Entropy transfer process and plasma turbulent transport in non-axisymmetric systems

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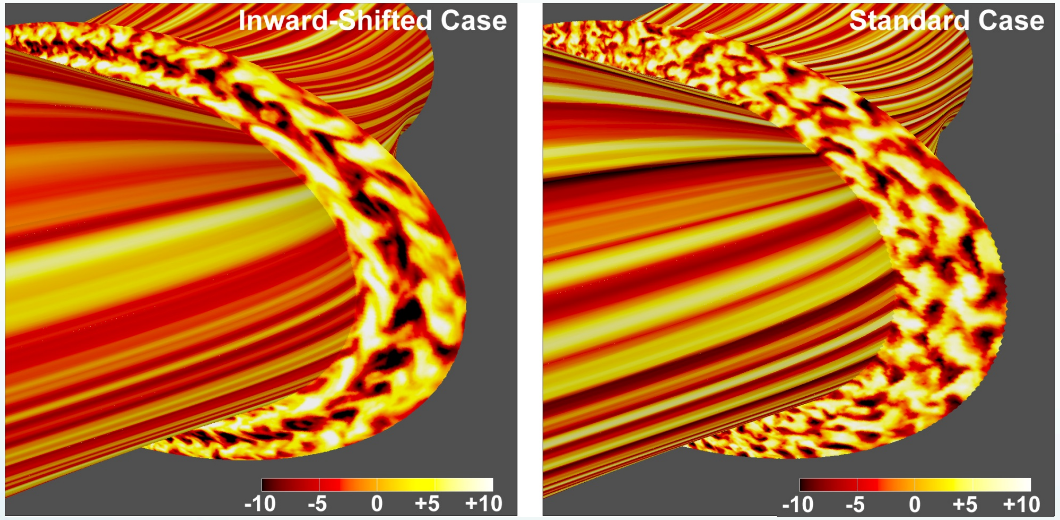
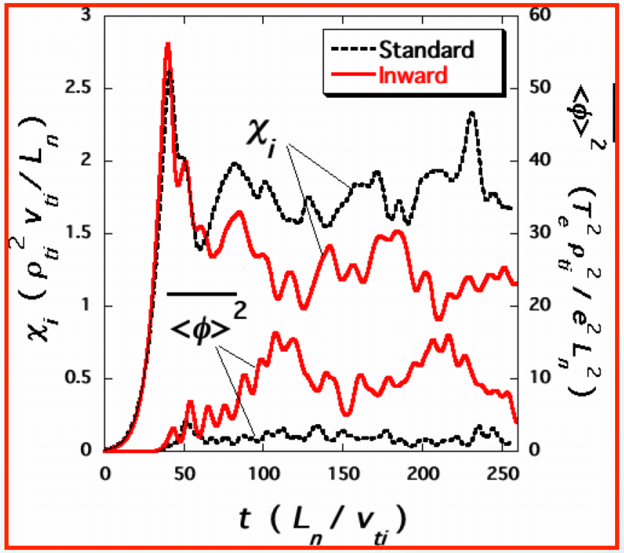
Orbit Optimization Leading to Turbulent Transport Reduction

- The enhanced ZFs in the inward-shifted LHD plasma regulate the ITG turbulence and transport.

[Watanabe, Sugama, Ferrando-Margalet, PRL 2008]





- Gyrokientic simulations with real geometries of LHD experimental equilibrium condition are demanded !
  - GKV-X simulations (M. Nunami, this conference)

Entropy Balance and Transfer

- A quadratic functional of *δf*, that is, *δS*, is a measure of fluctuation, "entropy variable"

$$\delta S_{\mathrm{i}\boldsymbol{k}_\perp} = \left\langle \int d\boldsymbol{v} \frac{|\delta f^{(\mathrm{g})}_{\mathrm{i}\boldsymbol{k}_\perp}|^2}{2F_{\mathrm{M}}} \right\rangle$$

- Production rate of *δS* balances with transport and dissipation

$$\frac{\partial}{\partial t}(\delta S_{\mathrm{i}\boldsymbol{k}_\perp} + W_{\boldsymbol{k}_\perp}) = L_{T_\mathrm{i}}^{-1} Q_{\mathrm{i}\boldsymbol{k}_\perp} + \mathcal{T}_{\mathrm{i}\boldsymbol{k}_\perp} + D_{\mathrm{i}\boldsymbol{k}_\perp}$$

