Renovation of diffusive transport code TASK/TR

Kyoto Universtiy Department of Nuclear Engeneering,

Tomoaki IKARI, Atsushi FUKUYAMA

The new TASK/TR

Basic equation

The basic equations still remain 1D diffusive transport equations; density, energy and magnetic diffusion eqautions.

$$\frac{\partial}{\partial t}(n_s V') = -\frac{\partial}{\partial \rho}(V'\Gamma_s) + S_s V'$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2} n_s T_s V'^{5/3}\right) = -V'^{2/3}\frac{\partial}{\partial \rho}(V' Q_s) + S_{Es} V'^{5/3}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial \psi}{\partial \rho}\right) = \frac{\partial}{\partial \rho}\left[\frac{\eta_\parallel}{\mu_0}\frac{I}{V'\langle R^{-2}\rangle}\frac{\partial}{\partial \rho}\left(\frac{V'}{I}\left\langle\frac{|\nabla\rho|^2}{R^2}\right\rangle\frac{\partial\psi}{\partial\rho}\right) - \frac{\eta_\parallel}{I\langle R^{-2}\rangle}\langle(J_{CD}+J_{BS})B\rangle\right]$$

Fluxes are:

$$\Gamma_s = \langle|\nabla\rho|\rangle n_s V_s - \langle|\nabla\rho|^2\rangle D_s \frac{\partial n_s}{\partial \rho}$$

$$Q_s = \frac{3}{2}\langle|\nabla\rho|\rangle n_s T_s V_{Es} - \langle|\nabla\rho|^2\rangle \chi_s \frac{\partial (n_s T_s)}{\partial \rho} - \langle|\nabla\rho|^2\rangle\left(\frac{3}{2}D_s - \chi_s\right) T_s \frac{\partial n_s}{\partial \rho}$$

D: particle diffusion coef.   *χ*: thermal diffusion coef.
ρ: norm. radius   < > denotes the flux surface average.

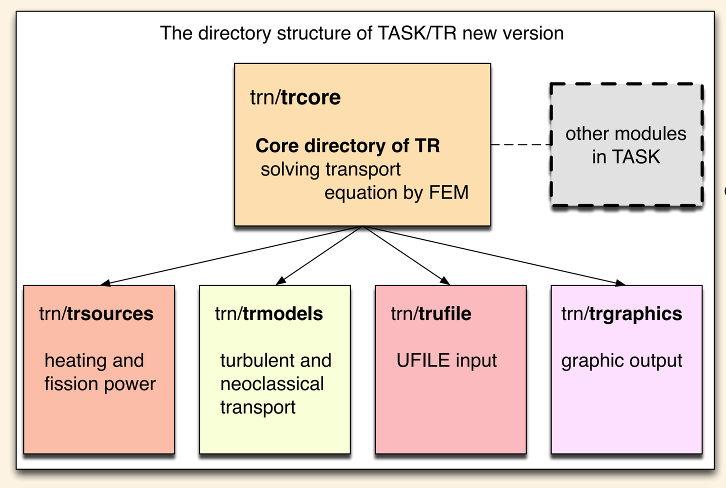
Finite Element Method

- The core part of caluculation has been written based on FEM from scratch.
- FEM makes it easy to vary the width between each grid point.

Multi species

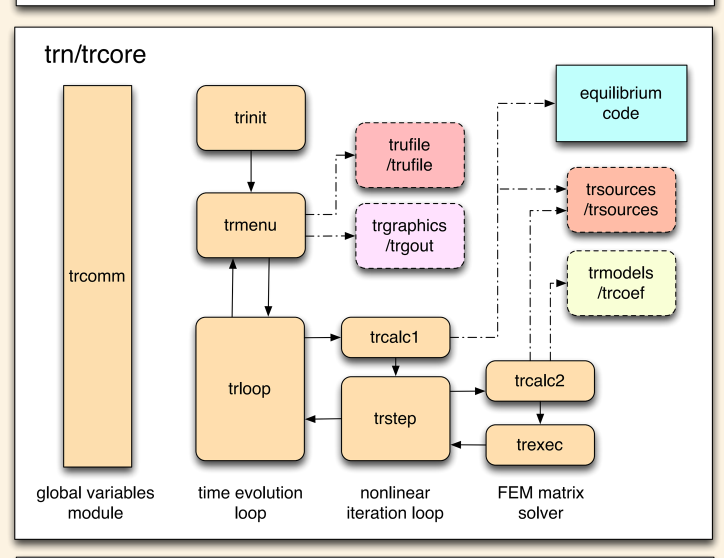
- New version can treat electron, ions, impurities, neutrals in the same manner.
- Fast and slow particles are regarded as different particle species even if they are the same ones.

