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Collisionality dependence of a shielding factor of a beam driven current
ビーム駆動電流における遮蔽因子の衝突率依存性

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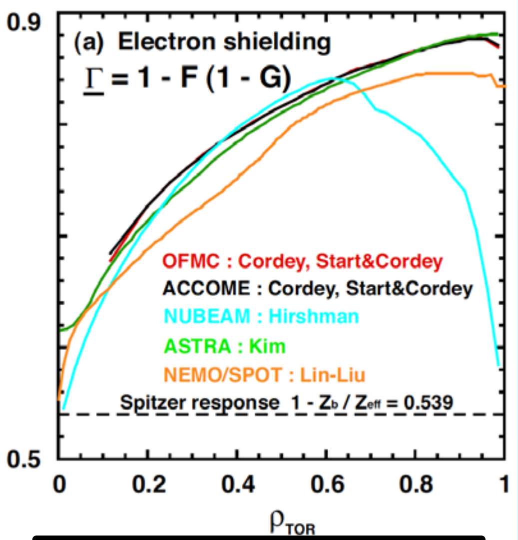
Background

- There is of growing importance in accurately estimating the neutral beam current drive (NBCD) for obtaining a fully current-driven, steady-state plasma.
- **Physical mechanism of NBCD**
  - Tangential NBI typically produces the fast-ion circulating current.
  - At the same time, electrons tend to be dragged by the fast ions and cancel the current.
  - §Ohkawa clarified that the effective NBCD was obtained in an impure plasma [NF 1970].
  - ¶Also, the existence of **trapped electrons** reduces the electron circulating current.

Shielding factor Γ: $\Gamma \equiv \frac{j_{\mathrm{b}}}{j_{\|\mathrm{f}}} = 1 - \overset{\S}{\boxed{F}}(1 - \overset{\P}{\boxed{G}})$ where typically $F = Z_{\mathrm{b}}/Z_{\mathrm{eff}}$

- The *G* factor stems from the neoclassical transport.
  - This is because trapped particles are connected to the neoclassical transport.
- Many models for the *G* factor have been proposed, but all of these models have been derived in the banana regime, except one model.
  - They do **not include the collisionality**.

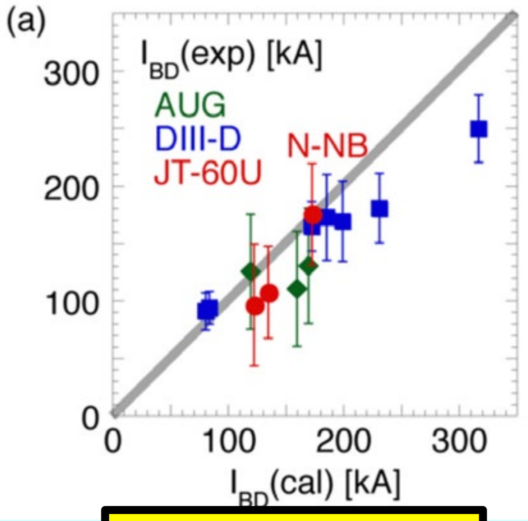


Oikawa IAEA2008

Background cont'd.

$$\nu_{*\mathrm{e}} = \left(\omega_{\mathrm{Te}}\tau_{\mathrm{ee}}\varepsilon^{3/2}\right)^{-1} \propto n_{\mathrm{e}} / T_{\mathrm{e}}^{2}$$

- Experimentally, we already found that the NBCD calculation is prone to overestimation of the driven current.
- This research is motivated by the idea that **the collisionality dependence of the shielding factor would be one of the candidates that can partly fill the gap between measured and calculated NBCD.**



Suzuki NF2011

- The Matrix Inversion (MI) method [Kikuchi PPCF1995] solves the momentum balance equations to calculate neoclassical transport coefs.
  - This is based on the moment approach proposed by Hirshman and Sigmar [NF1981]
  - Recently, the Shaing's viscosity model [PoP 1995] has been incorporated into MI, called the MI-S method, like the NCLASS module [Houlberg PoP 1997].
- Hirshman and Sigmar showed that adding the friction coefs. of fast ions to the moment approach gave the shielding factor.
  - Unfortunately, this derivation included some mistakes and implicit assumptions.

Derive and examine the collisionality dependent shielding factor using MI-S

Electron moment equations

The electron momentum and heat balance eqs. based on the moment approach

viscosity friction

beam momentum and heat friction forces

$$\left\langle \boldsymbol{B}\cdot\nabla\cdot\overset{\leftrightarrow}{\Pi}_{\mathrm{e}}\right\rangle = \langle BR_{\mathrm{ei}}\rangle + \boxed{\langle BR_{\mathrm{eb}}\rangle},$$

$$\left\langle \boldsymbol{B}\cdot\nabla\cdot\overset{\leftrightarrow}{\Theta}_{\mathrm{e}}\right\rangle = \langle BH_{\mathrm{ei}}\rangle + \boxed{\langle BH_{\mathrm{eb}}\rangle},$$

$$R_{\mathrm{eb}} = (m_{\mathrm{e}}Z_{\mathrm{b}}^2/\tau_{\mathrm{ee}})n_{\mathrm{b}}u_{\parallel\mathrm{b}}$$

$$H_{\mathrm{eb}} = -(3/2)R_{\mathrm{eb}}$$

bare minimum

Solving this simultaneous equation will give

the bootstrap current as well as the beam driven current.