- In kinetic plasma turbulence, *δS* is transferred in the phase space
  - Generation of fine velocity-space structures
  - Zonal flow and turbulence interactions

$$Q_{\mathrm{i}\boldsymbol{k}_\perp} = \mathrm{Re}\left\langle v_{\mathrm{ti}} \int d\boldsymbol{v}\, \delta f^{(\mathrm{g})}_{\mathrm{i}\boldsymbol{k}_\perp} \left(\frac{m_\mathrm{i} v_\parallel^2 + 2\mu B}{2T_\mathrm{i}} \right) i k_y \rho_{\mathrm{ti}} \frac{e\delta\psi^*_{\boldsymbol{k}_\perp}}{T_\mathrm{i}} \right\rangle$$

$$D_{\mathrm{i}\boldsymbol{k}_\perp} = \mathrm{Re}\left\langle \int d\boldsymbol{v}\, \mathcal{C}[h_{\mathrm{i}\boldsymbol{k}_\perp}] \frac{h^*_{\mathrm{i}\boldsymbol{k}_\perp}}{F_\mathrm{M}} \right\rangle$$

Entropy Transfer Function T_k

- Entropy transfer function describes nonlinear ExB interactions among turbulence fluctuations and zonal flows. (Sugama et al. PoP 2009; Nakata et al. PoP 2012)

$$\mathcal{T}_{\mathrm{i}\boldsymbol{k}_\perp} = \sum_{\boldsymbol{q}_\perp} \sum_{\boldsymbol{p}_\perp} \delta_{\boldsymbol{k}_\perp+\boldsymbol{p}_\perp+\boldsymbol{q}_\perp,0} \mathcal{J}_{\mathrm{i}}[\boldsymbol{k}_\perp|\boldsymbol{p}_\perp,\boldsymbol{q}_\perp]$$

$$\mathcal{J}_i[\boldsymbol{k}_\perp|\boldsymbol{p}_\perp,\boldsymbol{q}_\perp] = \left\langle \frac{c}{B}\boldsymbol{b}\cdot(\boldsymbol{p}_\perp\times\boldsymbol{q}_\perp)\int d\boldsymbol{v}\frac{1}{2F_{\mathrm{M}}}\mathrm{Re}[\delta\psi_{\boldsymbol{p}_\perp}h_{\mathrm{i}\boldsymbol{q}_\perp}h_{\mathrm{i}\boldsymbol{k}_\perp} - \delta\psi_{\boldsymbol{q}_\perp}h_{\mathrm{i}\boldsymbol{p}_\perp}h_{\mathrm{i}\boldsymbol{k}_\perp}]\right\rangle$$

h_k: non-adiabatic part of $\delta f^{(g)}$

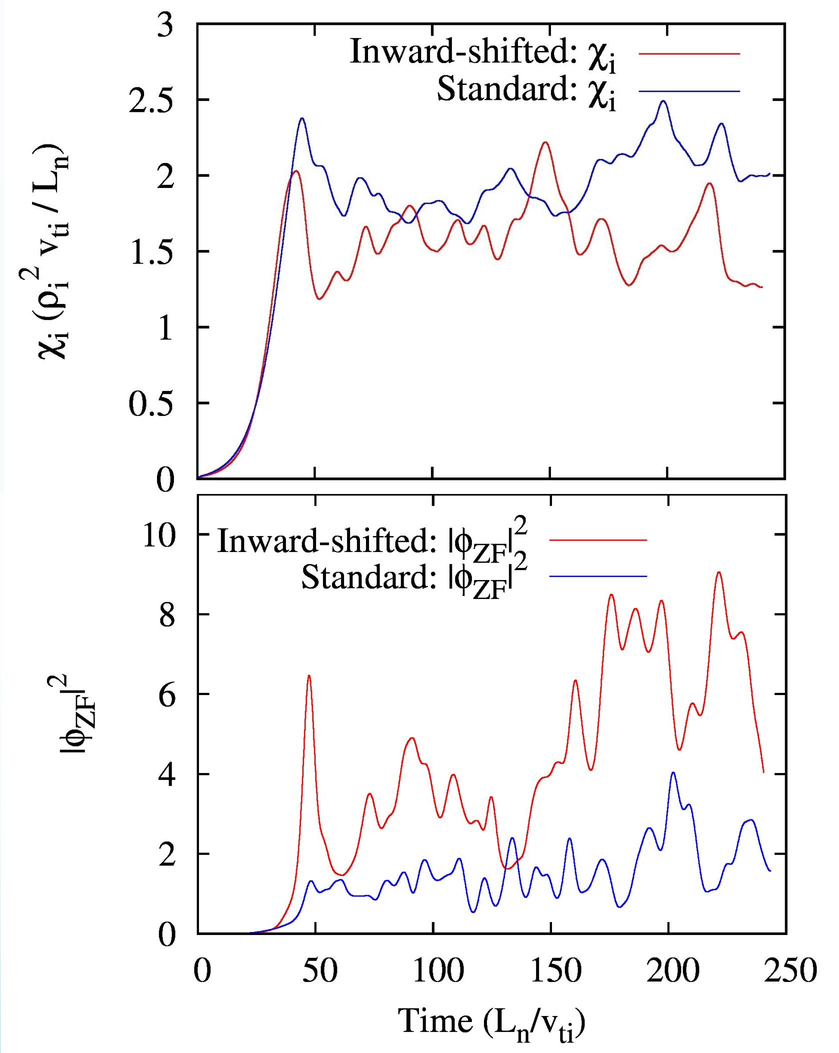
- Detailed balance relation holds for the triad interaction

