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2D full wave analysis of wave structure by TASK/WF2

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Motivation
Formulation
Code validation
Analysis of wave structure in LATE
Summary

1. Motivation

- For spherical tokamak (ST), the use of electron cyclotron heating and current drive (ECH/ECCD) is planned in order to start up plasma and ramp up current without central Ohmic solenoid.

- In the heating and current ramp up process of ST, however, the plasma density becomes higher and we cannot heat the central part of plasma by EC waves due to the wave cutoff.

- Owing to the finite Larmor radius (FLR) effects, the EC waves can be converted into electron Bernstein (EB) waves near the upper hybrid resonance (UHR) layer.

- The EB waves can propagate into higher-density region and heat the central part of plasma.

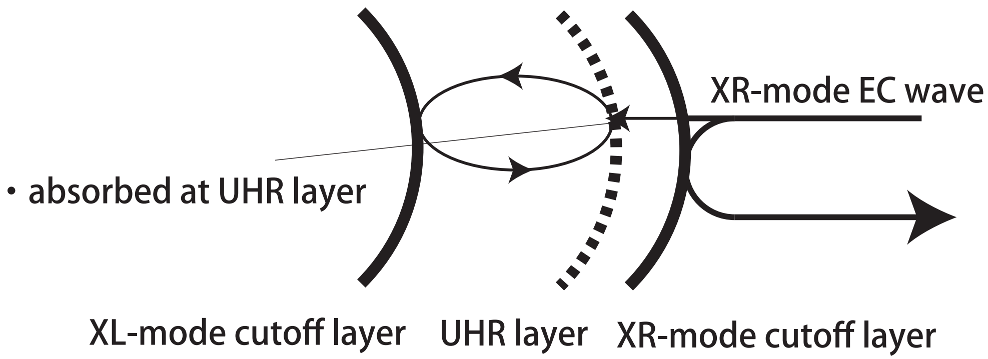
- **Final purpose of this analysis:**

  quantitative analysis of propagation and absorption of EC and EB waves in a high-density ST

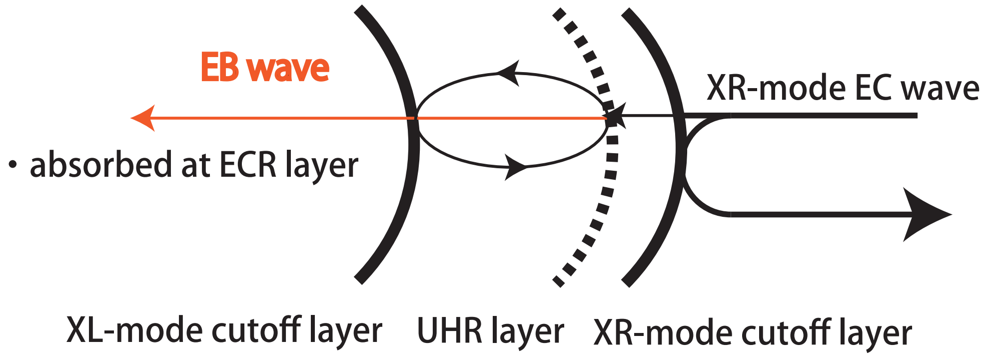
2. Mode conversion

- perpendicular XR-mode injection

Cold Plasma (without FLR effects)

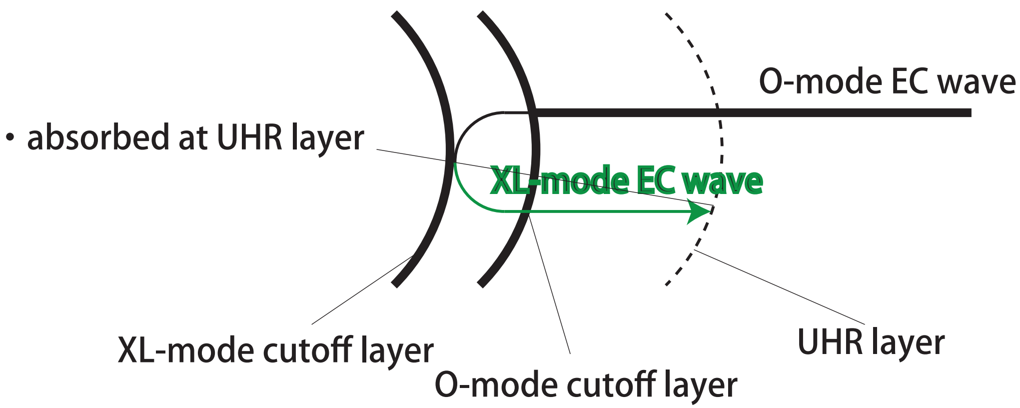


Hot Plasma (with FLR effects)

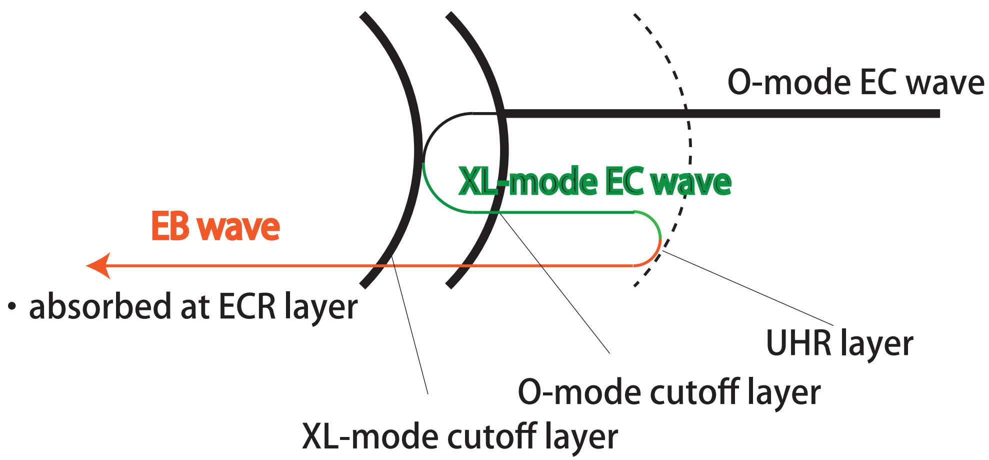


- non-perpendicular O-mode injection

Cold Plasma (without FLR effects)



Hot Plasma (with FLR effects)



3. High resolution full wave analysis

- For high density STs, **full wave analysis is necessary** owing to the existence of evanescent layers and mode conversion to EB waves.
- EB wave have very large $k_\perp$ ($k_\perp$: the wave number perpendicular to the static magnetic field) and **we need high spacial resolution in a 2D plane**.
- We have developed the 2D full wave code using finite element method (FEM) with mixed basis functions, TASK/WF2.

