Entropy balance in electromagnetic gyrokinetic simulations

電磁的ジャイロ運動論的シミュレーションにおけるエントロピーバランス

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Motivation

Electromagnetic effects on turbulent transport play important roles in high beta plasmas. Rapid parallel motions of kinetic electrons bring some numerical difficulties to electromagnetic gyrokinetic simulation.

Recently, we extended the GKV code for electromagnetic gyrokinetic simulations. Here, we develop numerical methods to treat kinetic electrons and discuss benefits of the methods.

Summary

We presented two methods for electromagnetic gyrokinetic simulations.

- An electromagnetic gyrokinetic code is successfully developed.
- The outflow boundary condition reduces numerical oscillations in the field-aligned coordinate.
- The entropy conservative difference substantially reduces numerical errors in entropy balance.

δf gyrokinetic equations

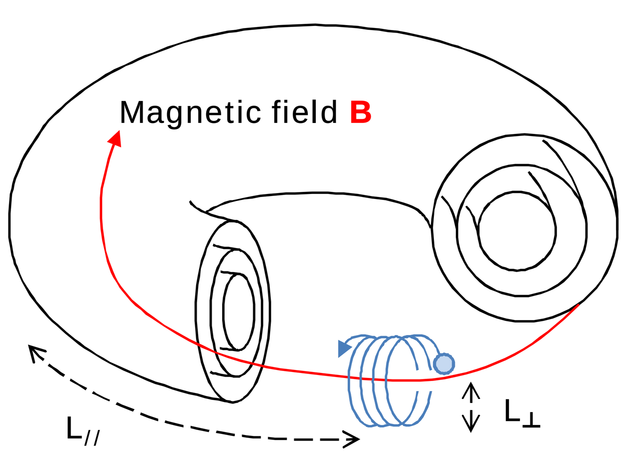
In a gyrokinetic framework, rapid gyrations are asymptotically decoupled from slow transport phenomena, and then reduced equations are,

Vlasov Eq. $$\left[\frac{\partial}{\partial t}+v_{/\!/}\nabla_{/\!/}+i\omega_{\mathrm{sd}}-\frac{\mu\nabla_{/\!/}B}{m_{\mathrm{s}}}\frac{\partial}{\partial v_{/\!/}}\right]\hat{f}_{\mathrm{s}\mathbf{k}}+N_{\mathrm{s}\mathbf{k}}=S_{\mathrm{s}\mathbf{k}}+C_{\mathrm{s}\mathbf{k}}$$

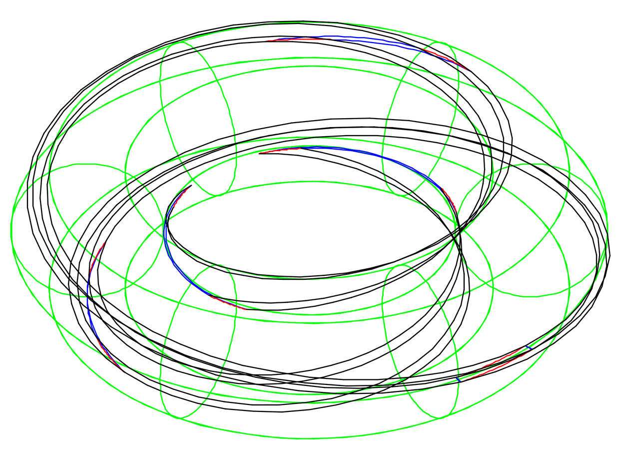
Poisson Eq. $$\left[k_\perp^2+\frac{1}{\varepsilon_0}\sum_{\mathrm{s=i,e}}\frac{e_{\mathrm{s}}^2 n_{\mathrm{s}}}{T_{\mathrm{s}}}(1-\Gamma_{0\mathrm{s}\mathbf{k}})\right]\hat{\phi}_{\mathbf{k}}=\frac{1}{\varepsilon_0}\sum_{\mathrm{s=i,e}}e_{\mathrm{s}}\hat{n}_{\mathrm{s}\mathbf{k}}$$