$$\mathcal{J}_{\mathrm{i}}[\boldsymbol{k}_\perp|\boldsymbol{p}_\perp,\boldsymbol{q}_\perp] + \mathcal{J}_{\mathrm{i}}[\boldsymbol{p}_\perp|\boldsymbol{q}_\perp,\boldsymbol{k}_\perp] + \mathcal{J}_{\mathrm{i}}[\boldsymbol{q}_\perp|\boldsymbol{k}_\perp,\boldsymbol{p}_\perp] = 0$$

[Nakata, Watanabe, and Sugama PoP (2012)]

Entropy Transfer Analysis Applied to Helical Configurations

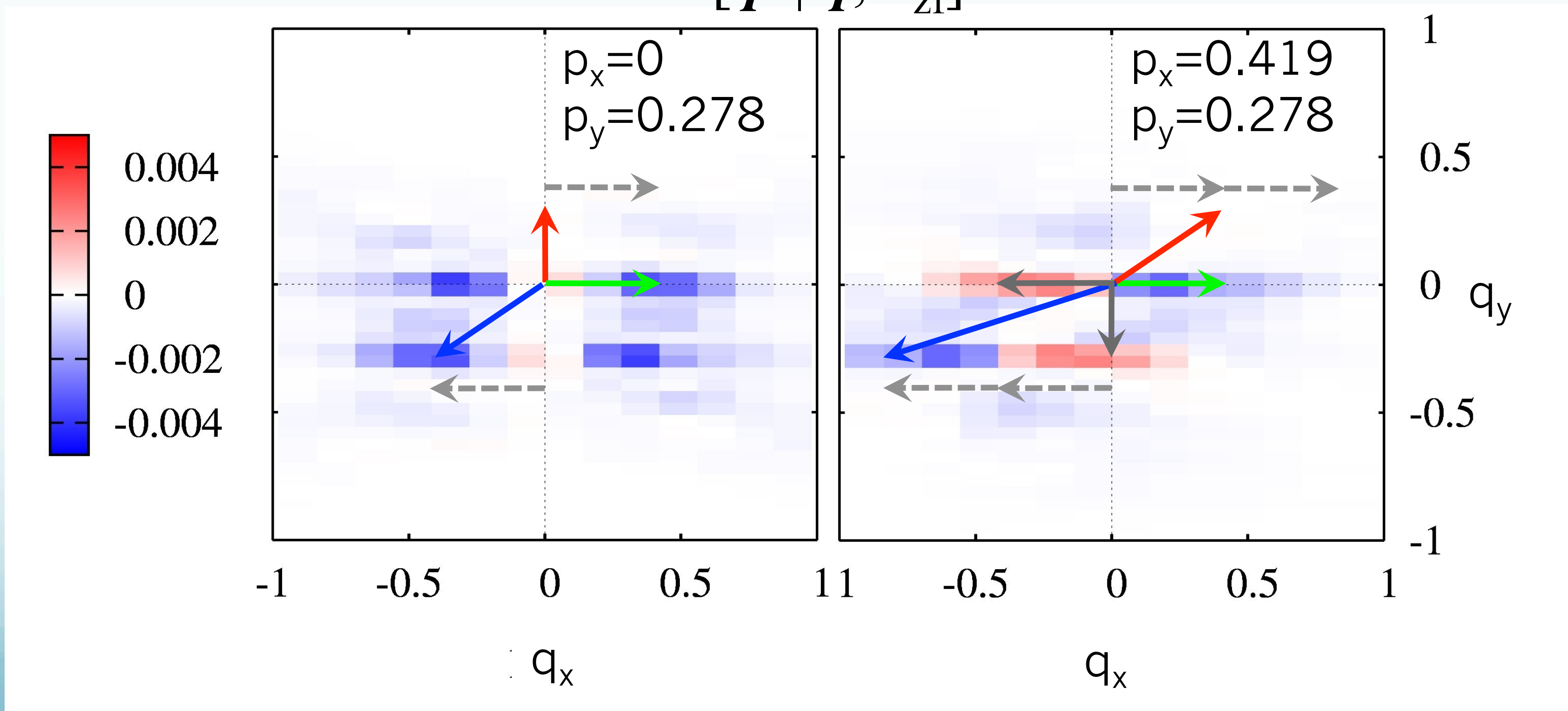
- ITG turbulent transport simulations with the entropy transfer analysis for model LHD configurations.
- Inward-shifted case
  - Stronger ZFs and lower χ_i
- Standard case
  - Weaker ZFs and higher χ_i