4. Formulation for TASK/WF2

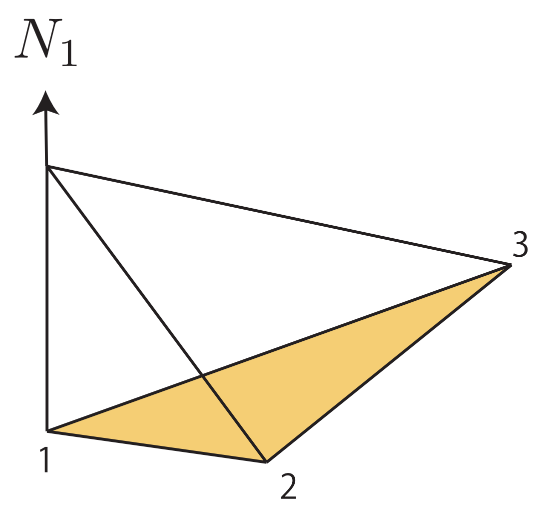
- **Wave electric field**: $\tilde{\boldsymbol{E}}(\boldsymbol{r},t) = \boldsymbol{E}(\boldsymbol{r})\mathrm{e}^{i\omega t}$
- **Used coordinate**: Cylindrical coordinate (r,ϕ,z)
- We expand the wave electric field to a Fourier series in the toroidal direction ϕ as

$$\begin{aligned}\boldsymbol{E}(r,\phi,z) &= \sum_{n_\phi=-\infty}^{\infty} \boldsymbol{E}_{n_\phi}(r,z)e^{in_\phi\phi} \\ &= \sum_{n_\phi=-\infty}^{\infty} \{E_{rn_\phi}(r,z)\hat{\boldsymbol{r}} + E_{\phi n_\phi}(r,z)\hat{\boldsymbol{\phi}} + E_{zn_\phi}(r,z)\hat{\boldsymbol{z}}\}\mathrm{e}^{in_\phi\phi}.\end{aligned}$$

- **Boundary condition on the edge of an element**:
  - **Normal component** can be **discontinuous**.
  - **Tangential component** is **always continuous**.

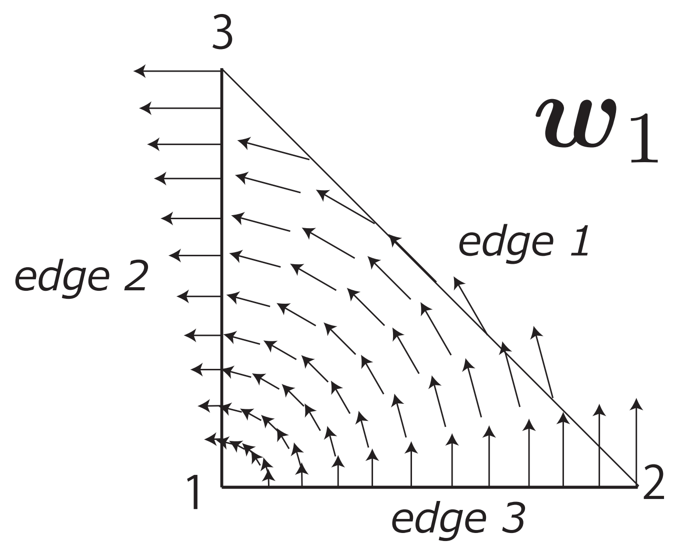
- We use scalar and vector basis functions in order to satisfy the boundary condition.

Scalar basis function



- $N_1 = 1$ on the node 1.
- $N_1 = 0$ on the other nodes.

Vector basis function



- component tangential to the edge 1 is 1 on the edge 1.
- component normal to the edge 2 is 0 on the edge 2.
- component normal to the edge 3 is 0 on the edge 3.
- the magnitude of $\boldsymbol{w}_1$ is 0 on the node 1.

- **Discretization of $\boldsymbol{E}(\boldsymbol{r})$**:

$$\boldsymbol{E}(\boldsymbol{r}) = \left(\sum_{i=1}^{3} E_{\mathrm{p}i} \boldsymbol{w}_i(r,z) + \sum_{i=1}^{3} E_{\mathrm{t}i} N_i(r,z) \hat{\boldsymbol{\phi}} \right) \mathrm{e}^{i n_\phi \phi}$$

E_{pi}	:	the tangential component of $\boldsymbol{E}$ on the edge i
E_{ti}	:	the toroidal component of $\boldsymbol{E}$ on the node i

- We consider a particular toroidal mode n_ϕ.