∝ diamagnetic particle and heat flows

Ignorable

which consists of terms proportional to $\langle Bu_{\parallel j}\rangle = \hat{u}_{j\theta}\langle B^2\rangle$

Solving it so that $\langle Bq_{\parallel\mathrm{e}}\rangle/p_{\mathrm{e}}$ vanishes yields

Electron parallel current regarding NBCD

$$\langle Bu_{\parallel\mathrm{e}}\rangle = \boxed{\frac{\ell_{11}^{\mathrm{ee}}(\hat{\mu}_3^{\mathrm{e}} - \ell_{22}^{\mathrm{ee}}) + \ell_{12}^{\mathrm{ee}}(\hat{\mu}_2^{\mathrm{e}} + \ell_{12}^{\mathrm{ee}})}{(\hat{\mu}_3^{\mathrm{e}} - \ell_{22}^{\mathrm{ee}})(\hat{\mu}_1^{\mathrm{e}} - \ell_{11}^{\mathrm{ee}}) - (\hat{\mu}_2^{\mathrm{e}} + \ell_{12}^{\mathrm{ee}})^2}}\left(-\langle B^2\rangle\sum_j \frac{Z_j^2 n_j}{Z_{\mathrm{eff}}n_{\mathrm{e}}}\hat{u}_{j\theta} - \frac{Z_{\mathrm{b}}^2 n_{\mathrm{b}}}{Z_{\mathrm{eff}}n_{\mathrm{e}}}\langle Bu_{\parallel\mathrm{b}}\rangle\right)$$

$$\equiv -\boxed{\gamma}\left(\frac{Z_{\mathrm{b}}^2 n_{\mathrm{b}}}{Z_{\mathrm{eff}}n_{\mathrm{e}}}\langle Bu_{\parallel\mathrm{b}}\rangle + \langle B^2\rangle\sum_j \frac{Z_j^2 n_j}{Z_{\mathrm{eff}}n_{\mathrm{e}}}\hat{u}_{j\theta}\right)$$

Derive shielding factor

This equation expresses the parallel electron flow driven mainly by the parallel beam-ion flow due to NBI, or in other words the beam driven electron flow.

Substituting it into the parallel current $\langle Bj_\parallel \rangle = \sum_{k=\mathrm{e},j,\mathrm{b}} Z_k |e| n_k \langle B u_{\parallel k} \rangle$ yields the beam driven component of the parallel current, namely, the beam driven current as follows:

$$\langle Bj_\mathrm{b} \rangle = Z_\mathrm{b}|e|n_\mathrm{b} \langle Bu_{\parallel \mathrm{b}} \rangle \left[1 + \gamma \frac{Z_\mathrm{b}}{Z_\mathrm{eff}} + \frac{\sum_j Z_j^2 n_j \langle B^2 \rangle \hat{u}_{j\theta}}{Z_\mathrm{b} n_\mathrm{b} \langle Bu_{\parallel \mathrm{b}} \rangle} \left(\frac{\gamma}{Z_\mathrm{eff}} + \frac{\sum_j Z_j n_j \hat{u}_{j\theta}}{\sum_j Z_j^2 n_j \hat{u}_{j\theta}} \right) \right]$$

It would be found that (1+γ) is equivalent to the bootstrap current coefficient L_{31}^e, i.e.

$$\boxed{1 + \gamma = L_{31}^\mathrm{e}} = \frac{(\hat{\mu}_3 - \ell_{22}^\mathrm{ee})\hat{\mu}_1^\mathrm{e} - (\hat{\mu}_2^\mathrm{e} + \ell_{12}^\mathrm{ee})\hat{\mu}_2^\mathrm{e}}{(\hat{\mu}_3^\mathrm{e} - \ell_{22}^\mathrm{e})(\hat{\mu}_1^\mathrm{e} - \ell_{11}^\mathrm{ee}) - (\hat{\mu}_2^\mathrm{e} + \ell_{12}^\mathrm{ee})^2}$$

This fact has already been found by Lin-Liu and Hinton [PoP 1997] in a different manner.

Thus, we finally have

Shielding factor

$$\boxed{\Gamma \equiv \frac{\langle Bj_\mathrm{b} \rangle}{Z_\mathrm{b}|e|n_\mathrm{b} \langle Bu_{\parallel \mathrm{b}} \rangle} = 1 - \frac{Z_\mathrm{b}}{Z_\mathrm{eff}} + \left(\frac{Z_\mathrm{b}}{Z_\mathrm{eff}} + \frac{n_\mathrm{e} \langle B^2 \rangle \hat{u}_{i\theta}}{Z_\mathrm{b} n_\mathrm{b} \langle Bu_{\parallel \mathrm{b}} \rangle} \right) L_{31}^\mathrm{e}}$$

Add $\langle BR_\mathrm{eb} \rangle$ & $\langle BH_\mathrm{eb} \rangle$

solely

$Z_\mathrm{b}/Z_\mathrm{eff}$ term

Any other term is purely neoclassical!

Matrix Inversion proposes two kinds of shielding factor models.

The ion poloidal flow with the order of $\mathcal{O}(m_\mathrm{e}/m_\mathrm{i})^{1/2}$ is smaller than the parallel beam-ion flow, and thus the second term in the brackets is negligible [H&S NF 1981].

MI-S analytic model

$$\boxed{\Gamma = 1 - \frac{Z_\mathrm{b}}{Z_\mathrm{eff}}(1 - L_{31}^\mathrm{e}) = 1 - \frac{Z_\mathrm{b}}{\underbrace{Z_\mathrm{eff}}_{F}}\left[1 - \underbrace{\frac{(\hat{\mu}_3 - \ell_{22}^\mathrm{ee})\hat{\mu}_1^\mathrm{e} - (\hat{\mu}_2^\mathrm{e} + \ell_{12}^\mathrm{ee})\hat{\mu}_2^\mathrm{e}}{(\hat{\mu}_3^\mathrm{e} - \ell_{22}^\mathrm{e})(\hat{\mu}_1^\mathrm{e} - \ell_{11}^\mathrm{ee}) - (\hat{\mu}_2^\mathrm{e} + \ell_{12}^\mathrm{ee})^2}}_{G=L_{31}}\right]}$$

The Lin-Liu model [PoP 1997] exploits the analytical expressions of the viscosities valid solely for the banana regime from [Hirshman NF 1988].

In contrast, MI-S is capable of estimating the collisionality-dependent viscosities valid for all collisionality regime, because it solves the momentum and heat balance equations to obtain the parallel flows for each species including beam ions.

More fundamentally, using MI-S gives us the fast-ion circulating current and the beam driven current as follows:

$$\langle Bj_{\parallel\mathrm{f}}\rangle = Z_\mathrm{b}|e|n_\mathrm{b}[(\overleftrightarrow{M} - \overleftrightarrow{L})^{-1}]_\mathrm{bb}S_{\parallel\mathrm{b}},$$
$$\langle Bj_\mathrm{b}\rangle = \sum_{k=\mathrm{e},j,\mathrm{b}} Z_k|e|n_k[(\overleftrightarrow{M} - \overleftrightarrow{L})^{-1}]_{k\mathrm{b}}S_{\parallel\mathrm{b}},$$

⇨

MI-S model

$$\boxed{\Gamma \equiv \frac{\langle Bj_\mathrm{b}\rangle}{\langle Bj_{\parallel\mathrm{f}}\rangle} = \frac{\sum_{k=\mathrm{e},j,\mathrm{b}} Z_k n_k[(\overleftrightarrow{M} - \overleftrightarrow{L})^{-1}]_{k\mathrm{b}}}{Z_\mathrm{b}n_\mathrm{b}[(\overleftrightarrow{M} - \overleftrightarrow{L})^{-1}]_\mathrm{bb}}.}$$

Even in the banana limit, the MI-S model can be adopted by introducing the banana viscosities solely.