Ampere Eq. $$k_\perp^2\hat{A}_{/\!/\mathbf{k}}=\mu_0\sum_{\mathrm{s=i,e}}e_{\mathrm{s}}\hat{u}_{/\!/\mathrm{s}\mathbf{k}}$$

Charged-particle gyromotion



Flux-tube simulation domain



The GKV code[1], originally developed for electrostatic turbulent simulations, solves time evolution of perturbations in a flux-tube simulation domain.

We extend the GKV code for electromagnetic gyrokinetic simulations with kinetic ions and electrons.

[1] T.-H. Watanabe and H. Sugama, Nucl. Fusion **46**, 24 (2006).

We note that the gyrokinetic Eqs. satisfy the entropy balance relation,

$$\frac{d}{dt}\left(\sum_{\mathrm{s=i,e}}S_{\mathrm{s}}+W_{\mathrm{E}}+W_{\mathrm{M}}\right)=\sum_{\mathrm{s=i,e}}\left(\frac{T_{\mathrm{s}}\Gamma_{\mathrm{s}}^{\mathrm{A}}}{L_{p\mathrm{s}}}+\frac{Q_{\mathrm{s}}^{\mathrm{A}}}{L_{T\mathrm{s}}}+D_{\mathrm{s}}\right)$$

which describes that the particle fluxes Γ_s and the heat flux Q_s act like sources of the perturbed energy (the entropy variables S_s, the electric energy W_E and magnetic energy W_M) and the collisional dissipation D_s act like sinks.

Numerical methods

We employs the numerical methods same with the original GKV code,

- Discretization in x and y: Fourier expansion
- Discretization in z, $v_{/\!/}$ and μ: Finite difference
- Time integration: 4th-order Runge-Kutta-Gill method

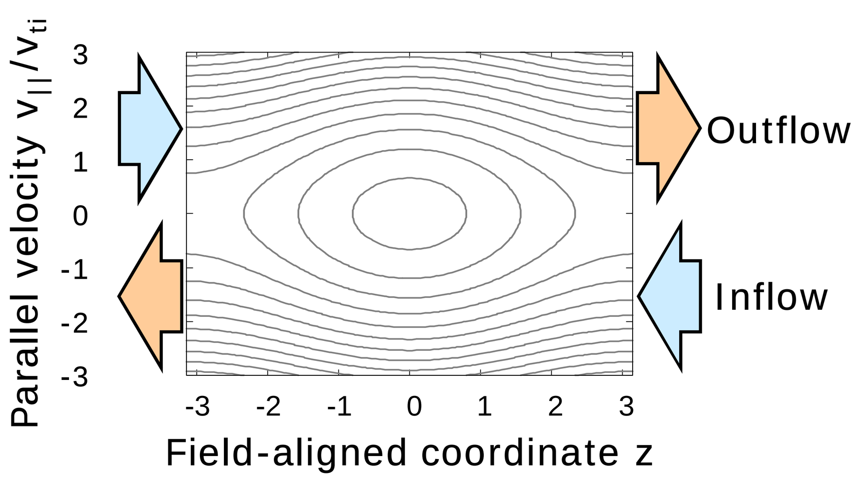
and develops special methods to treat kinetic electrons.