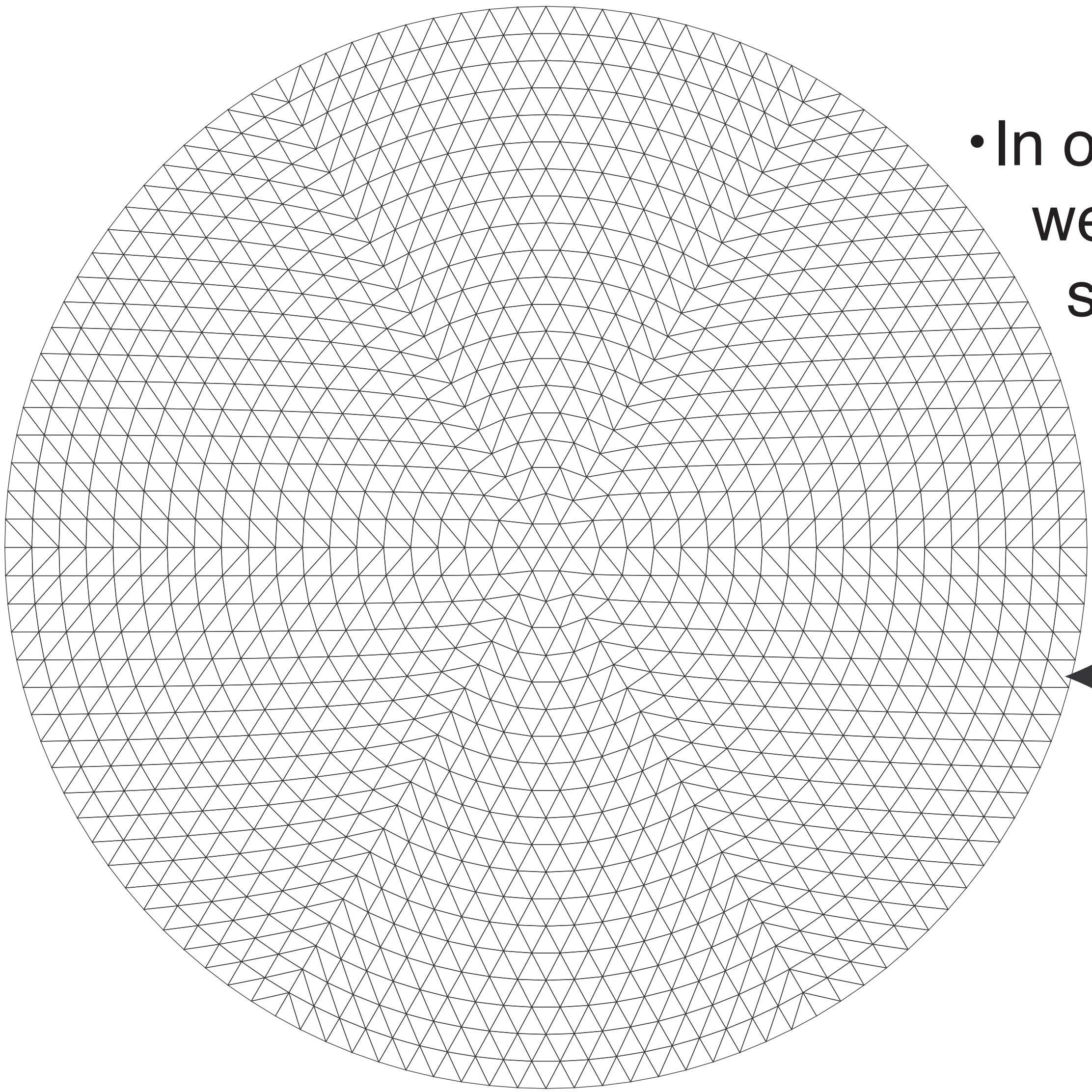
- We use weighted residual method and Galarkin's method.

- **Weak form of the wave equation**:

$$\int_V (\nabla \times \boldsymbol{F}) \cdot (\nabla \times \boldsymbol{E}) \mathrm{d}V + \int_S (\boldsymbol{F} \times \nabla \times \boldsymbol{E}) \cdot \boldsymbol{n} \mathrm{d}S$$
$$= \frac{\omega^2}{c^2} \int_V (\boldsymbol{F} \cdot \overleftrightarrow{\epsilon} \cdot \boldsymbol{E}) \mathrm{d}V + i\omega\mu_0 \int_V \boldsymbol{F} \cdot \boldsymbol{J}_{\mathrm{ext}} \mathrm{d}V$$

$\boldsymbol{F}$	:	weight function
$\overleftrightarrow{\epsilon}$	:	dielectric tensor
$\boldsymbol{J}_{\mathrm{ext}}$	:	external current

5. Mesh



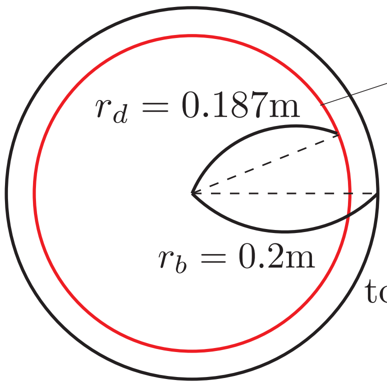
- In order to compare with TASK/WM, we use a circular poloidal cross section in this poster.

- For TASK/WF2, we divide mesh almost evenly.

