Development of RWM analysis code for rotating plasmas
(RWM = Resistive Wall Mode)

J. Shiraishi, N. Aiba, and M. Yagi (JAEA)

Acknowledgements: M.S. Chu and L.L. Lao (GA)
T. Ozeki (JAEA)
Why RWM? What’s RWM? How to stabilize RWM?

Stabilization of RWM is inevitable for advanced tokamaks aiming at steady state high-\(\beta\) operation such as JT-60SA.

- Originates from low-\(n\) external kink modes (timescale \(\sim \tau_A\))
- Ideal wall \(\rightarrow\) stabilization of ideal external kink
- Resistive wall \(\rightarrow\) slow down kink instability to timescale of eddy current decay time in the resistive wall \(\rightarrow\) RWM (timescale \(\sim \tau_w\))
- Many theoretical/experimental research on rotational stabilization of RWM

As a basis of quantitative RWM study, we need to develop a numerical code for RWM in realistic tokamak geometry including plasma rotational effects.
Introduction – RWM codes in tokamak geometry

- **MARS-F** (Chu PoP05), **MARS-K** (Liu NF09), **CarMa** (Liu PoP09)
  - Linearized resistive MHD, perturbative toroidal rotation, kinetic effects, 3D wall, feedback.
- **NMA** (Chu NF03)
  - Linearized ideal MHD, feedback.
- **MISK** (Berkery PRL11)
  - Linearized ideal MHD, without rotation, kinetic effects
- **VALEN** (Bialek PoP01)
  - Linearized ideal MHD, without rotation, 3D wall, and feedback
- **RWMaC/MINERVA**
  - We develop a new RWM code. It has some advantages: (1) perturbative poloidal rotation (Aiba PoP11) (2) equilibrium change by toroidal rotation (3) initial value approach
Energy balance in plasma – wall – vacuum system (Shiraishi PoP10)

\[ W_{RWM} = K + U + W_p - W_d + W_{OV} + W_{IV} + D_W = 0 \]
We compute all metric quantities in IV in a new coordinate system \((s, \theta, \phi)\).

We use a “thin-shell approximation,” which indicates that normal magnetic field is continuous across the wall.
Governing equations in resistive wall

Governing equations are the pre-Maxwell equations and the Ohm’s law.

\[ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \quad \partial_t \mathbf{B} = -\eta \nabla \times \mathbf{J} \]

\( \eta \) : volume resistivity of wall

We introduce a “current potential” and magnetic “scalar” potentials in vacuum:

\[ \mathbf{J}(s, \theta, \phi, t) = \nabla \times \nabla \kappa(\theta, \phi, t) \delta(s - s_w) \]

\[ \mathbf{B} = \nabla \chi^{(\pm)} \]

in OV (IV)

Integration of the Ampere’s law on the wall yields a “jump” condition for magnetic field.

\[ \chi^{(w+)}(\theta, \phi, t) - \chi^{(w-)}(\theta, \phi, t) = \mu_0 \kappa(\theta, \phi, t) \quad (1) \]

The superscript \((w)\) indicates the limit to the resistive wall.

Integration of the Faraday’s law gives a “diffusion equation” for magnetic energy.

\[ \frac{\Delta \nabla s}{\eta} \frac{\partial B^{(n)}}{\partial t} = L \kappa \quad (2) \]

\( \Delta \) : wall width measured by \( s \)

\( B^{(n)} = \mathbf{B} \cdot \hat{n} \) : normal \( \mathbf{B} \) on wall

\( L \) : elliptic operator (shown in the next page)
Method of eigenfunction expansion

We invoke a property of operator $L$ by considering following eigenvalue problem on wall:

$$L\kappa = \omega |\nabla s| \kappa$$  \hspace{1cm} (3)\hspace{1cm}$$L\bullet = -\nabla \times [\nabla \times (\nabla s \times \nabla \bullet)]$$