Yet another shielding factor model with collisionality dependence

We could readily obtain the collisionality dependent Γ factor using the analytical expression if we had a simple, accurate expression of the viscosities valid over the whole collisionality domain.

The problem is that we never knew such expressions! **However…**

By looking at the L_{31} coef. that is essentially equivalent to the G factor, we find that a set of formulae for calculating the bootstrap current proposed by **Sauter [PoP 1999]** includes the collisionality dependent L_{31} coefficient as follows:

Fitted L_{31} model

$$\boxed{L_{31} = F_{31}(X = f^{31}_{\rm teff}) = \left(1 + \frac{1.4}{Z+1}\right) X - \frac{1.9}{Z+1}X^2 + \frac{0.3}{Z+1}X^3 + \frac{0.2}{Z+1}X^4,}$$

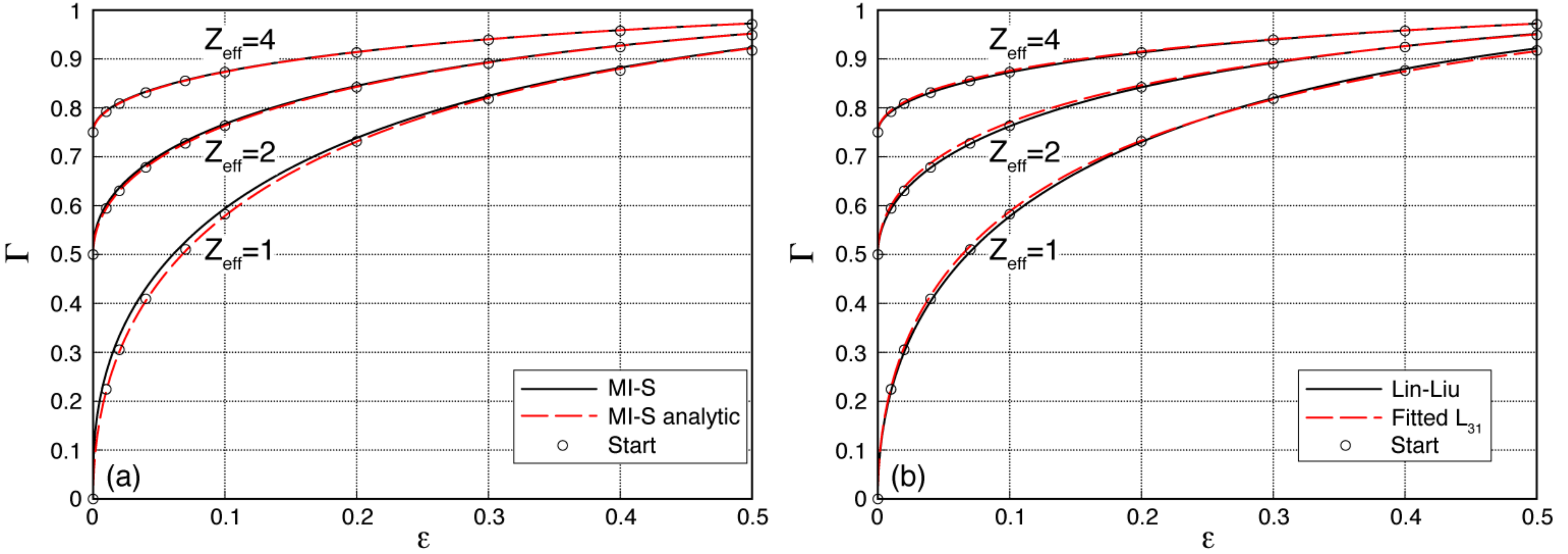
$$f^{31}_{\rm teff}(\nu_{*e}) = \frac{f_t}{1 + (1 - 0.1 f_t)\nu_{*e}^{1/2} + 0.5(1 - f_t)\nu_{*e} Z^{-1}},$$

The model is based on numerical results of a code CQLP, solving the Fokker-Planck equation with the full, linearized collision operator.

This L_{31} clearly includes the collisionality dependence solely through $f^{31}{}_{teff}$.

Comparison of the <u>collisionless</u> shielding factor models

ν_{*e}→0, Z_b=1, nearly concentric circular equilibrium, $f_t=1.46\varepsilon^{1/2}-0.46\varepsilon^{3/2}$