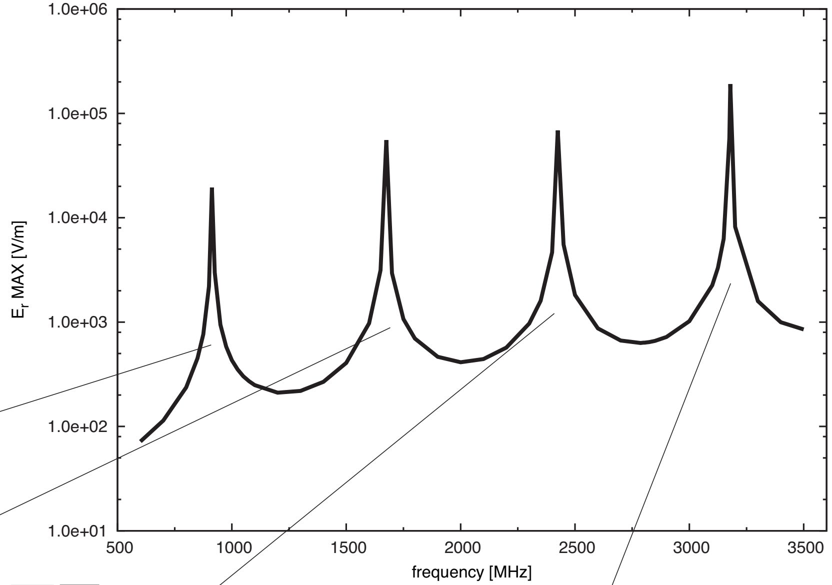
Mesh example: 1261 nodes, 2400 elements

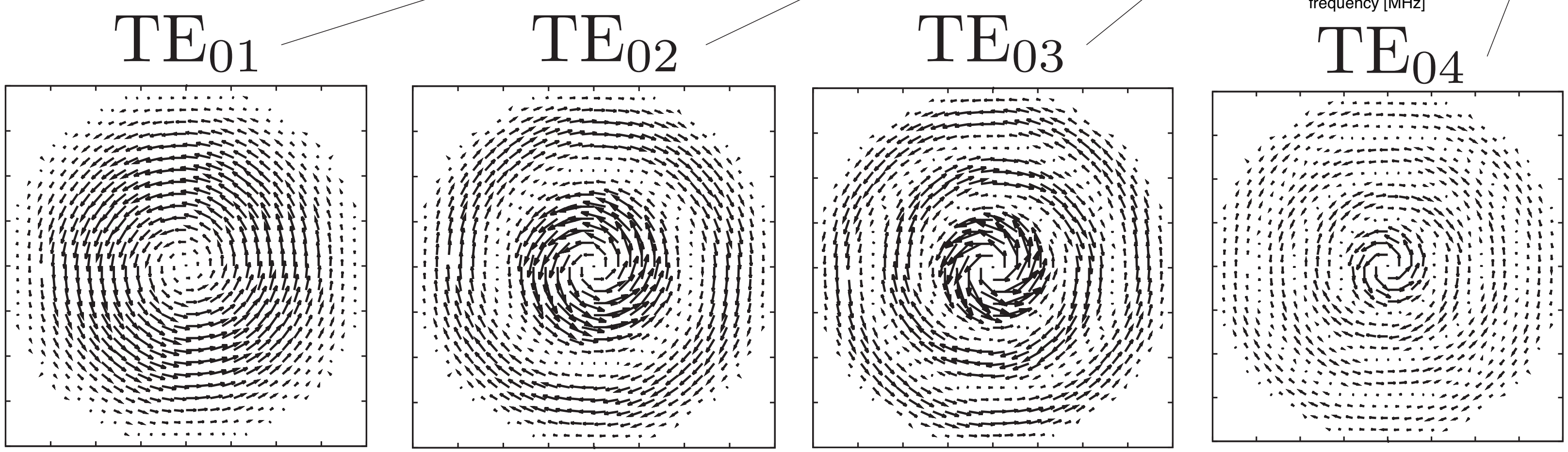
6. Eigenmodes of circular waveguide

- Antenna & used parameters



major radius	R	$10^4\mathrm{m}$
minor radius	a	$0.16\mathrm{m}$
toroidal mode number	n_ϕ	0

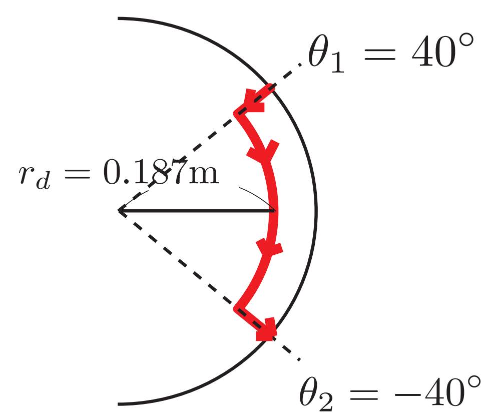




	TE_{01}	TE_{02}	TE_{03}	TE_{04}
Analytic	914.2MHz	1674MHz	2427MHz	3179MHz
Numerical	912.5MHz	1675MHz	2425MHz	3179MHz

- We confirmed that TASK/WF2 can analyze eigenmodes of circular waveguide.