Entropy Transfer Function for Inward-Shifted Case

- Strong ZF-turbulence interactions in the inward-shifted case drives the successive entropy transfer.

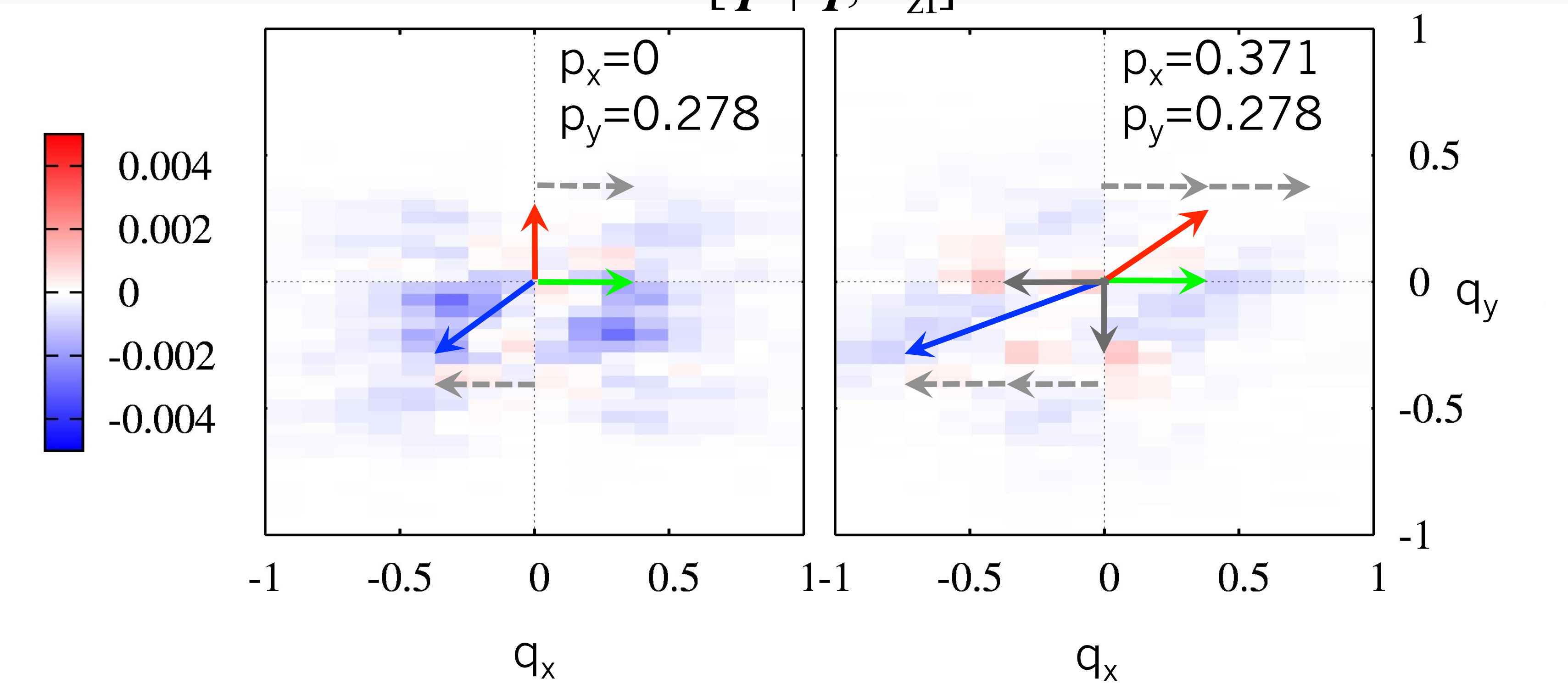
$J[\boldsymbol{p} \mid \boldsymbol{q}, \boldsymbol{k}_{zf}]$