Directory structure



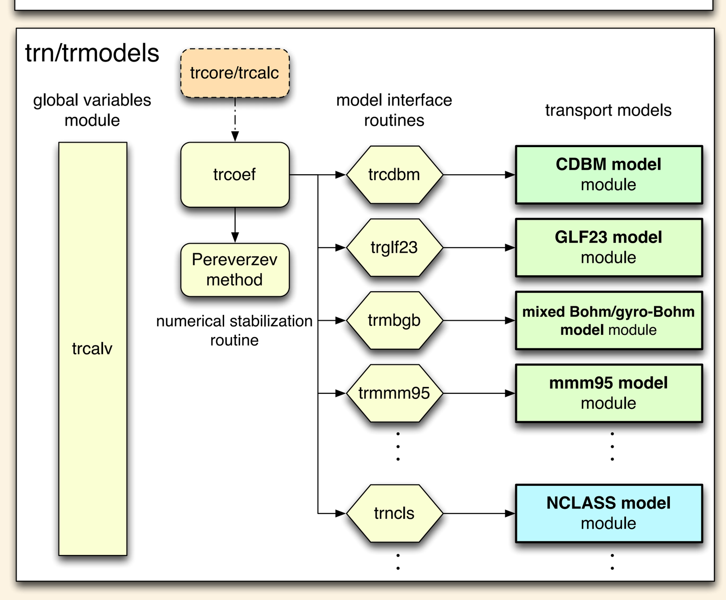
overall view

- Source codes are seperated into 5 dirs. according to their functions.



the core directory

- Global variables module
- Main execution part consists of trloop, trstep and trexec.



the directory of transport models

- Several models can be called in the same manner through the interface routines.

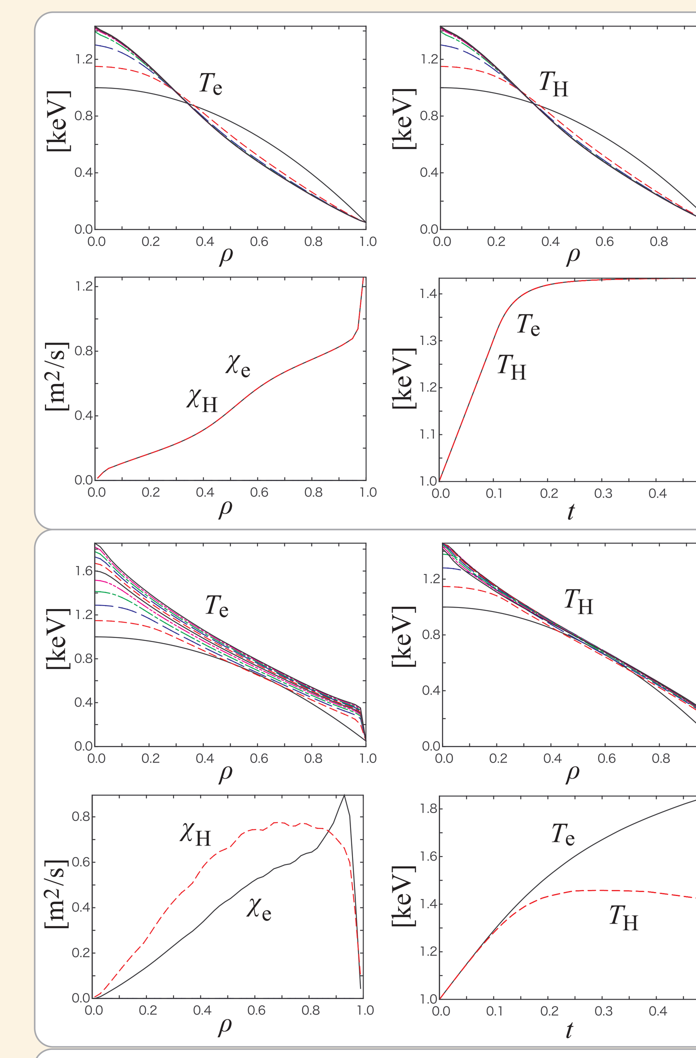
And others

- Portable structure of souce term calculation
- More turbulent models has been implemented.
- Simple and tidy code structure for given profiles; experimental and artificial profiles.