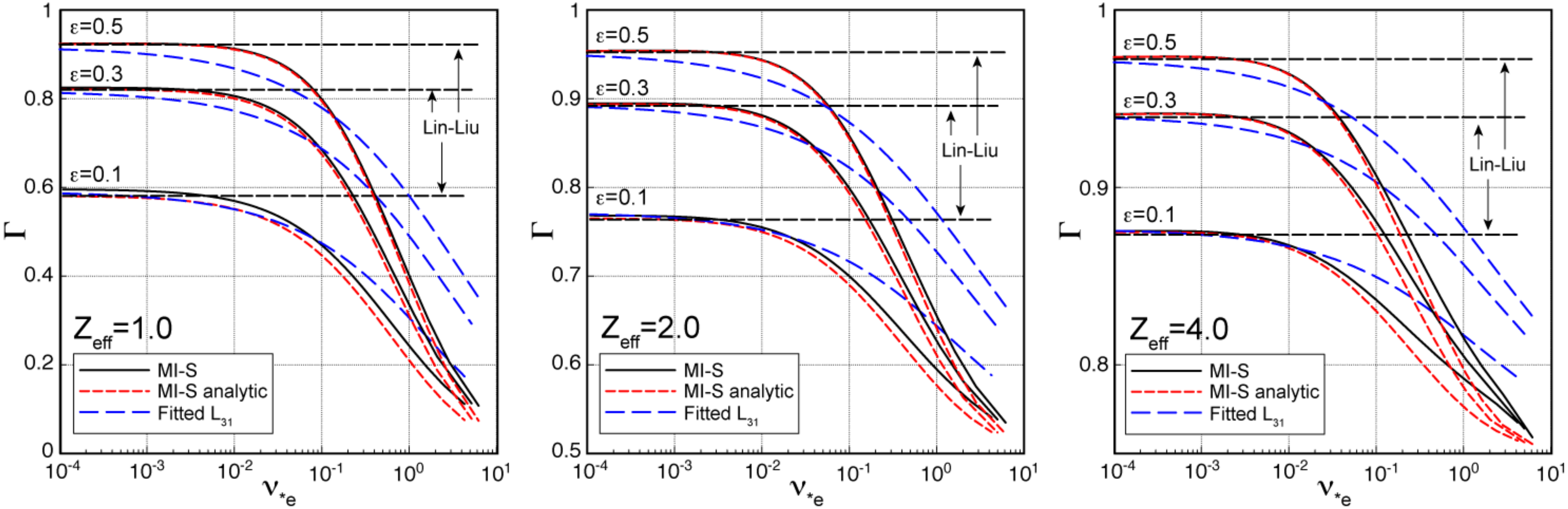


Comparison of the <u>MI-S</u>, <u>MI-S analytic</u>, <u>Lin-Liu</u> and <u>fitted L_{31}</u> models against <u>Start and Cordey</u> model

- Very good agreement among all
- Γ increases as ε or Z_{eff} increases.
  - ✓ This tendency coaxes us into employing off-axis NBI in order to maximize the NBCD.
- Even in a pure plasma (Z_{eff}=1), we have finite NBCD due to the trapped electron effect (*G*).
- Slight deviation of the MI-S model seems to be due to effects of ions.

Collisionality dependence of shielding factor models

$\nu_{*e}\to 0$, $Z_b=1$, nearly concentric circular equilibrium, $f_t=1.46\varepsilon^{1/2}-0.46\varepsilon^{3/2}$



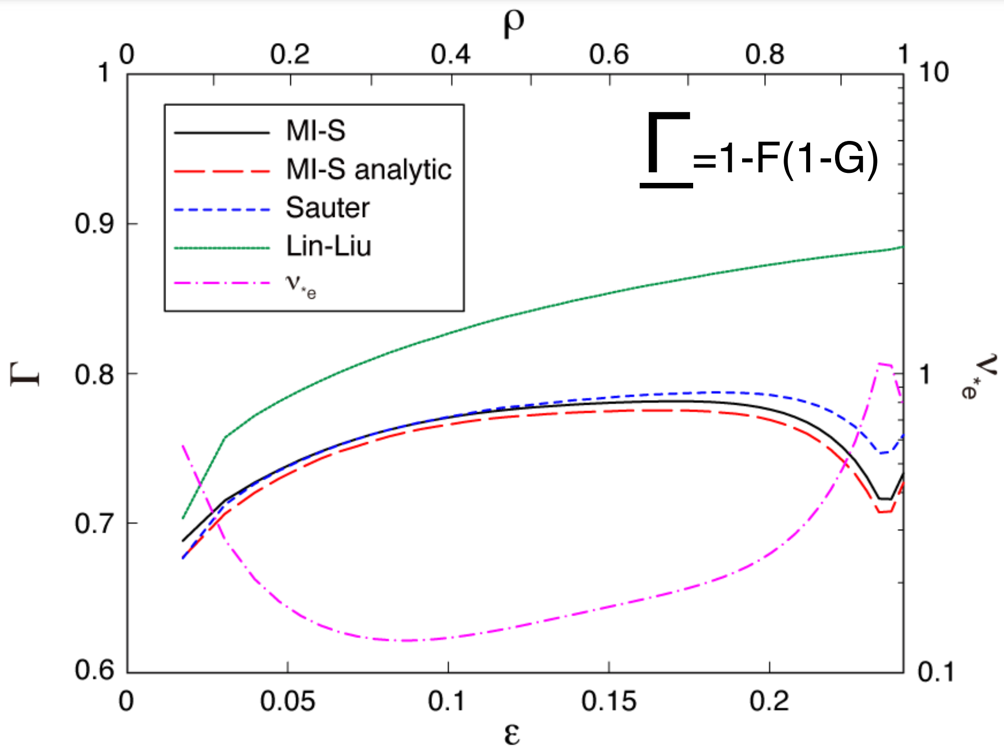
Examine the coll. dependence of the MI-S, MI-S analytic and fitted L_{31} models

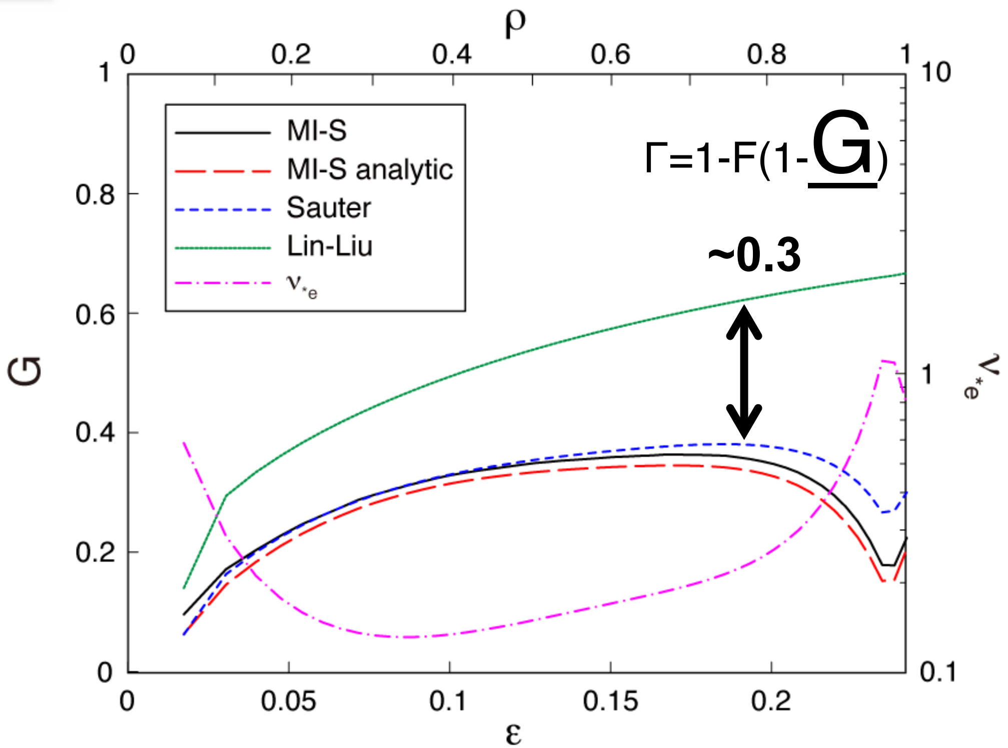
- Γ certainly converges to its collisionless value as $\nu_{*e}\to 0$.
- Γ decreases as ν_{*e} increases, especially for low Z_{eff} cases.
  - ✓ Irrespective of collisionality, the range of Γ becomes narrower as Z_{eff} increases.
- There appears some difference in the dependence of Γ.
  - ✓ The MI-S models predict that Γ is almost independent of ν_{*e} up to $\nu_{*e}\approx 10^{-2}$.

