_{w,\ell}^2 \int_0^\infty dx \mathcal{R}_{a1\ell} \left[u^\varphi + 2.0\sigma \left| \frac{1}{e} \frac{dT_a}{d\chi} \right| \right]$$

$$\mathcal{R}_{ay\ell} = \frac{1}{2} \frac{n^2 (1 + (\frac{\ell}{2})^2) \frac{\nu_a}{2\epsilon} x (x - \frac{5}{2})^y e^{-x}}{\left[\ell \frac{\pi\sqrt{\epsilon}}{4\sqrt{2}} \omega_{ta} \sqrt{x} - n\omega_E - n\sigma \frac{q^3}{4\epsilon} (\omega_{ta}^2 / \omega_{ga}) x \right]^2 + \left[(1 + (\frac{\ell}{2})^2) \frac{\nu_a}{2\epsilon} \right]^2 x^{-3}}.$$

In calculation, a rough approx. for ω_t is adopted. Direction of the precession is prescribed by σ, but it may be wrong when $E \times B$ rotation is very fast.

Consideration of viscosity from flow shear (1)

Definition of viscosity

Momentum balance equation : $\frac{\partial}{\partial t}(mn\mathbf{u}) = -\nabla\cdot\overleftrightarrow{\mathbf{P}} + en\mathbf{u}\times\mathbf{B} + en\mathbf{E} + \mathbf{F} + \mathbf{S}_m$

FORTEC-3D: Evaluate the covariant product of it ($\boldsymbol{e}_\zeta = \partial\boldsymbol{x}/\partial\zeta$) in **Boozer coordinates**

$$\left\langle \frac{\partial}{\partial t}(mn\mathbf{e}_\zeta\cdot\mathbf{u})\right\rangle = -\boxed{\left\langle \mathbf{e}_\zeta\cdot\nabla\cdot\overleftrightarrow{\mathbf{P}}\right\rangle} + \langle \mathbf{J}\times\mathbf{B}\cdot\mathbf{e}_\zeta\rangle + T_\zeta$$

$$\overleftrightarrow{\mathbf{P}} \simeq \overleftrightarrow{\mathbf{P}}_{CGL} = p_0(\psi)\overleftrightarrow{\mathbf{I}} + \delta P_{||}\mathbf{bb} + \delta P_\perp(\overleftrightarrow{\mathbf{I}} - \mathbf{bb})$$

In Analytic formula : Evaluate $\boldsymbol{B}_t\cdot$ product of it in **Hamada coordinates**; use viscous tensor instead of pressure tensor

p and Π are defined on the frame moving with the mean flow $\boldsymbol{u}$.

$$\overleftrightarrow{\mathbf{\Pi}} = \overleftrightarrow{\mathbf{P}} - p\overleftrightarrow{\mathbf{I}} - mn\mathbf{uu}$$

Momentum balance equation expressed in somewhat different way ($\boldsymbol{B}_t = \psi'\nabla V\times\nabla\theta$)

$$\left\langle mn\frac{\partial}{\partial t}(\mathbf{B}_t\cdot\mathbf{u}) + mn\mathbf{B}_t\cdot\mathbf{u}\cdot\nabla\mathbf{u}\right\rangle = -\boxed{\left\langle \mathbf{B}_t\cdot\nabla\cdot\overleftrightarrow{\mathbf{\Pi}}\right\rangle} + \langle \mathbf{J}\times\mathbf{B}\cdot\mathbf{B}_t\rangle + T_\zeta$$

Be careful about the position of the partial derivative

This convective derivative term is usually neglected.

Consideration of viscosity from flow shear (2)

Viscosity tensor when the mean flow is not small :

$$\nabla \cdot \overleftrightarrow{\mathbf{P}} = \nabla \cdot \overleftrightarrow{\mathbf{P}}_{CGL} + \nabla \cdot (mn\mathbf{u}\mathbf{u}) + (others)$$

FORTEC-3D:

$$\simeq \nabla p_0 + \nabla \cdot (\delta \overleftrightarrow{\mathbf{P}}_{\parallel} + \delta \overleftrightarrow{\mathbf{P}}_{\perp}) + \nabla \cdot [mn(\mathbf{u}_{\parallel}\mathbf{u}_E + \mathbf{u}_E\mathbf{u}_{\parallel} + \mathbf{u}_E\mathbf{u}_E)]$$

$$(\mathbf{u}_E = \mathbf{E} \times \mathbf{B}/B^2)$$

Effect of parallel flow shear ($mn\mathbf{u}_{\parallel}\mathbf{u}_{\parallel}$) is included here in the δf simulation

ExB flow shear viscosity is not considered in present FORTEC-3D simulation

⇩

may affect when $M_p \sim 1$ or $u_{||}/v_{th} \sim 1$.

Analytic formula:

$$\nabla \cdot \overleftrightarrow{\mathbf{\Pi}} = \nabla \cdot (\overleftrightarrow{\mathbf{P}} - p\overleftrightarrow{\mathbf{I}} - mn\mathbf{u}\mathbf{u}) \quad \text{where } \mathbf{u} = \mathbf{u}_{\parallel} + \mathbf{u}_E$$

Since the viscosity tensor is defined on the moving frame, large flow effect is excluded from Π. However, the effect of convective derivative term $mn\mathbf{u} \cdot \nabla \mathbf{u}$ is omitted from the momentum balance equation!

In both models, flow shear viscosity should be taken into account when $u/v_{th} \ll 1, M_p \ll 1$ approximations are not valid.

Summary

➢ **FORTEC-3D code was applied to calculate neoclassical toroidal viscosity in tokamak plasmas with asymmetric magnetic field.**

➢ **Benchmark tests were carried out with the combined analytic formula.**

➢ **In $E_r = 0$ limit :**

✓ Strong peak profile of NTV at the resonant flux surface.

✓ Smaller NTV compared with asymptotic analytic theories.

✓ Good agreement with Park's combined analytic formula.

➢ **In finite-E_r cases :**

✓ As $|E_r/E_{r0}|$ (E_{r0} : force-balance value) increases, absolute value of NTV increases in both calculations both in FORTEC-3D and the combined formula.

✓ Radial profile of NTV shows significant difference b/w two calculations when $E_r/E_{r0} \gg 1$. In FORTEC-3D simulations, a twin-peak profile of NTV appears around the resonant rational flux surface as $|E_r/E_{r0}|$ increases.

✓ Parallel flow shear in FORTEC-3D seems to affect the NTV when $|E_r|$is large.

Future Tasks

- Find the physical reason for the formation of the twin-peak NTV profile when $|E_r|$ is large.
  - What is missing in the combined analytic formula?
  - Why sheared parallel flow develops in FORTEC-3D simulation?
  - What happens if multi-helicity perturbations are applied?
- Investigate the collisionality dependence of NTV when $E_r \neq 0$.
  - Is NTV really scales as $\nu \sim \sqrt{\nu}$ as analytic theory predicts?
- Is finite-orbit-width effects (included only in the δf simulation) on NTV important?