Entropy Transfer Function for Standard Case

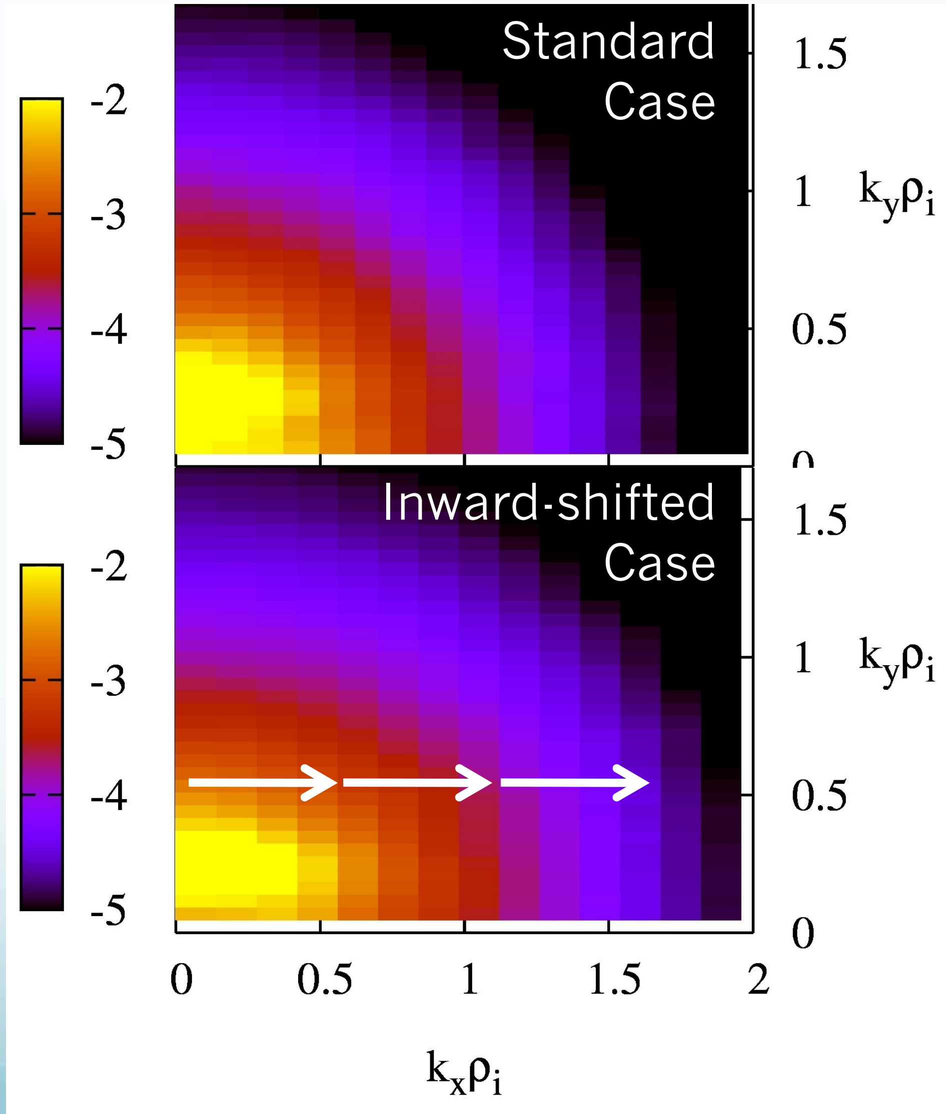
- ZF-turbulence interaction is weaker in the standard case. Successive entropy transfer is not clearly found.

