Development of multi-species model in local gyrokinetic turbulence simulations

Motoki Nakata and Yasuhiro Idomura

Japan Atomic Energy Agency

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Introductions

- In burning plasma experiments, such as ITER, multi-species effects on turbulent transport are crucial not only for confinement performance but also for burning efficiency.

- A minimal turbulence system for the burning plasma is ITG-TEM driven turbulence which is composed of Deuterium, Tritium, electrons, and Helium-ash (or impurity ions).

→ In addition to heat transport, the kinetic (non-adiabatic) response of electrons leads to the particle transport.

- Some earlier studies work on quasilinear calculations and nonlinear simulations for limited cases, e.g.,
Estrada-Mila (quasilinear particle flux of He-ash/alpha particles: PoP2006),
Camenen (profile-shear effects on ITG-TEM: NF2011),
Casson(rotation effects on quasilinear flux in ITG-TEM: PoP2010),
Dannert(nonlinear GK-simulations for pure-TEM: PoP2005).

Extension to GKV code for multi-species

- Original version (Watanabe NF2006):
  - -- electrostatic GK model with the quasi-neutrality condition
  - -- local fluxtube model with tokamak (s-alpha) and helical geometries
  - -- single ion/electron species with adiabatic electrons/ions: ITG-ae, ETG-ai
  - -- entropy balance and transfer diagnostics

- Extended version:
  - -- enable to treat any number of particle species: MPI-parallelization
  - -- enable to switch kinetic electron responses and the hybrid model (adiabatic responses are assumed except for trapped particles)
  - -- enable to switch self-consistent interactions and the passive limit (test-particle model) for impurity ions

- Another extensions:
  - -- Realistic helical equilibrium with VMEC (Nunami: PFR2010)
  - -- Semi-Lagrangian ASIRK time integrator (Maeyama: PFR2011)

Gyrokinetic eqs. for multi-species

- GKE

$$s = \{\mathrm{e}, \mathrm{D}, \mathrm{T}, \mathrm{He}, \cdots\}$$

$$\left[\frac{\partial}{\partial t} + v_\parallel \boldsymbol{b}\cdot\nabla + i\boldsymbol{k}_\perp\cdot\boldsymbol{v}_{ds} - \mu(\boldsymbol{b}\cdot\nabla\Omega_s)\frac{\partial}{\partial v_\parallel}\right]\delta f_{s\boldsymbol{k}_\perp} + \mathcal{N}(\delta f_{s\boldsymbol{k}_\perp}, \delta\psi_{s\boldsymbol{k}_\perp})$$

$$= F_{Ms}\left[i\boldsymbol{k}_\perp\cdot\boldsymbol{v}_{*Ts} - i\boldsymbol{k}_\perp\cdot\boldsymbol{v}_{ds} - v_\parallel\boldsymbol{b}\cdot\nabla\right]\frac{e_s\delta\psi_{s\boldsymbol{k}_\perp}}{T_s} + C_s(h_{s\boldsymbol{k}_\perp})$$

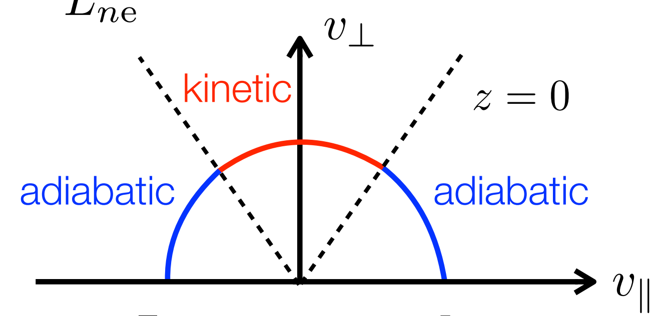
- Poisson equation

$$k_\perp^2\delta\phi_{\boldsymbol{k}_\perp} = 4\pi\sum_s e_s\left[\int d\boldsymbol{v}J_{0s}\delta f_{s\boldsymbol{k}_\perp} - n_{s0}\frac{e_s\delta\phi_{\boldsymbol{k}_\perp}}{T_s}(1-\Gamma_{0s})\right]$$