Preliminary caluculation

We solved energy transport eqn. with four turbulent trasnport models.
In this results, neoclassical transport is not included.
Parameters: R = 3.0 [m], a = 1.0 [m], parabolic heating profile.

Turbulent models



CDBM model [2]

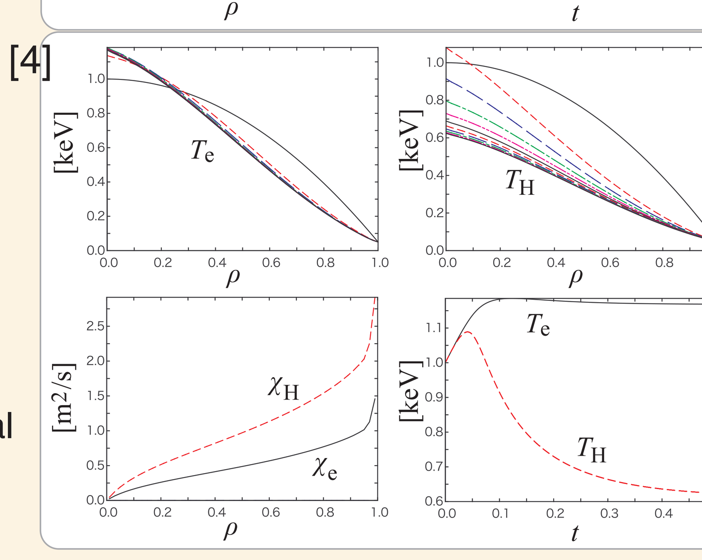
- Current diffusion driven by electron viscosity
- The amplitude of fluctuations is determined by the stability limit where the growth rate becomes zero.

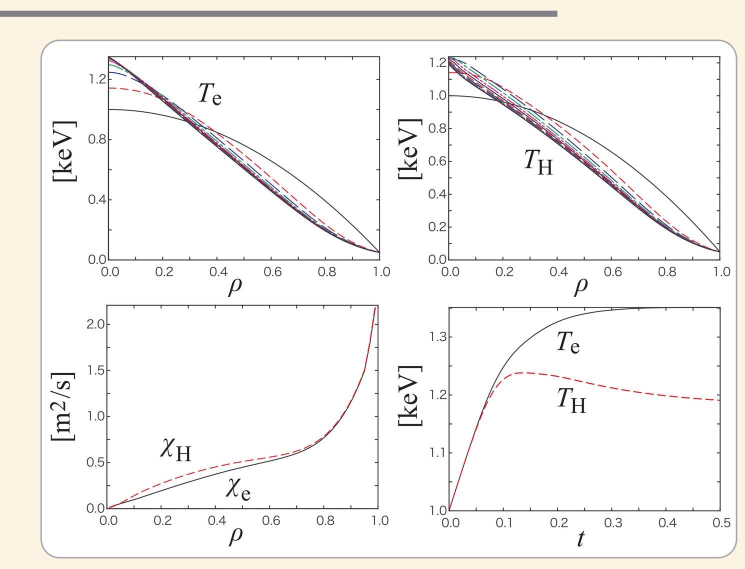
GLF23 model [3]

- Based on ITG mode and TEM
- A numerically stiff (caritical temperature gradient model)
- Eigenvalue problem for spectral of 20 modes.

mixed Bohm/gyro-Bohm model [4]

- gyro-Bohm like transport : Heat diffusion coefficient χ is proportional to the normalized radius ρ^*.
- Bohm like transport : Turbulent scale length is proportiional minor radius a.





MMM95 model [5]

This model is a combination of theory baced transport models:

- Weiland model (ITG, TEM)
- Guzdar-Drake model (drift-resistive balloonig modes)
- kinetic ballooning modes.