$J\,[\,\boldsymbol{p}\,|\,\boldsymbol{q},\,\boldsymbol{k}_{zf}]$



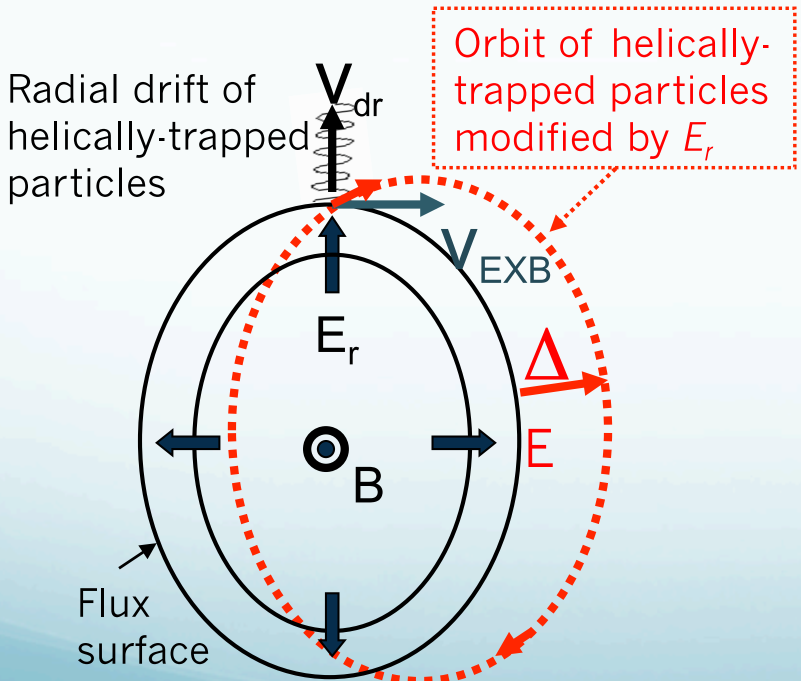
Spectrum Broadening Through Successive Entropy Transfer

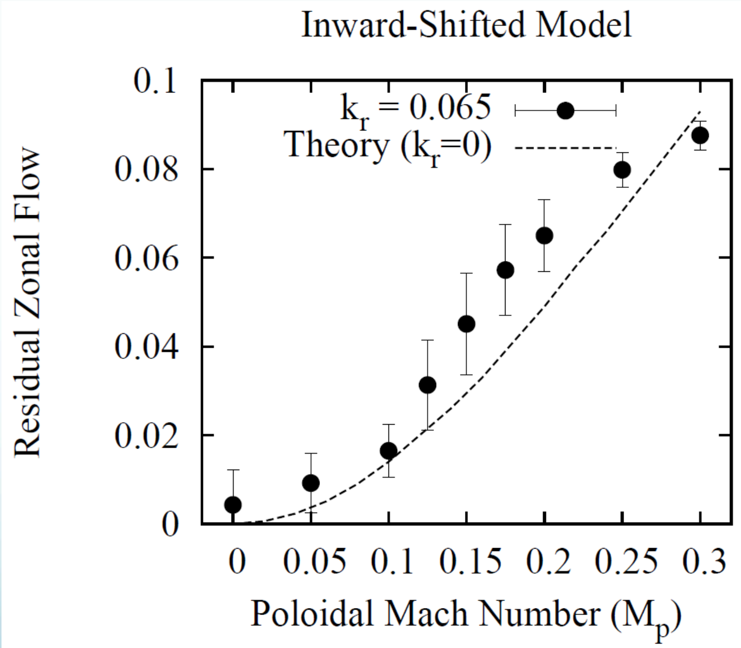
- In the inward-shifted case, the fluctuation spectrum expands into higher-k_r space due to the successive entropy transfer by zonal flows.
- Standard case:
  $\sigma^2 = <k_r^2> = 0.917$
- Inward-shifted case:
  $\sigma^2 = <k_r^2> = 0.991$



Enhancement of Zonal Flow Response by E_r

- The equilibrium-scale radial electric field E_r generated by the neoclassical transport
  - improves collisionless particle orbits, and
  - simultaneously **enhances the ZF response**





[Watanabe et al. NF (2011)]

How to couple ZF computation with turbulence simulation?