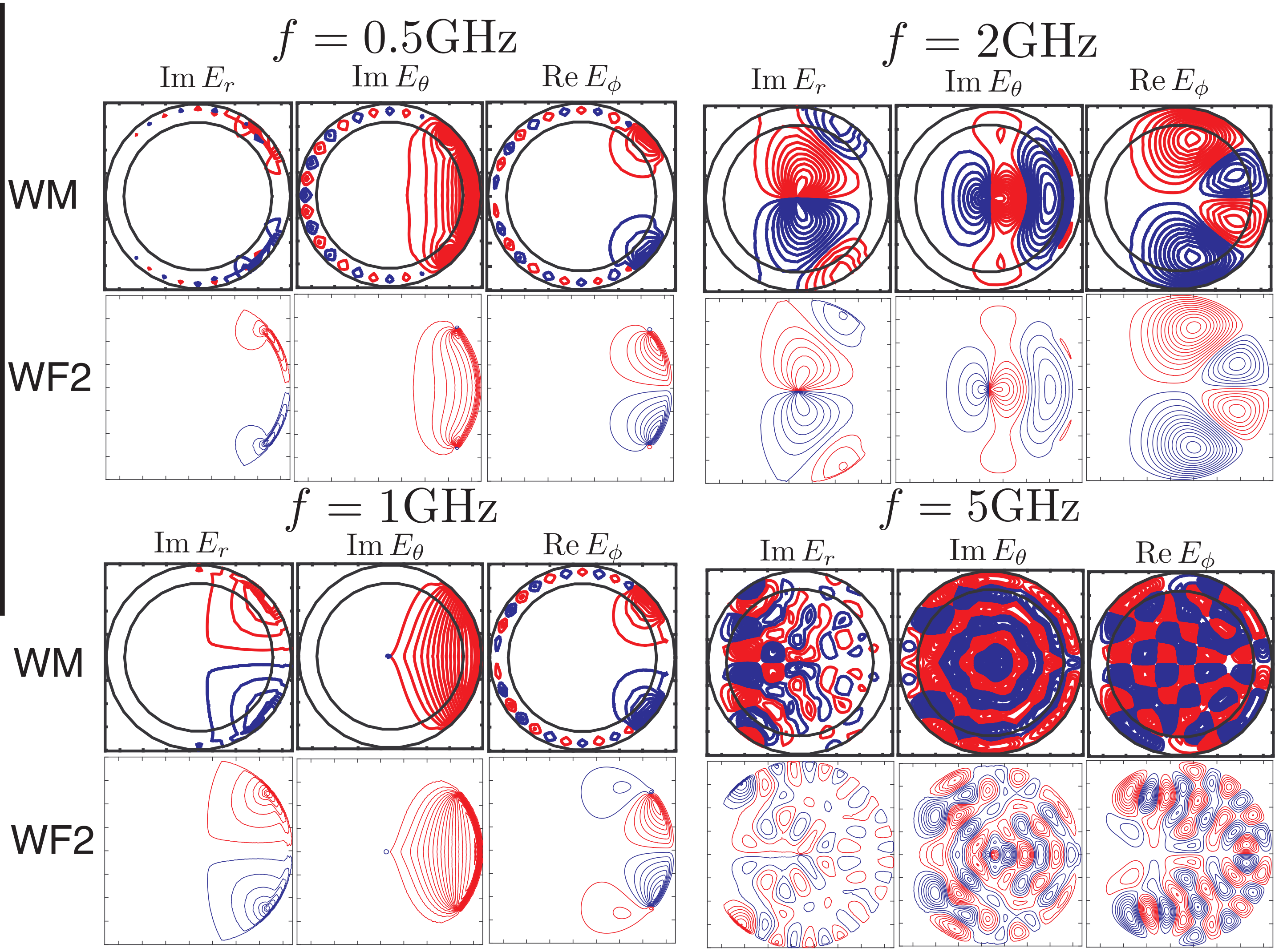
7. Benchmark test with TASK/WM



major radius	R	0.22m
minor radius	a	0.16m
toroidal mode number	n_ϕ	8

Resolution

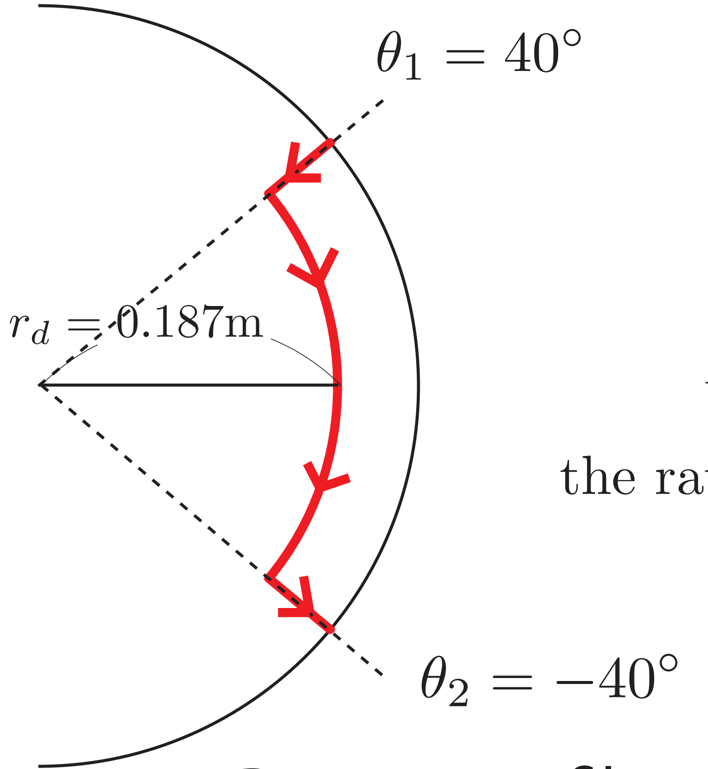
- WF2: 25350 elements
- WM:
  - radial direction: 100 points
  - poroidal direction: 32 modes



- TASK/WM and WF2 have different spacial resolution, so there are some differences in their results.

8. Antenna and used parameters

- Antenna

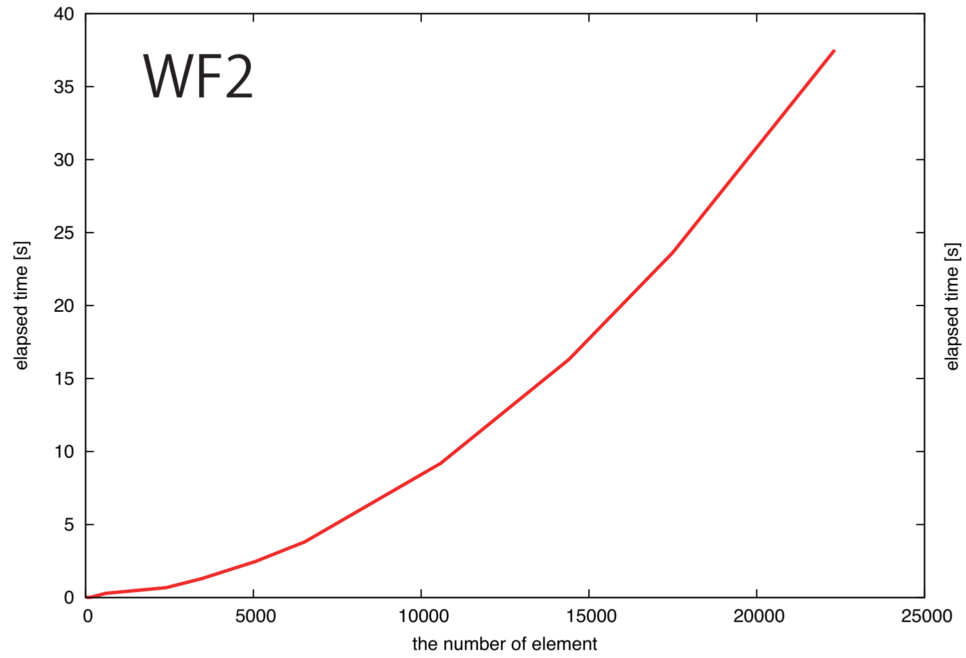


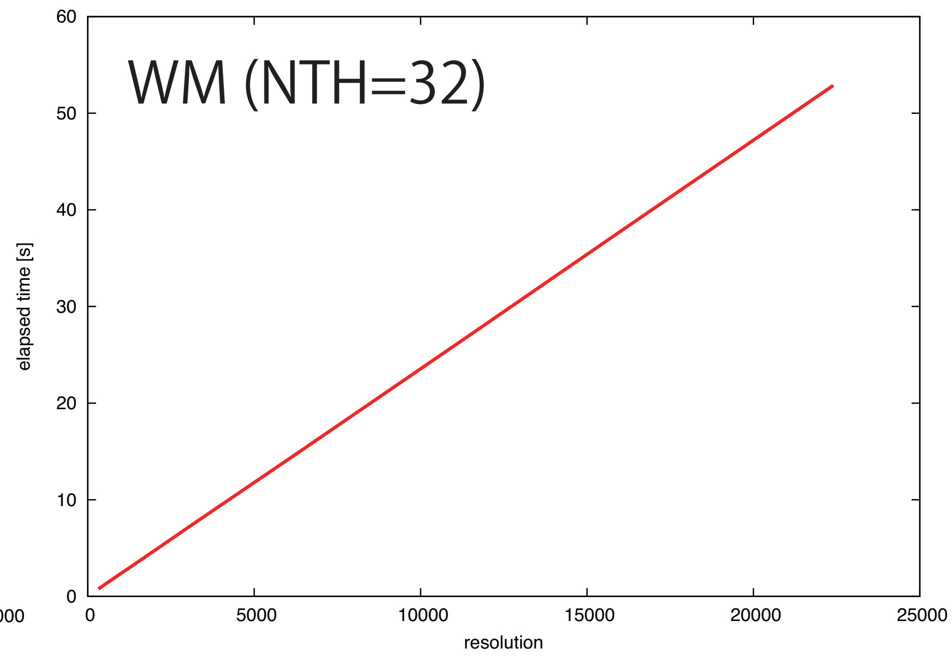
- Used parameters (**LATE**: Kyoto University)

major radius	R	0.22m
minor radius	a	0.16m
wave frequency	f	5GHz
the number of elements	NEMAX	25350
the ratio of collision frequency to ω	ν/ω	1.0×10^{-3}

- Density profile is parabolic.
- Collisional cold plasma model is used for the dielectric tensor.
  - Mode conversion of EC wave into EB wave does not occur.