Using collisionless models does always overestimate a driven current!

JT-60U #45687 t=12.5s

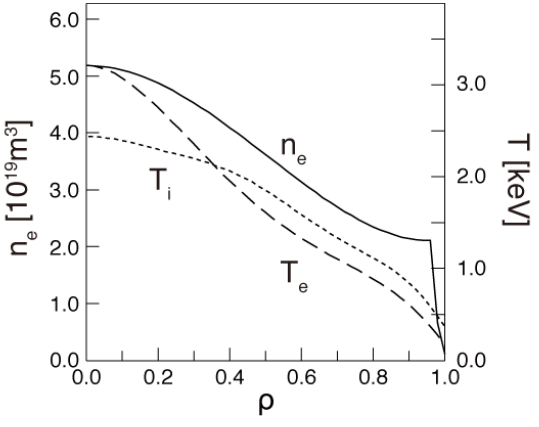
Courtesy of H. Urano

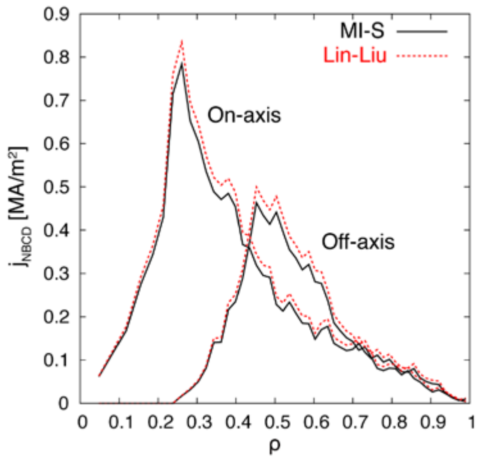




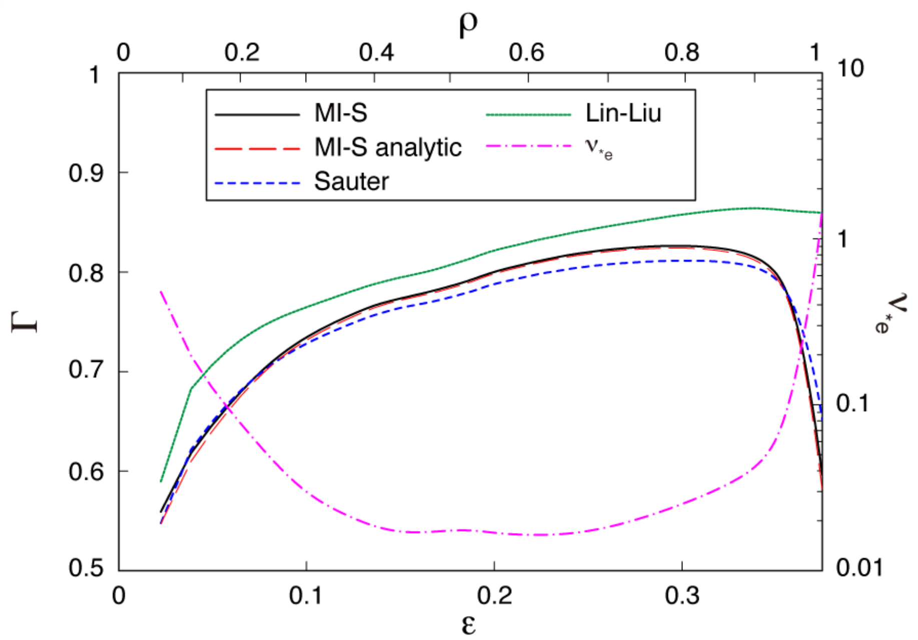
R=3.19 m, a=0.781 m, B_T=2.71 T, I_p=1.05 MA, Z_{eff}=2.84, $<n_e>$=2.90e19 m^{-3}
4 P-NBI of 85keV: bal-perp 3.94 MW, co-tang onax 1.9 MW, co-tang offax 1.84 MW

model	tot [kA]	%	onax [kA]	offax [kA]
MI-S	82.9	91.7	47.2	36.9
MI-S analytic	82.3	91.0	46.9	36.6
Sauter	83.3	92.1	47.4	37.1
Lin-Liu	90.4	100	51.3	40.4

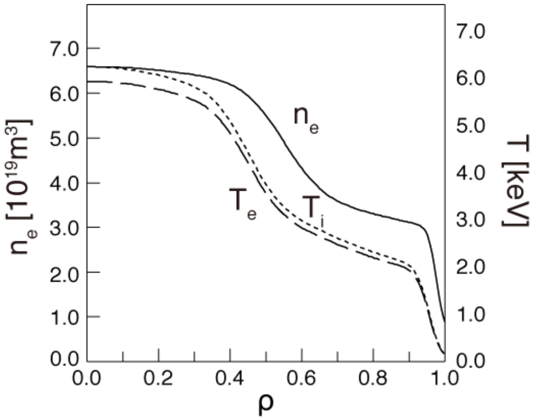




JT-60SA LSN SS 2.3MA



Courtesy of S. Ide



model	[MA]	%
MI-S	0.529	96.5
MI-S analytic	0.528	96.3
Sauter	0.523	95.4
Lin-Liu	0.548	100