Outflow boundary condition along the field-aligned coordinate

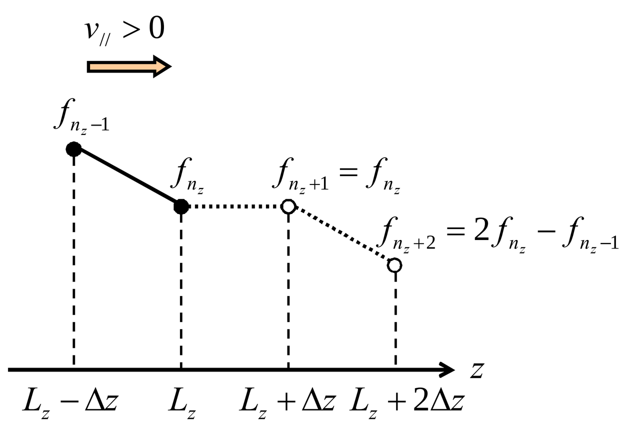
Instabilities drive perturbations at the bad curvature region (z=0), and the electron perturbation is rapidly advected along the field-aligned coordinate z.

Outflow boundary condition is employed to reduce numerical oscillations at the edge of the field-aligned coordinate.

Trajectories of particle parallel motions



Schematic picture of the outflow boundary condition



The outflow boundary introduces artificial sinks in the entropy balance,

$$E_{\mathrm{s}}=-\sum_{\mathbf{k}}\frac{1}{\int\sqrt{g}dz}\left[\int\frac{v_{/\!/}F_{\mathrm{sM}}}{2T_{\mathrm{s}}}\left|\frac{T_{\mathrm{s}}\hat{f}_{\mathrm{s}\mathbf{k}}}{F_{\mathrm{sM}}}+e_{\mathrm{s}}J_{0\mathrm{s}\mathbf{k}}\hat{\phi}_{\mathbf{k}}\right|^2dv^3\right]_{z=-L_z}^{z=L_z}$$

Entropy conservative difference for parallel motions

Parallel dynamics can be written as an incompressible form,

$$-v_{/\!/}\nabla_{/\!/}\hat{f}_{\mathrm{s}\mathbf{k}}+\frac{\mu\nabla_{/\!/}B}{m_{\mathrm{s}}}\frac{\partial\hat{f}_{\mathrm{s}\mathbf{k}}}{\partial v_{/\!/}}-\frac{e_{\mathrm{s}}F_{\mathrm{sM}}}{T_{\mathrm{s}}}v_{/\!/}\nabla_{/\!/}J_{0\mathrm{s}\mathbf{k}}\hat{\phi}_{\mathbf{k}}=\frac{1}{m_{\mathrm{s}}}\left\{\frac{T_{\mathrm{s}}\hat{f}_{\mathrm{s}\mathbf{k}}}{F_{\mathrm{sM}}}+e_{\mathrm{s}}J_{0\mathrm{s}\mathbf{k}}\hat{\phi}_{\mathbf{k}},F_{\mathrm{sM}}\right\}_{/\!/}$$

where $\{f,g\}_{/\!/}=\nabla_{/\!/}f\partial_{v_{/\!/}}g-\partial_{v_{/\!/}}f\nabla_{/\!/}g$. Application of Morinishi's scheme[2] guarantees the entropy conservative property,

$$\sum_{\mathbf{k}}\left\langle\int\left(\frac{T_{\mathrm{s}}\hat{f}_{\mathrm{s}\mathbf{k}}^*}{F_{\mathrm{sM}}}+e_{\mathrm{s}}J_{0\mathrm{s}\mathbf{k}}\hat{\phi}_{\mathbf{k}}^*\right)\frac{1}{m_{\mathrm{s}}}\left\{\frac{T_{\mathrm{s}}\hat{f}_{\mathrm{s}\mathbf{k}}}{F_{\mathrm{sM}}}+e_{\mathrm{s}}J_{0\mathrm{s}\mathbf{k}}\hat{\phi}_{\mathbf{k}},F_{\mathrm{sM}}\right\}_{/\!/}dv_{/\!/}\right\rangle=E_{\mathrm{s}}$$

where <・> denotes the flux-surface average. E_s is non-zero when the outflow boundary condition is employed.

[2] Y. Morinishi, et al., J. Comp. Phys. **143**, 90 (1998).