9. Computational time (one core)



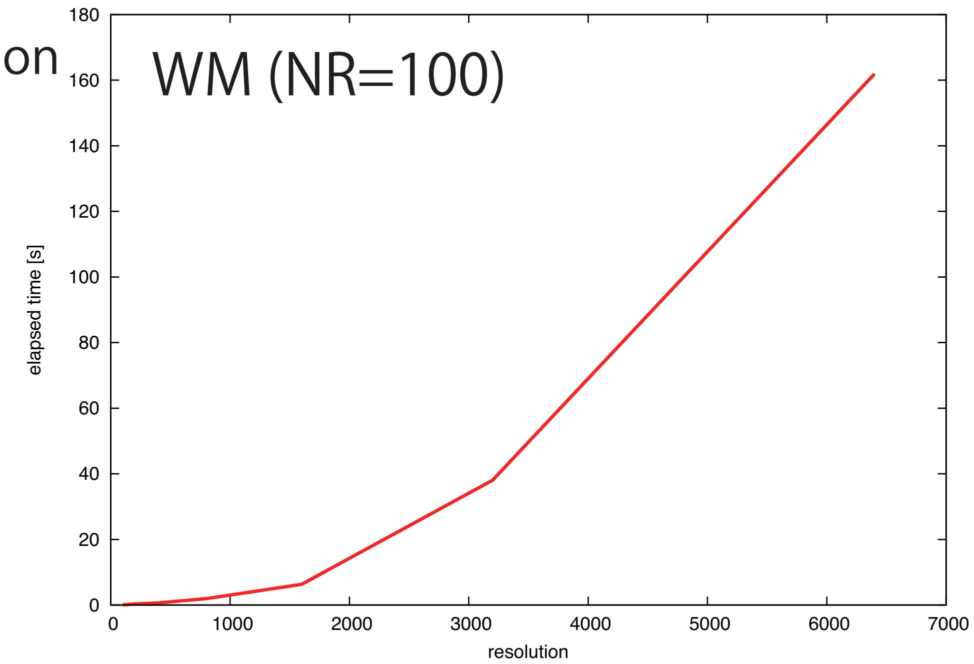


- NR: the number of points in radial direction
- NTH: maximam poloidal mode number

- For WM, resolution is defined as

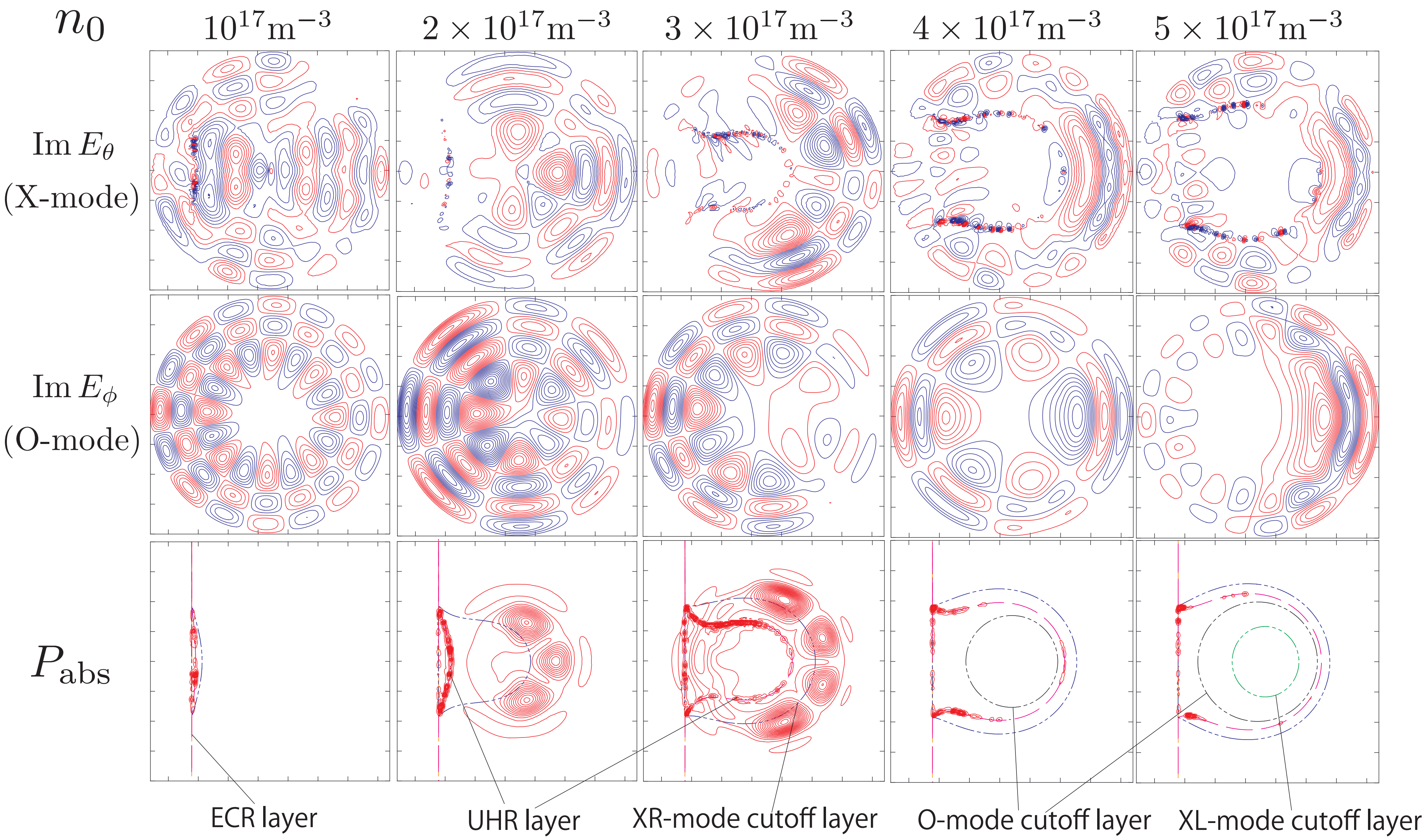
  resolution = NR×NTH

- In order to obtain high resolution in poloidal direction, WM requires longer computation time than WF2.