R=2.95 m, a=1.12 m, B_T=1.72 T, I_p=2.3 MA

Z_{eff}=2.0, $\langle n_e \rangle$=3.03e19 m^{-3}

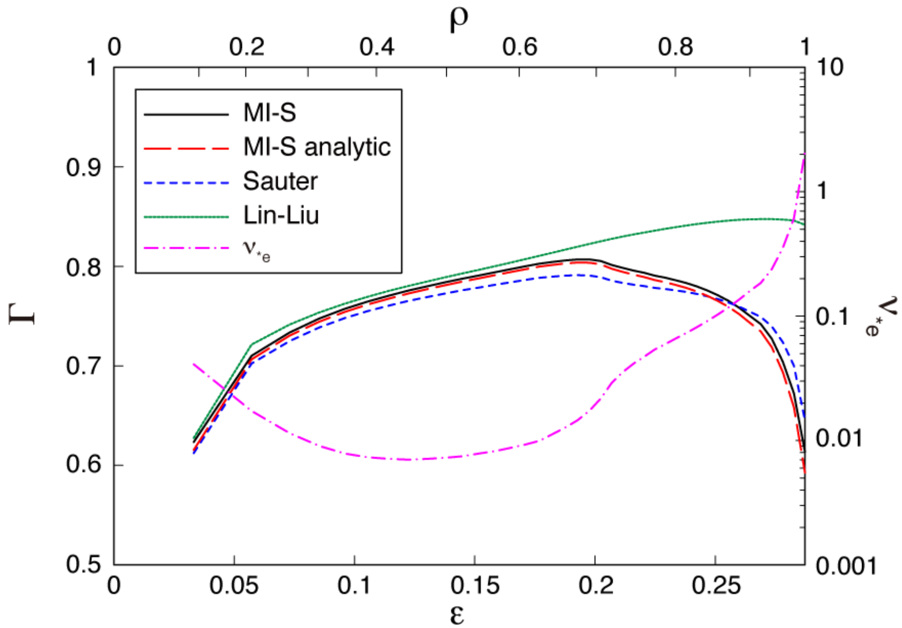
2 N-NBI of 500keV: co-tang 10 MW

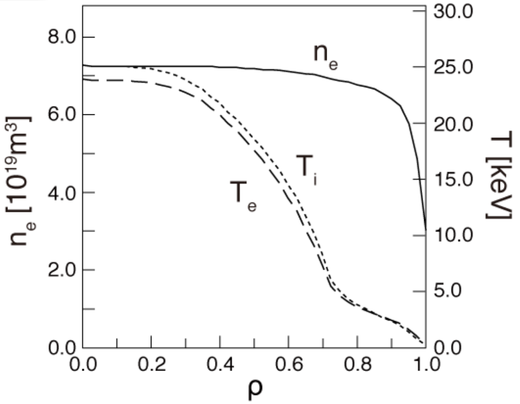
12 P-NBI of 85keV: bal. 20 MW

ITER 9MA SS DT scenario§

Courtesy of T. Oikawa

§A R Polevoi et al 2002 Proc. 19th IAEA FEC (Lyon) CT/P-08





model	[MA]	%
MI-S	1.75	99.4
MI-S analytic	1.74	98.9
Sauter	1.72	97.7
Lin-Liu	1.76	100

R=6.35 m, a=1.85 m, B_T=5.3 T, I_p=9 MA

Z_{eff}=2.17, $\langle n_e \rangle$=6.74e19 m^{-3}

1 N-NBI of 1MeV: co-tang 33 MW

Conclusions

The collisionality dependence of the NBCD shielding factor has been investigated.

- ✓ The MI-S models newly proposed can not only reproduce the collisionless shielding factor Γ but also estimate the collisionality dependent Γ.
  - ➢ The MI-S model is the only one that can incorporate effects of ions self-consistently.
  - ➢ The choice of a set of friction coefs. will alter results.
- ✓ It is found that the Sauter BS current model can be used for estimating the collisionality dependent Γ as a simple, analytic formula.
- ✓ **Collisionality always acts as decreasing the *G* factor and the resultant Γ.**
  - ➢ **The increase in ε and the decrease in Z_{eff} enhance this tendency.**
- ✓ **It is subsequently expected that this effect is emphasized when an off-axis NBI is employed rather than an on-axis NBI.**
- ✓ **The collisionality effect must be included in any shielding factor models, an effect which can partly fill the gap between expts. and calculations.**
  - ➢ **It would become prominent in current experiments rather than future burning plasmas that have lower collisionality.**

Additional materials

Basic formulae and coefs. for neoclassical parallel transport theory

mom balance eqs.

$$\left\langle \boldsymbol{B}\cdot\nabla\cdot\overset{\leftrightarrow}{\Pi}_{\rm e}\right\rangle = \langle BR_{\rm ei}\rangle + \langle BR_{\rm eb}\rangle\,,$$
$$\left\langle \boldsymbol{B}\cdot\nabla\cdot\overset{\leftrightarrow}{\Theta}_{\rm e}\right\rangle = \langle BH_{\rm ei}\rangle + \langle BH_{\rm eb}\rangle\,,$$

where

neoclassical viscous stress

$$\left\langle \boldsymbol{B}\cdot\nabla\cdot\overset{\leftrightarrow}{\Pi}_j\right\rangle = \langle B^2\rangle\left(\hat\mu_1^j\hat u_{j\theta} + \hat\mu_2^j\frac{2\hat q_{j\theta}}{5p_j}\right),$$
$$\left\langle \boldsymbol{B}\cdot\nabla\cdot\overset{\leftrightarrow}{\Theta}_j\right\rangle = \langle B^2\rangle\left(\hat\mu_2^j\hat u_{j\theta} + \hat\mu_3^j\frac{2\hat q_{j\theta}}{5p_j}\right),$$

parallel flows

$$\langle Bu_{\parallel j}\rangle = \langle BV_{1j}\rangle + \hat u_{j\theta}\langle B^2\rangle\,,$$
$$\frac{2}{5}\frac{\langle Bq_{\parallel j}\rangle}{p_j} = \langle BV_{2j}\rangle + \frac{2\hat q_{j\theta}}{5p_j}\langle B^2\rangle\,,$$

$$R_{\rm eb} = (m_{\rm e}Z_{\rm b}^2/\tau_{\rm ee})n_{\rm b}u_{\parallel\rm b}$$
$$H_{\rm eb} = -(3/2)R_{\rm eb}$$