We can easily prove that

(a) $L$ is positive, i.e., $\forall \omega > 0, \forall \omega \in \mathbb{R}$

(b) Eigenfunctions belonging to different eigenvalues are orthogonal

$$\int_{S_w} \kappa_j^* \kappa_k |\nabla s| \sqrt{g} \, d\theta d\phi = 0$$

By (2), we can expand $B^{(n)}$ and $\kappa$ on wall as

$$B^{(n)}(\theta, \phi, t) = \sum_{j=1}^{N_p} a_j(t) \hat{\kappa}_j(\theta, \phi)$$ \hspace{1cm} (4)

$$\kappa(\theta, \phi, t) = \frac{R_0^3 \Delta}{\eta} \sum_{j=1}^{N_p} \frac{1}{\omega_j} \frac{da_j}{dt} \hat{\kappa}_j(\theta, \phi)$$ \hspace{1cm} (5)

$\hat{\kappa}_j$ : normalized eigenfunction

$N_p$ : poloidal mode number
Energy balance

To get energy balance of the system, we multiply (1) by \( (1/2\mu_0)B^{(n)*}|\nabla s|\sqrt{g}d\theta d\phi \) and integrate them on wall.

\[
\frac{1}{2\mu_0} \int_{S_w} \chi^{(w+)} B^{(n)*} |\nabla s| \sqrt{g}d\theta d\phi - \frac{1}{2\mu_0} \int_{S_w} \chi^{(w-)} B^{(n)*} |\nabla s| \sqrt{g}d\theta d\phi = \frac{1}{2} \int_{S_w} \kappa B^{(n)*} |\nabla s| \sqrt{g}d\theta d\phi
\]

The RHS gives energy dissipation in the resistive wall, which by (4) and (5) can be written as

\[
\frac{1}{2} \int_{S_w} \kappa B^{(n)*} |\nabla s| \sqrt{g}d\theta d\phi = \frac{R_s^5 \Delta}{\eta} \sum_{j=1}^{N_p} 1 \text{ } \frac{a_j^*}{\omega_j} \frac{da_j}{dt} = D_w \quad (6)
\]

After some manipulation, we get energy balance:

\[ W_{RWM} = W_{OV} + W_{IV} + D_W + \frac{1}{2\mu_0} \int_{S_p} \chi^{(p+)} Q^*_e \cdot \hat{n} dS = 0 \quad (7) \]

\[ W_{IV(OV)} = \frac{1}{2\mu_0} \int_{IV(OV)} |\nabla \chi|^2 dV \text{ : vacuum magnetic energy} \]

Plasma response

\[ W_{OV} = \frac{1}{2\mu_0} \int_{S_w} \kappa B^{(n)*} |\nabla s| \sqrt{g}d\theta d\phi \]
Plasma response

We employ Frieman-Rotenberg equation (Frieman, RMP 60) as a plasma model, linearized ideal MHD equation including equilibrium rotation.

\[
\rho_0 \frac{\partial^2 \xi}{\partial t^2} + 2 \rho_0 (V_0 \cdot \nabla) \frac{\partial \xi}{\partial t} = F \xi \quad F : \text{generalized force operator}
\]

Energy balance in plasma leads to

\[
\frac{1}{2\mu_0} \int_{S_p} \kappa^{(p)} Q_e^* \cdot \hat{n} dS = K + U + W_p - W_d + \frac{1}{2} \int_{S_p} \left[ \left( \frac{Q \cdot B}{\mu_0} + p \right) \xi^* - \left( \frac{Q_e \cdot B_e}{\mu_0} \right) \xi^* \right] \cdot \hat{n} dS
\]

\[
K = \frac{1}{2} \int \xi^* \cdot \rho_0 \frac{\partial^2 \xi}{\partial t^2} dV \quad : \text{kinetic energy}
\]

\[
U = \frac{1}{2} \int \xi^* \cdot 2 \rho_0 (V_0 \cdot \nabla) \frac{\partial \xi}{\partial t} dV \quad : \text{energy associated with convective term}
\]