Numerical stabilization

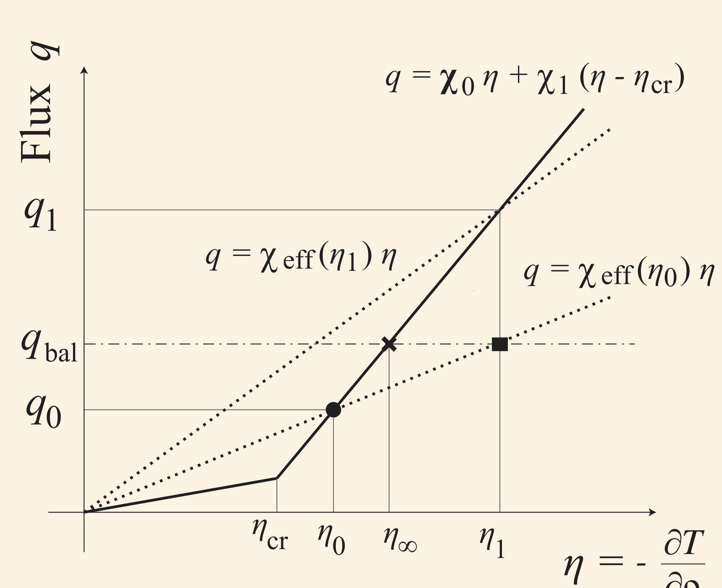
Stiff problem

$$\frac{\partial T}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\chi\frac{\partial T}{\partial \rho}\right) + S.$$

Flux of stiff model

$$q = \chi_0\eta + \chi_1(\eta - \eta_{cr})$$

$$\chi_{eff} = q/\eta$$



When χ is calculated by stiff model, the non-linear iteration does not converge for large time steps. Then the size of time step must be very small for stable calculation.

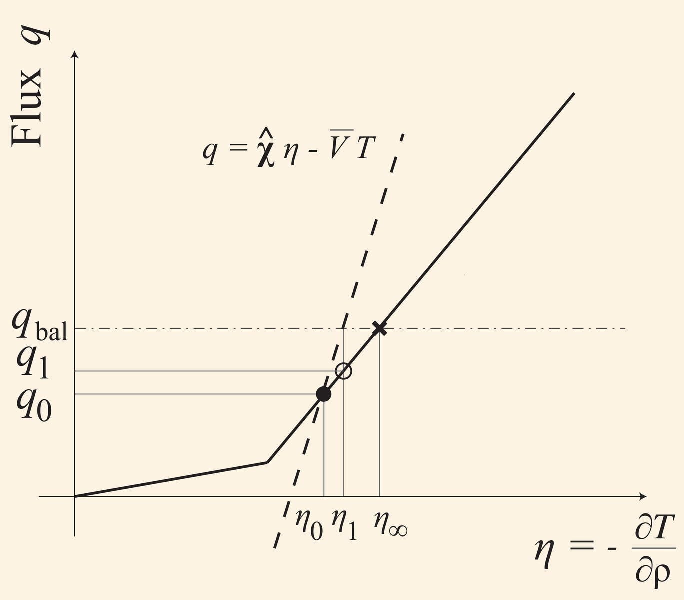
→ **very time consuming**

Numerical stabilization scheme [1]

$$\frac{\partial T}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\hat{\chi}\frac{\partial T}{\partial \rho} - \rho\bar{V}T\right) + S \qquad \hat{\chi} = \chi + \bar{\chi}, \quad \bar{V} = \bar{\chi}\frac{(\partial T/\partial \rho)}{\rho}$$

This eqn. is equivalent to the former one mathematically.

The flux-gradient relation of this eqn. are illustrated by the broken line in the right fig.



In numerical calculation $\bar{V}$ is evaluated at a previous time step, then additional non-zero term contributes the numerical stability.

Mathematical equivalence

Typical trasnport equations

$$\frac{\partial}{\partial t}(V'X_n) = \frac{\partial}{\partial \rho}\left[V'\langle|\nabla\rho|^2\rangle\sum_{n'} D_{nn'}\frac{\partial X_n}{\partial \rho} - V'\langle|\nabla\rho|\rangle\sum_{n'}V_{nn'}X_{n'}\right] + V'S_n$$

are formulated as element equation of FEM as follow.

$$\overleftrightarrow{L}_i \boldsymbol{X}_{n\,i}^{k+1} + (\Delta t)\sum_{n'}(\overleftrightarrow{R}_{Di} - \overleftrightarrow{R}_{Vi})\boldsymbol{X}_{n'\,i}^{k+1} = \overleftrightarrow{L}_i \boldsymbol{X}_{n\,i}^{k} + (\Delta t)\overleftrightarrow{R}_{Si}\boldsymbol{S}_{n\,i}$$

Considering the mathematical equivalence of the systems in steady state, additional terms must vanish in nodal equations in FEM.

$$\overleftrightarrow{R}_{\bar{D}i2\,1}X_i + \overleftrightarrow{R}_{\bar{D}i2\,2}X_{i+1} = \overleftrightarrow{R}_{\bar{V}i2\,1}X_i + \overleftrightarrow{R}_{\bar{V}i2\,2}X_{i+1} \qquad (*)$$

Assuming an expression for $\bar{V}$ as follow,

$$\bar{V}_{n\,i}^j = \bar{D}_{n\,i}^j\frac{X_{n\,i+1}^j - X_{n\,i}^j}{X_{\mathrm{ave}\,i}^j h_i}$$

we can determined the expression for X_{ave} with the use of (*).