friction forces

$$\begin{pmatrix}\langle BR_{\rm ei}\rangle\\ \langle BH_{\rm ei}\rangle\end{pmatrix} = \begin{pmatrix}\ell_{11}^{\rm ee} & -\ell_{12}^{\rm ee}\\ -\ell_{21}^{\rm ee} & \ell_{22}^{\rm ee}\end{pmatrix}\begin{pmatrix}\langle Bu_{\parallel\rm e}\rangle\\ \dfrac{2\langle Bq_{\parallel\rm e}\rangle}{5p_{\rm e}}\end{pmatrix} + \sum_j\begin{pmatrix}\ell_{11}^{{\rm e}j} & -\ell_{12}^{{\rm e}j}\\ -\ell_{21}^{{\rm e}j} & \ell_{22}^{{\rm e}j}\end{pmatrix}\begin{pmatrix}\langle Bu_{\parallel j}\rangle\\ \dfrac{2\langle Bq_{\parallel j}\rangle}{5p_j}\end{pmatrix}$$

approx. friction coefs.

$$\ell_{11}^{\rm ee} = -\frac{m_{\rm e}n_{\rm e}}{\tau_{\rm ee}}Z_{\rm eff},$$
$$\ell_{11}^{{\rm e}j} = \frac{m_{\rm e}n_{\rm e}}{\tau_{\rm ee}}\frac{Z_j^2n_j}{n_{\rm e}} = -\ell_{11}^{\rm ee}\frac{Z_j^2n_j}{Z_{\rm eff}n_{\rm e}},$$
$$\ell_{12}^{\rm ee} = \ell_{21}^{\rm ee} = \frac{3}{2}\ell_{11}^{\rm ee},$$

$$\ell_{12}^{{\rm e}j} = 0,$$
$$\ell_{21}^{{\rm e}j} = \frac{3}{2}\ell_{11}^{{\rm e}j},$$
$$\ell_{22}^{\rm ee} = -\frac{m_{\rm e}n_{\rm e}}{\tau_{\rm ee}}\left(\sqrt{2}+\frac{13}{4}Z_{\rm eff}\right) = \frac{\ell_{11}^{\rm ee}}{Z_{\rm eff}}\left(\sqrt{2}+\frac{13}{4}Z_{\rm eff}\right),$$
$$\ell_{22}^{{\rm e}j} = 0.$$

Shaing's neoclassical viscosity model

$$\begin{Bmatrix} \hat{\mu}_1 \\ \hat{\mu}_2 \\ \hat{\mu}_3 \end{Bmatrix} = \frac{8}{3\pi^{1/2}} \int_0^\infty \mathrm{d}x\, x^4 \exp(-x^2) \begin{Bmatrix} 1 \\ (x^2 - \frac{5}{2}) \\ (x^2 - \frac{5}{2})^2 \end{Bmatrix} \frac{K_\mathrm{B} K_\mathrm{PS}}{K_\mathrm{B} + K_\mathrm{PS}},$$