Availability of the schemes for electromagnetic simulations

Plasma parameters: $L_n/L_{Ti}=3.1$, $L_n/L_{Te}=0$, $R/L_n=2.22$, $r/R=0.18$, $q=1.4$, $s=0.786$, $m_e/m_i=5.669\times10^{-4}$, $T_e=T_i$, $\beta_i=1\%$, $\nu_i=2\times10^{-3}v_{ti}/L_n$, $\nu_e=\nu_i\times\sqrt{m_i/m_e}$.

Numerical settings for linear runs: $-11.9<k_x\rho_{ti}<11.9$, $k_y\rho_{ti}=0.2$, $-\pi<z<\pi$, $-4v_{ts}<v_{/\!/}<4v_{ts}$, $0<\mu B/T_s<8$, $N_{kx}=25$, $N_{ky}=1$, $N_z=32$, $N_v=64$, $N_\mu=16$, $\Delta t=0.002L_n/v_{ti}$;

for nonlinear runs: $-50.9\rho_{ti}<x<50.9\rho_{ti}$, $-62.8\rho_{ti}<y<62.8\rho_{ti}$, $-\pi<z<\pi$, $-4v_{ts}<v_{/\!/}<4v_{ts}$, $0<\mu B/T_s<8$, $N_x=288$, $N_y=72$, $N_z=32$, $N_v=64$, $N_\mu=16$, adaptive time steps.

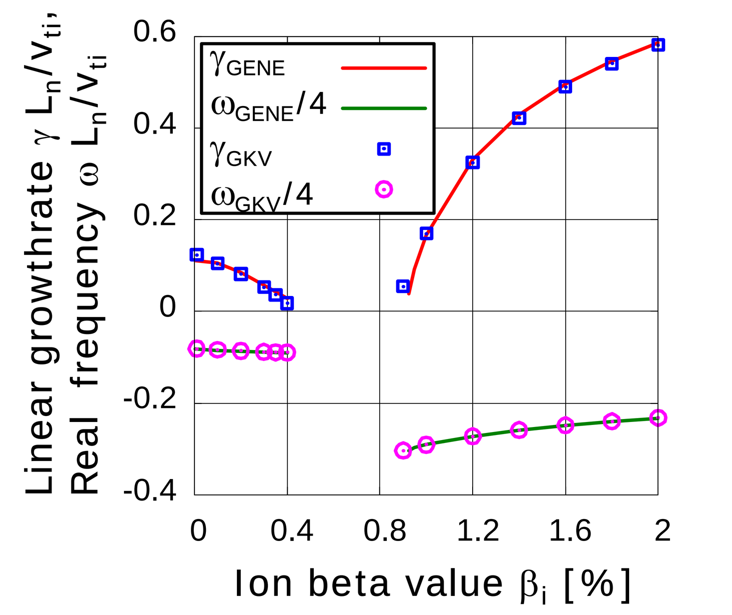
Linear simulations

First of all, the developed electromagnetic GKV code is compared with the GENE code[3].

The electromagnetic GKV code shows good agreements with the GENE code.