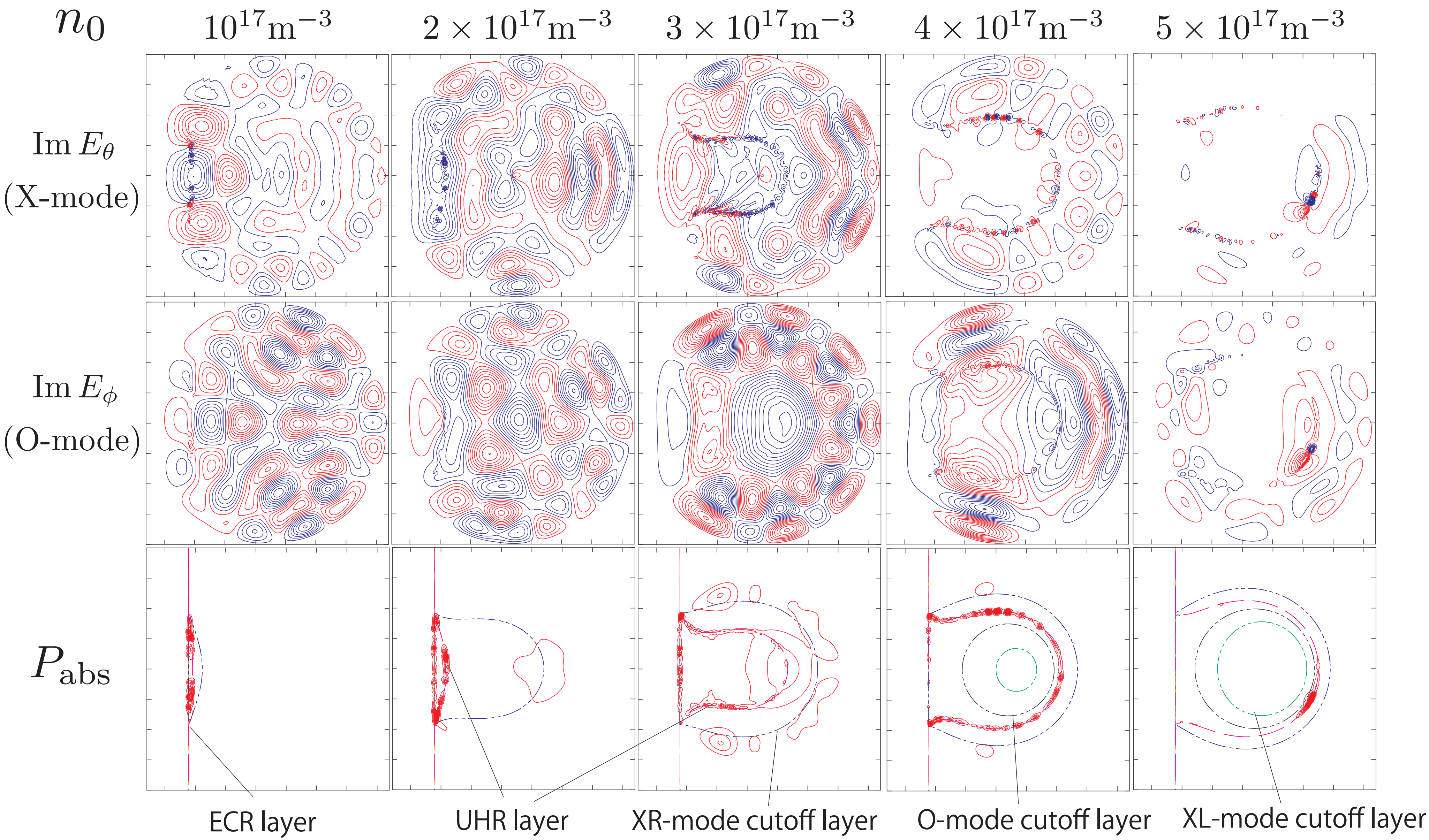
10. Density dependence ($n_\phi = 0$)