- In the ZF response calculation with E_r, α-dependence of the confinement field strength B should be introduced, *i.e.*,

$$\left[\frac{\partial}{\partial t} + v_\parallel \hat{\boldsymbol{b}}\cdot\nabla + i\boldsymbol{k}_r\cdot\boldsymbol{v}_d - \mu\hat{\boldsymbol{b}}\cdot\nabla\Omega\frac{\partial}{\partial v_\parallel} + \omega_\theta q\frac{\partial}{\partial\alpha}\right]\delta f = -i\boldsymbol{k}_r\cdot\boldsymbol{v}_d\frac{e\langle\psi\rangle}{T_i}F_M$$

$$\alpha = \varsigma - q\theta$$

$$\omega_\theta = -\frac{cE_r}{r_0 B_0}$$

  - A non-local treatment in the toroidal direction, because of the α dependence of operators, such as $\mathbf{v}_d(\alpha)$ etc.

- Mixture of the typical scale-lengths of turbulence $\sim\rho_i$ and non-axisymmetric geometry $\sim r_0/Mq$ (M : toroidal period of $|B|$)

- An alternative idea:
  - Scale separation between ZF and turbulence for the field-line label coordinate α (while the same scale in the radial direction)
  - Analogy to the "radial scale-separation" that is applied in δf-GK

Scale Separation of Zonal Flows and Turbulence

- The gyrokinetic equation for the perturbed ion gyrocenter distribution function, δf (h: non-adiabatic part)

$$\left[\frac{\partial}{\partial t} + v_{\parallel}\mathbf{b}\cdot\nabla + \mathbf{v}_d\cdot\nabla - \frac{\mu}{m}(\mathbf{b}\cdot\nabla B)\frac{\partial}{\partial v_{\parallel}}\right]\delta f + \frac{c}{B_0}\{\Phi, \delta f\} = (\mathbf{v}_* - \mathbf{v}_d - v_{\parallel}\mathbf{b})\cdot\frac{e\nabla\Phi}{T}F_M + C(h)$$

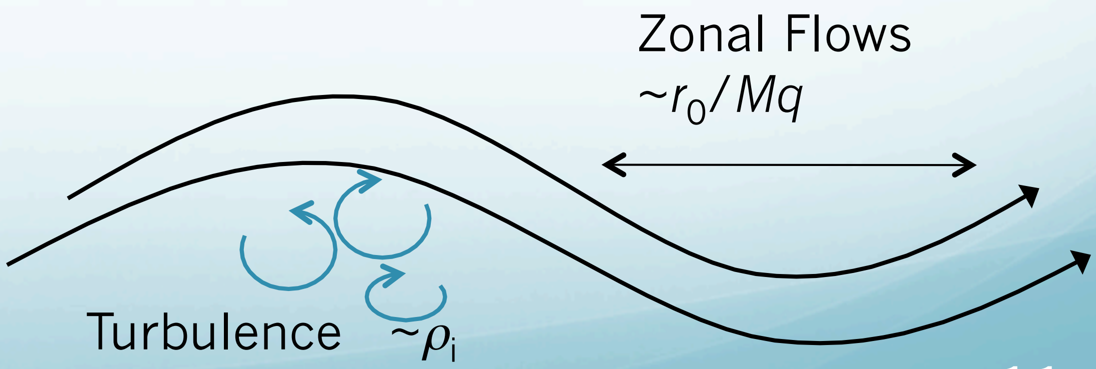
$$\delta f_{\mathbf{k}} = -\frac{e\Phi_{\mathbf{k}}}{T}F_M + h_{\mathbf{k}}$$

- Postulate the spatial dependence on slow ($\alpha' = \alpha$) and fast (y) coordinates

$$\delta f = \hat{f}(x, \alpha', z, v_{\parallel}, \mu) + \tilde{f}(x, y, z, v_{\parallel}, \mu; \alpha') \qquad \Phi = \hat{\Phi}(x, \alpha', z, \mu) + \tilde{\Phi}(x, y, z, \mu; \alpha') + \Phi_0$$

- Flux tube coordinates

$$\begin{cases} x = r - r_0 \\ y = (r_0/q_0)(q\theta - \zeta) \\ z = \theta \end{cases}$$