Thus (7) yields energy balance for the resistive wall-plasma system:

\[
W_{RWM} = K + U + W_p - W_d + W_{OV} + W_{IV} + D_W = 0 \quad (8)
\]

MINERVA (Aiba, CPC09) calculates these terms by FEM and Fourier decomposition.
“RWMaC” modules compute $W_{IV(OV)}$ and $D_W$

Inner vacuum and outer vacuum: $W_{IV(OV)}$
Governance equation: Laplace equation for $\chi$
(magnetic scalar potential $B = \nabla \chi$)
Numerical scheme: FEM for IV
FEM or Green’s function method for OV

Resistive wall: $D_W$
Governance equation: diffusion equation for $\kappa$
current potential $J = (\nabla s \times \nabla \kappa) \delta(s - s_{wall})$
Numerical scheme: FEM

Boundary conditions on resistive wall and plasma surface:
Continuity of normal magnetic field + natural boundary condition
Implementation of \textsc{RWMaC} in \textsc{MINERVA}

The \textsc{RWMaC} module solves electromagnetic problems in the vacuum and wall. The \textsc{MINERVA} [Aiba CPC (2009)] solves linear plasma dynamics with equilibrium toroidal rotation.

\textsc{MINERVA}/\textsc{RWMaC} has been benchmarked with \textsc{NMA} (Chu NF03) using Solov’ev equilibrium.

\textsc{MINERVA}/\textsc{RWMaC} has some advantages (1) include poloidal rotation effect perturbatively (Aiba PoP11) (2) include MHD equilibrium change induced by toroidal rotation. (3) can solve initial value problem.
Benchmark between MINERVA/RWMaC and MARS–F using Solov’ev circle without rotation

We start benchmarking from Solov’ev circle without rotation to remove numerical errors in numerical computation of the Grad-Shafranov equation.

\[ \beta_N = 3.3, B_{axis} = 1\text{T}, I_p = 0.17\text{MA}, \]
\[ q_0 = 1.2, q_a = 1.4 \]

Benchmark succeeded for the marginal wall position of external kink, and the RWM growth rates even for large wall decay time.
Benchmark of MINERVA/RWMaC with MARS–F using up-down asymmetric MHD equilibrium with toroidal rotation

MINERVA/RWMaC has been benchmarked with MARS–F (Chu PoP95) with rigid toroidal rotation under assumption that toroidal rotation does not affect MHD equilibrium.

\[ \beta_N = 2.63, B_{axis} = 1.7T \]

\[ I_p = 1.1\text{MA} \]

Benchmark succeeded for frequency of \( O(1\%) \) of Alfven frequency in RWM growth rate, critical rotation, location of stable window.

(Many thanks to L.L. Lao and M.S. Chu)
Application to RWM analysis in JT-60SA

By modeling the wall shape, we start RWM study in JT-60SA configuration.

\[ \beta_N = 3.3, \quad B_{axis} = 1.7T \]
\[ I_p = 2.3 \text{MA} \]

Pressure and \( q \) profile

n=1 RWM growth rate vs. wall time

Estimated RWM growth rate \( \gamma \sim 0.6 \text{kHz} \) for stabilizing plate

Rigid rotation requires \( \Omega \sim 0.025 \omega_A \) to fully stabilize RWM.
Summary

• We have
  • developed RWMaC modules that solve electromagnetic problems in the vacuum and resistive wall.
  • implemented RWMaC modules in linear MHD code, MINERVA (with rotation) and MARG2D (w/o rot, inertia).
  • benchmarked MINERVA/RWMaC against NMA (w/o rotation) and MARS–F (with rotation).
  • used MINERVA/RWMaC to study RWMs in JT-60SA high $\beta_N$ equilibrium.

• Based on MINERVA/RWMaC, we will
  • study how the equilibrium change induced by toroidal rotation affects RWMs.
  • implement kinetic effects.
  • implement the 3D wall structure.