- Lowest order quasi-neutrality (f_{Cs} is the charge-density fraction)

$$\sum_s Z_s n_{s0} = 0\,, \quad \sum_{s\neq \mathrm{e}} f_{Cs} = \sum_{s\neq \mathrm{e}} Z_s\frac{n_{s0}}{n_{\mathrm{e}0}} = 1\,, \quad \sum_{s\neq \mathrm{e}} f_{Cs}\frac{R_0}{L_{ns}} = \frac{R_0}{L_{ne}}$$

- Hybrid electron model for the density calculation



$$\int d\boldsymbol{v}J_{0\mathrm{e}}\delta f_{\mathrm{e}k_\perp} = \int d\boldsymbol{v}\Theta J_{0\mathrm{e}}\delta f_{\mathrm{e}k_\perp} + \frac{e\delta\phi_{\boldsymbol{k}_\perp}}{T_\mathrm{e}}\int d\boldsymbol{v}(1-\Theta)J_{0\mathrm{e}}^2F_{M\mathrm{e}}$$

$\Theta(z,\bar{v}_\parallel,\bar{\mu}) = 1\ (\kappa^2<1),\ 0\ (\kappa^2\geqslant 1)$, where $\kappa^2(z,\bar{v_\parallel},\bar{\mu}) = \left[\bar{v_\parallel}^2/2 + \bar{\mu}(\bar{\Omega}(z) - 1 + \epsilon_r)\right]/2\epsilon_r\bar{\mu}$

(Note that ky=0-modes are solved kinetically)

Gyrokinetic eqs. for multi-species cont.

- Entropy balance equation for multi-species turbulent plasmas:

$$\frac{\partial}{\partial t} T_{\mathrm{s}} \delta S_{\mathrm{s}k} + T_{\mathrm{s}} \delta R_{\mathrm{s}k} = T_{\mathrm{s}} \left(J_{1\mathrm{s}k} X_{1\mathrm{s}} + J_{2\mathrm{s}k} X_{2\mathrm{s}} \right) + T_{\mathrm{s}} \mathcal{T}_{\mathrm{s}k} + T_{\mathrm{s}} D_{\mathrm{s}k}$$

for each modes of species “s”

- Definitions:

Entropy variable

$$\frac{\partial}{\partial t} T_{\mathrm{s}} \delta S_{\mathrm{s}k} = \frac{\partial}{\partial t} T_{\mathrm{s}} \left\langle \int d\boldsymbol{v} \frac{|\delta f_{\mathrm{s}k}|^2}{2F_{M\mathrm{s}}} \right\rangle$$

Field energy

$$T_{\mathrm{s}} \delta R_{\mathrm{s}k} = \left\langle \mathrm{Re} \int d\boldsymbol{v} e_{\mathrm{s}} \delta\psi_{\mathrm{s}k} \frac{\partial \delta f^*_{\mathrm{s}k}}{\partial t} \right\rangle = \frac{\partial}{\partial t} T_{\mathrm{s}} W_{\mathrm{s}k} + T_{\mathrm{s}} \delta \tilde{R}_{\mathrm{s}k}$$

$$= \frac{\partial}{\partial t} T_{\mathrm{s}} \left\langle \frac{n_{\mathrm{s}} e_{\mathrm{s}}^2}{2T_{\mathrm{s}}} \left(1 - \Gamma_{\mathrm{s}k}\right) |\delta\phi_k|^2 \right\rangle + T_{\mathrm{s}} \left\langle \mathrm{Re} \frac{e_{\mathrm{s}} \delta\phi_k}{T_{\mathrm{s}}} \frac{\partial}{\partial t} \delta n^{(\mathrm{p})*}_{\mathrm{s}k} \right\rangle$$

, where $\displaystyle\sum_{\mathrm{s}} T_{\mathrm{s}} \delta \tilde{R}_{\mathrm{s}k} = \frac{\partial}{\partial t} \frac{1}{8\pi} \left\langle k_\perp^2 |\delta\phi_k|^2 \right\rangle,$

if the finite Debye length is considered, or this term is 0.