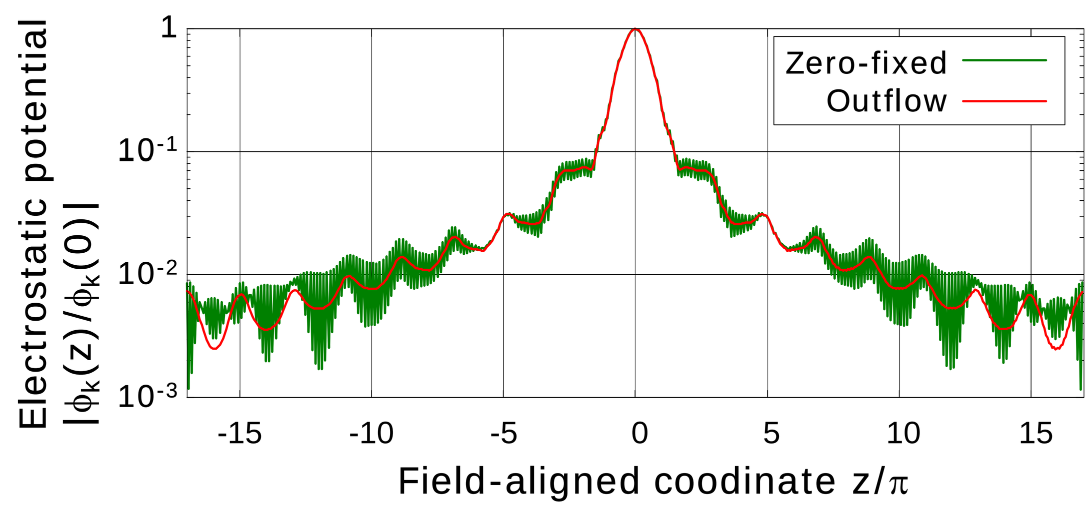
[3] F Jenko, et al., Phys. Plasmas **7**, 1904 (2000).

Benchmark test with the GENE code: Beta dependence of linear instabilities



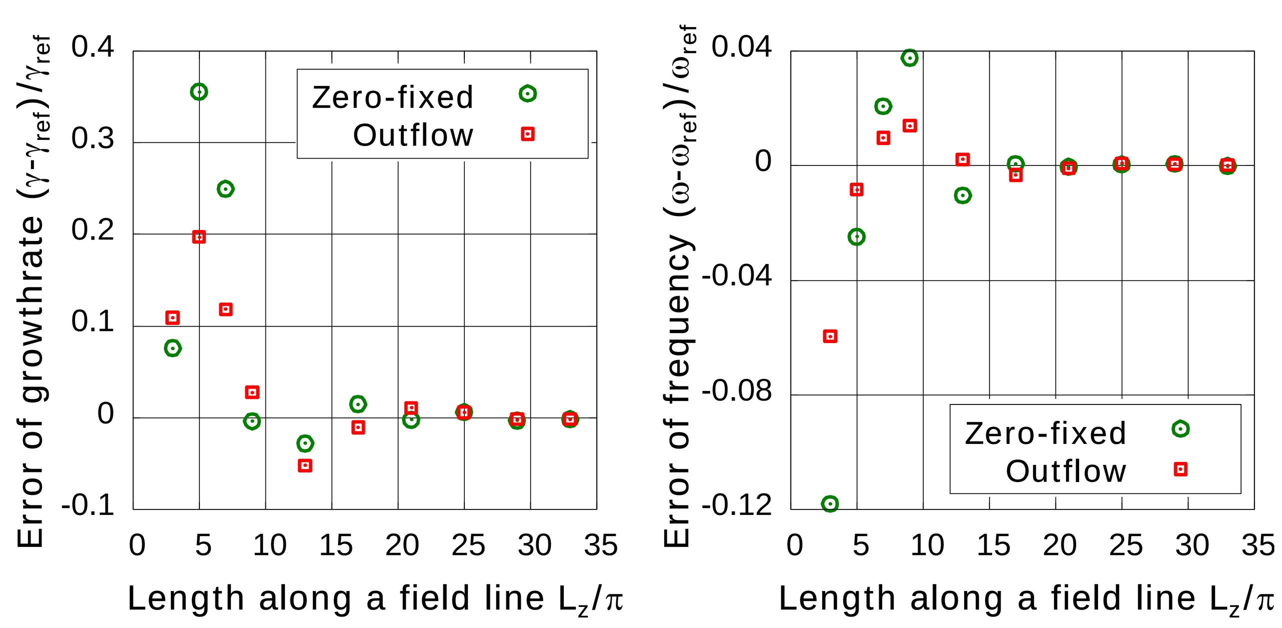
Following two tests are carried out to check the usefulness of the outflow boundary condition.