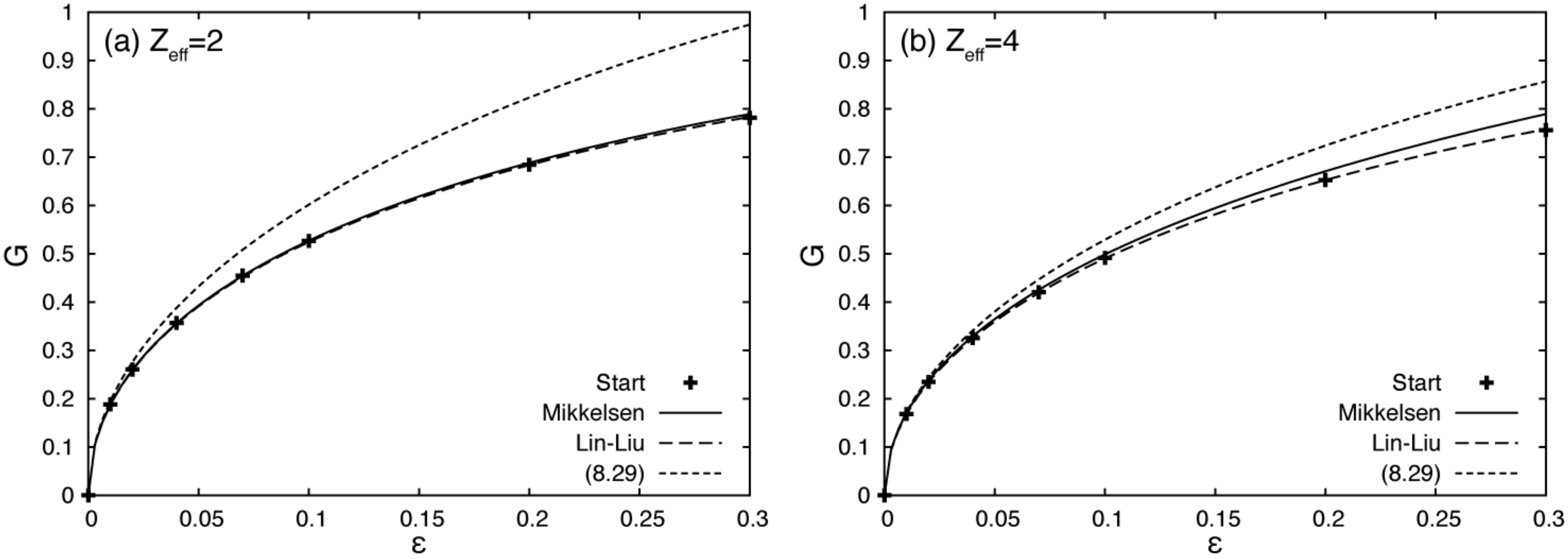
where

$$\begin{aligned}
K_\mathrm{B} &\equiv g \frac{\nu_\mathrm{D}}{S^{3/2}}, \\
K_\mathrm{PS} &\equiv \frac{3}{2} v_\mathrm{T}^2 x^2 \sum_{m=1}^{\infty} F_m \frac{\nu_T I_R^m}{\nu_T}, \\
F_m &\equiv \frac{2}{\langle B^2 \rangle \langle \boldsymbol{B} \cdot \nabla \theta \rangle} [\langle (\sin m\Theta)(\boldsymbol{n} \cdot \nabla B) \rangle \langle (\sin m\Theta)(\boldsymbol{B} \cdot \nabla \Theta)(\boldsymbol{n} \cdot \nabla B) \rangle \\
&\quad + \langle (\cos m\Theta)(\boldsymbol{n} \cdot \nabla B) \rangle \langle (\cos m\Theta)(\boldsymbol{B} \cdot \nabla \Theta)(\boldsymbol{n} \cdot \nabla B) \rangle], \\
\nu_T I_R^m &\equiv -\frac{3}{2} \left(\frac{\nu_T}{\omega_m} \right)^2 - \frac{9}{2} \left(\frac{\nu_T}{\omega_m} \right)^4 \\
&\quad + \left\{ \frac{1}{4} + \left[\frac{3}{2} + \frac{9}{4} \left(\frac{\nu_T}{\omega_m} \right)^2 \right] \left(\frac{\nu_T}{\omega_m} \right)^2 \right\} \frac{2\nu_T}{\omega_m} \tan^{-1} \left(\frac{\omega_m}{\nu_T} \right), \\
\omega_m &\equiv x v_\mathrm{T} (m \boldsymbol{n} \cdot \nabla \Theta), \\
\Theta(l_p) &\equiv \gamma \int_0^{l_p} \mathrm{d}l'_p \frac{B}{B_p}, \\
\gamma &= 2\pi \left(\int_0^{L_p} \mathrm{d}l'_p \frac{B}{B_p} \right)^{-1}.
\end{aligned}$$

The original moment approach uses the energy-space partitioning method to express the neoclassical viscosities.

Shaing proposed more appropriate analytic expression for viscosities in finite aspect ratio, which can reproduce all the asymptotic collisionality limits.

Comparison of the existing collisionless shielding factor models



Z_b=1, the model magnetic field $B \cong B_\phi = B_0/(1+\varepsilon\cos\theta)$, $f_t=1.46\varepsilon^{1/2}-0.46\varepsilon^{3/2}$

- Start and Cordey model [PoF 1980]
- Mikkelsen and Singer model [Nucl. Technol./Fusion 1983]
  - Fitted formula of the tabulated values of Start and Cordey
- Lin-Liu and Hinton model [PoP 1987]
- Eq. (8.29) in the review paper of Hirshman and Sigmar [NF 1981]

Possible reasons that produce the discrepancy between the MI-S and fitted L_{31} models

① Different approaches to obtain the shielding factor

② $Z=Z_{\mathrm{eff}}$ assumption exploited in the fitted L_{31} model may not be appropriate for some cases. This was pointed out in the original paper [Sauter PoP 1999].

③ The fitted L_{31} formula is a numerically-fitted function of simply ν_{*e}, f_t and Z and the applicability of the formula is not explicitly specified in the paper. When considering the collisional regime where $\nu_{*e} \gg \varepsilon^{-3/2}$, to leading order we have

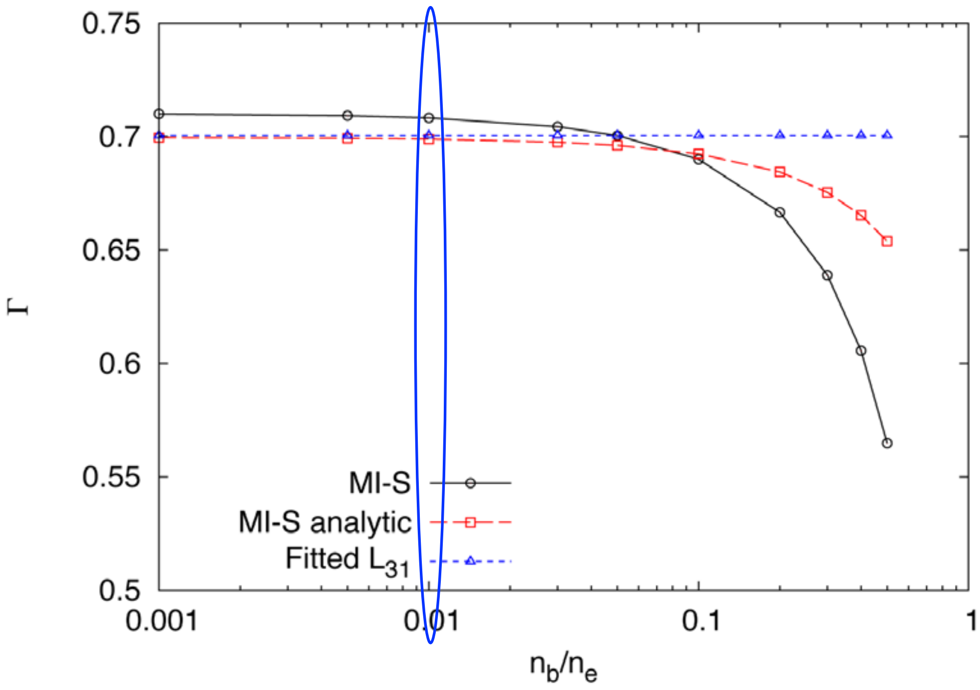
$$L_{31} \sim Zg\nu_{*e}^{-1} = Z\frac{f_t}{f_c}\nu_{*e}^{-1}.$$

For the moment approach, *g* does not participate in any transport coefs. in the collisional regime, because trapped particles no longer exist due to frequent collisions in this regime.

④ CQLP code uses the full, linearized collision operator, while in the collisional regime MI-S adopts the Shaing's viscosity model that was derived from a linearized DK eq. with a Krook operator [PoF B 1990, PoP 1996]. The discrepancy in the operator may cause about 20% error. In the banana regime, it does not matter.

An advantage of the MI-S model is to be able to include effects of ions.

So far, we have assumed n_b/n_e=0.01 for all simulations.



at ε=0.3
$\nu_{*e}\approx$0.076

- Using the current expressions of beam friction forces implicitly assumes n_b/n_e<<1.
- It is apparent that the MI-S models includes n_i and n_b inside, whereas the fitted L_{31} model does not.
- As n_b/n_e increases, Γ gradually decreases.
  - This tendency can be reproduced irrespective of collisionality, even in the collisionless limit.
- This decrease in Γ is due to the decrease in viscosities through the decreases in the 90° deflection frequency.