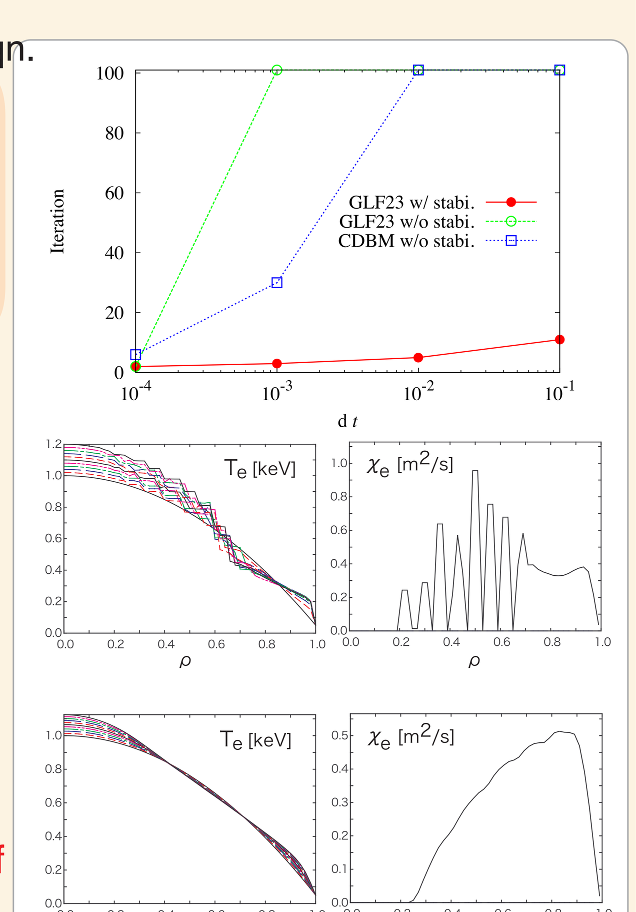
$$X_{\mathrm{ave}\,i}^j = \frac{\bar{D}_{n\,i}^j}{\bar{D}_{n\,i}^{j+1}}\frac{(2G_{1\,i} + G_{1\,i+1})X_{n\,i}^j + (G_{1\,i} + 2G_{1\,i+1})X_{n\,i+1}^j}{3(G_{2\,i} + G_{2\,i+1})}$$

Here superscript j denotes the value at previous time step, and subscript i the position in radial mesh.

Benchmarking by GLF23 model

We've solved energy trasnport eqn.

Parameters:
R = 3.0 m, a = 1.0 m
e, H plasma, $\bar{D} = 3.0$ [m²/s]
parabolic heating profile



Increase of dt without stabilization scheme leads to numerical oscillation, and iterations do not come to converge.

New scheme gives the smooth solutions and less iterations.
This scheme is very effective to solve transport eqn. with stiff models (GLF23, Weiland etc.).

Future Tasks

1.5D trasnport simulation

- solving magnetic diffusion equation.
- more realistic heating profile (commonalize heating module with TASK/TX)
- coupling with equilibrium code (TASK/EQ, EQU, VMEC)

Comparison with experimental results

- For existing large tokamaks, JT-60U, JET, TFTR etc. (from the International Multi-tokamak Confinement Profile Database)
- Compare deviations of incremental energies and temperature
- L-H transition(ETB) and formation of ITB

Comparison with TASK/TX

- TASK/TX is 1D dynamic transport code (solving toroidal and poloidal rotation).
- Compare the results and examine the effect of radial electric field and plasma rotation

References

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[2] K.Itoh, M.Yagi, S.-I.Itoh, et al., *Plasma Phys. Control. Fusion*, 35 (1993)
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[4] M.Erba, T.Aniel, V.Basiuk, A.Becoulet, X.Litaudon, *Nuclear Fusion*, 38, 1013 (1998)
[5] G.Bateman, A.H.Kritz, J.E.Kinsey, et al., *Phys Plasmas* 5 (1998) 1793