Profile of the eigenfunction of electrostatic potential along the field file



Outflow boundary reduces numerical oscillations in the field-aligned coordinate.

Convergence of the growthrate and frequency with the length along the field line



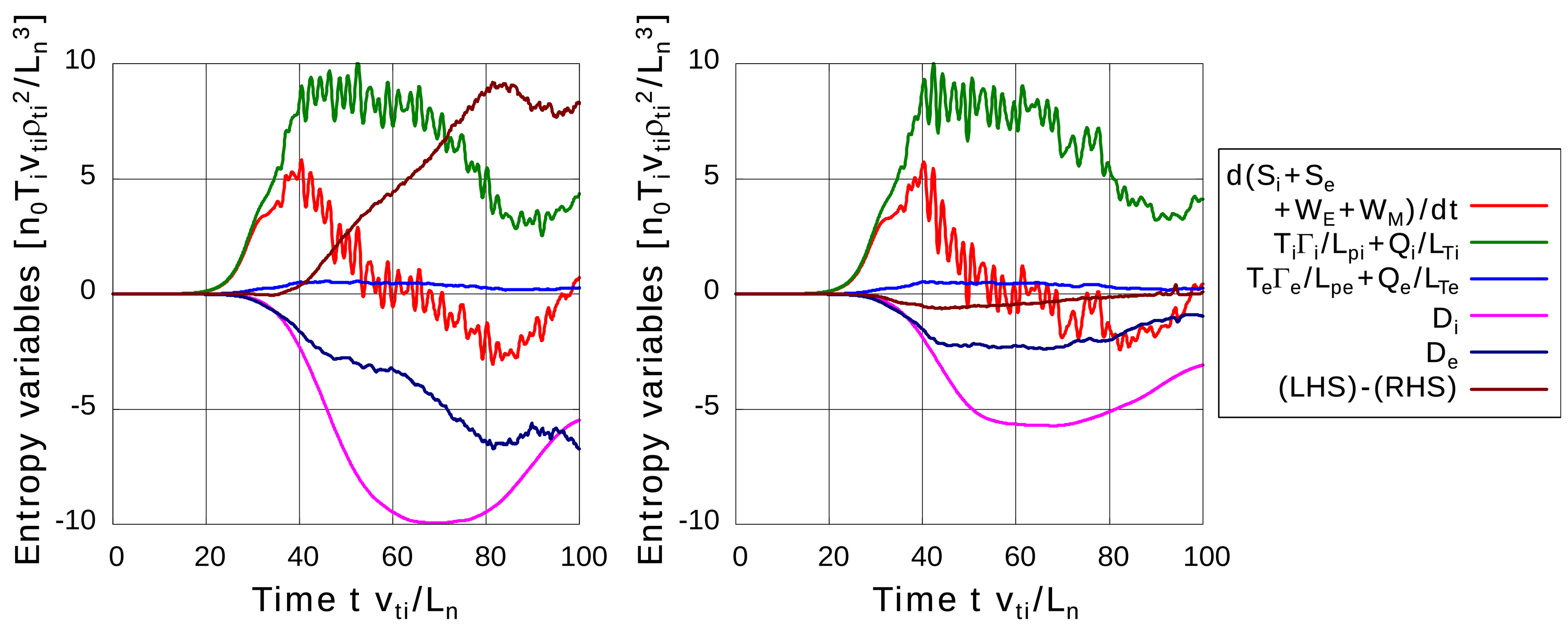
Outflow boundary improves the convergence with the length along the field line.

Nonlinear simulations

Using the outflow boundary condition, two nonlinear turbulent simulations are carried out: the one employs the 4th-order central finite difference, and the other employs the 4th-order entropy conservative difference.

Entropy conservative difference significantly reduces entropy balance error.

Time evolution of entropy variables by using (Left) central finite and (Right) entropy conservative differences



Numerical errors from the parallel dynamics act like an artificial source term, and enhance the collisional dissipation.