Flux-Tube Bundle Model for Multi-Scale Interactions of E_r, ZFs, and Turbulence

- Zonal flow components with α dependence (non-axisymmetry)

$$\left[\frac{\partial}{\partial t}+v_\parallel \mathbf{b}\cdot\nabla+v_{dx}\frac{\partial}{\partial x}-\frac{\mu}{m}(\mathbf{b}\cdot\nabla B)\frac{\partial}{\partial v_\parallel}-\omega_\theta q\frac{\partial}{\partial \alpha'}\right]\hat{f}=\left(-v_{dx}\frac{\partial\hat{\Phi}}{\partial x}-v_\parallel \mathbf{b}\cdot\nabla\hat{\Phi}\right)\frac{e}{T}F_M+C(\hat{h})+S_i^{ZF}$$

- Turbulence components in the ith flux tube at $\alpha'=\alpha_i$

$$\left[\frac{\partial}{\partial t}+v_\parallel \mathbf{b}\cdot\nabla+\mathbf{v}_d\cdot\nabla-\frac{\mu}{m}(\mathbf{b}\cdot\nabla B)\frac{\partial}{\partial v_\parallel}-\omega_\theta r_0\frac{\partial}{\partial y}\right]\tilde{f}+\frac{c}{B_0}\{\Phi,\delta f\}_i$$

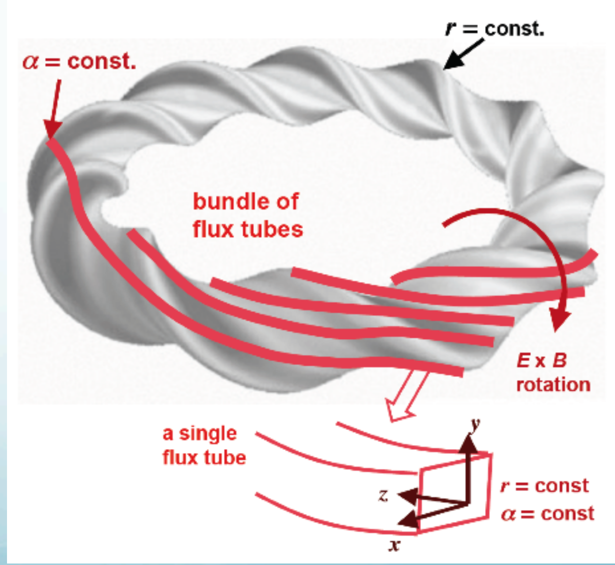
$$=\left(\mathbf{v}_*-\mathbf{v}_d-v_\parallel\mathbf{b}\right)\frac{e\nabla\Phi}{T}F_M+C(\tilde{h})-S_i^{ZF}$$

where

$$\frac{c}{B_0}\{\Phi,\delta f\}_i \Rightarrow \sum_{\mathbf{k}=\mathbf{k}'+\mathbf{k}''}\mathbf{k}'\times\mathbf{k}''\Phi_{i,\mathbf{k}'}\delta f_{i,\mathbf{k}''} \qquad S_i^{ZF}=-\frac{c}{B_0}\overline{\{\Phi,\delta f\}}_i$$

$$\delta f_{i,\mathbf{k}}=\begin{cases}\tilde{f}_i & \text{for } k_y\neq 0\\ \hat{f}(\alpha'=\alpha_i) & \text{for } k_y=0\end{cases}$$

Flux tube at $\alpha'=\alpha_i$ (i=0, 1, 2, ...)



Relationship with Conventional Models

- Linear dispersion of drift waves remains the same as that in the flux tube model, except for the Doppler shift with ω_θ
- Zonal flow response enhancement by E_r can also be reproduced
- Entropy balance relation

Turb. $$\frac{d}{dt}\left(\delta\tilde{S}_i + \tilde{W}_i\right) = L_T^{-1}Q_i + \tilde{D}_i - \tau^{ZF}$$

ZF $$\frac{\partial}{\partial t}\left(\delta\hat{S}_i + \hat{W}_i\right) - \omega_\theta q \frac{\partial}{\partial \alpha'}\left(\delta\hat{S}_i + \hat{W}_i\right) = \hat{D}_i + \tau^{ZF}$$

Total $$\frac{d}{dt}\left(\delta S + W\right) = L_T^{-1}Q + D$$