- $B_0 = 0.072$T



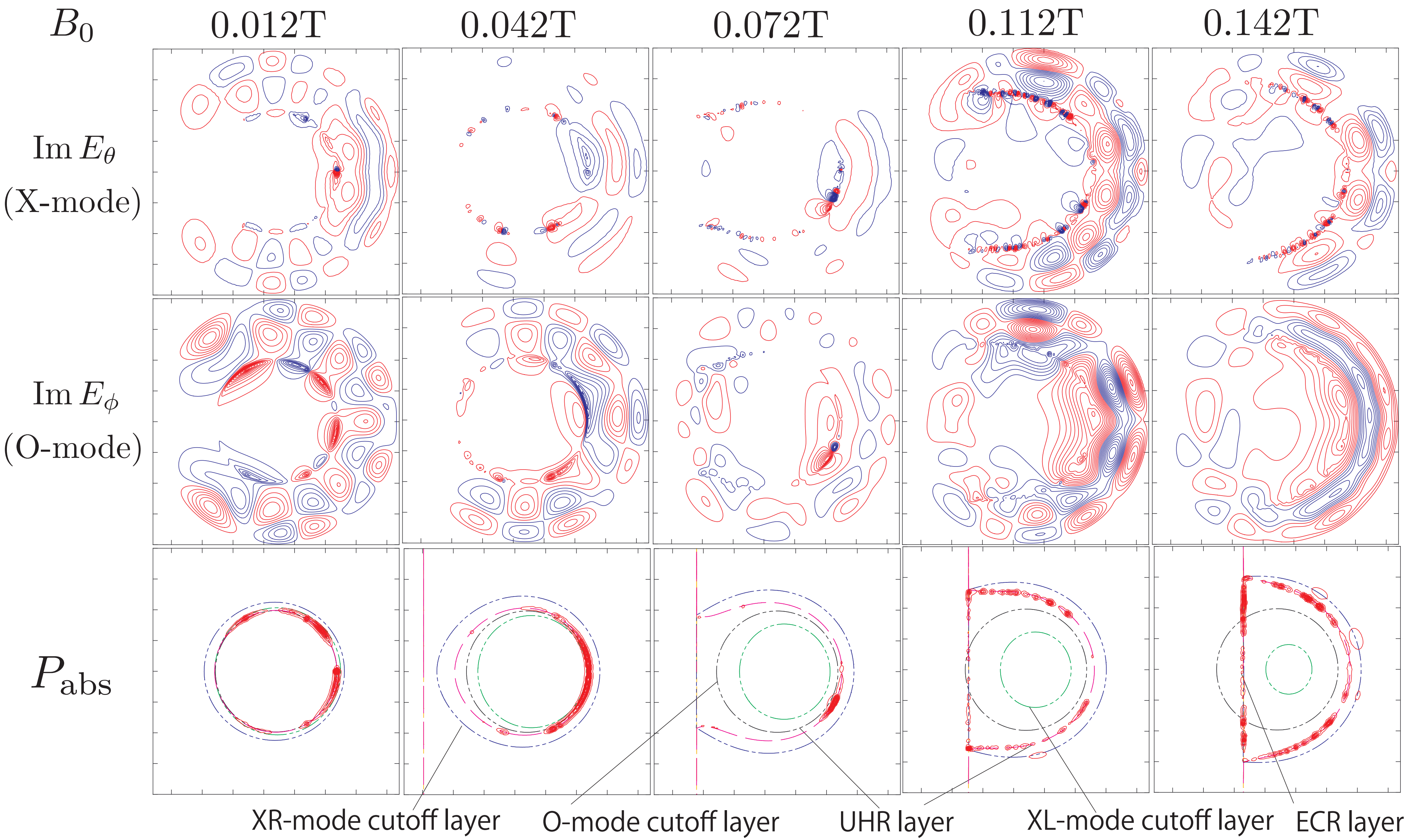
11. Density dependence ($n_\phi = 8$)

- $B_0 = 0.072$T

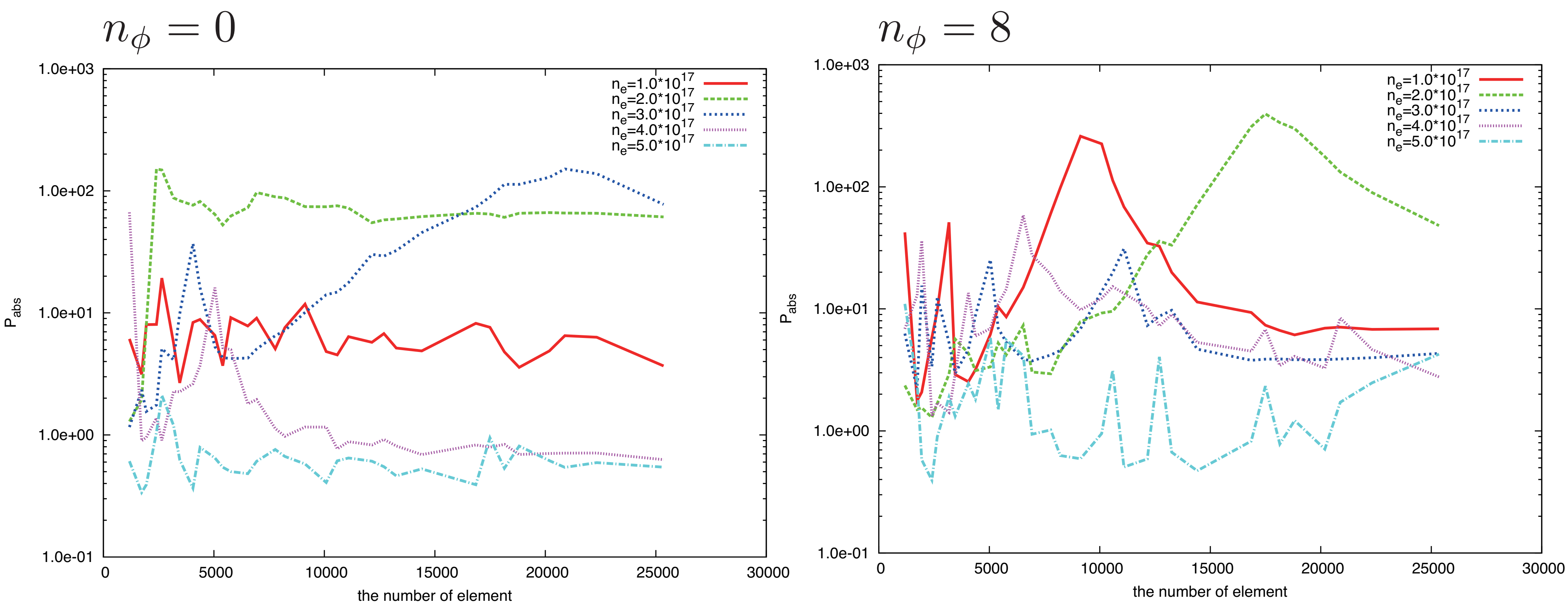


12. Magnetic field dependence

- $n_0 = 5 \times 10^{17}\text{m}^{-3}$



13. Numerical convergence



- Total absorbed power P_{tabs} is used as an index of convergence.
- In the present we use first order element. In order to obtain higher convergence, higher order element is probably required.

14. Summary

- In order to obtain high resolution in poroidal cross section, we have developed 2D full wave code using FEM, TASK/WF2.
- We analyzed eigenmodes of circular waveguide for n_ϕ and confirmed that the numerical solution is consistent with the analytic solution.
- We compared EC wave propagation in vacuum analyzed by TASK/WF2 to the one analyzed by TASK/WM and confirmed its conssistency
- We analyzed wave propagation in LATE and presented density dependence and magnetic field dependence of wave propagation.
- Owing to short wavelength wave converted near the UHR layer, numerical solution was not converged for 25000 elements.
- **Remaining issues**:
  - Waveguide excitation
  - Formulation and implementation of kinetic dielectric tensor
  - Optimization for parallel computing
  - Use of higher order element