Turbulent particle and heat fluxes

$$J_{1\mathrm{s}k} X_{1\mathrm{s}k} = \left\langle \mathrm{Re} \int d\boldsymbol{v} \frac{cT_{\mathrm{s}}}{e_{\mathrm{s}} B_0} \left(-ik_y \frac{e_{\mathrm{s}} \delta\psi_{\mathrm{s}k}}{T_{\mathrm{s}}} \right)^* \delta f_{\mathrm{s}k} \right\rangle L_{p_{\mathrm{s}}}^{-1}$$

$$J_{2\mathrm{s}k} X_{2\mathrm{s}k} = \left\langle \mathrm{Re} \int d\boldsymbol{v} \frac{cT_{\mathrm{s}}}{e_{\mathrm{s}} B_0} \left(\frac{m_{\mathrm{s}} v^2}{2T_{\mathrm{s}}} - \frac{5}{2} \right) \left(-ik_y \frac{e_{\mathrm{s}} \delta\psi_{\mathrm{s}k}}{T_{\mathrm{s}}} \right)^* \delta f_{\mathrm{s}k} \right\rangle L_{T_{\mathrm{s}}}^{-1}$$

Nonlinear (triad) entropy transfer (Nakata PoP2012)

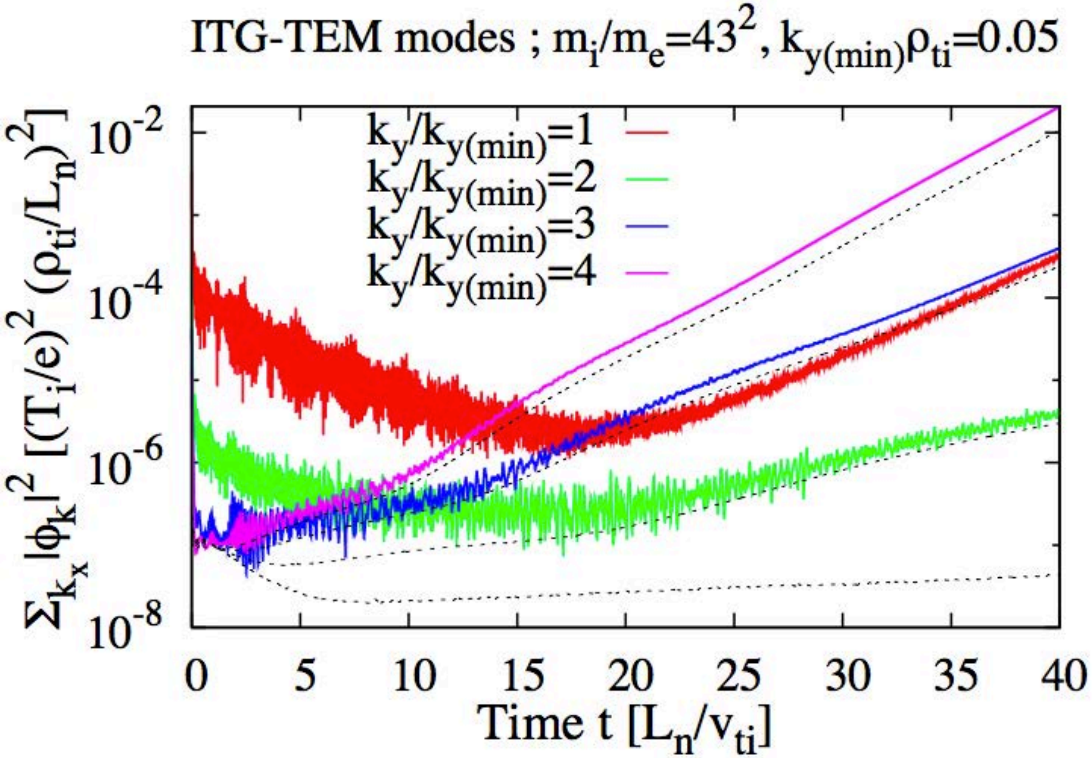
$$\mathcal{T}_{\mathrm{s}k} = \sum_q \sum_p \delta_{k+p+q,0} \mathcal{J}_{\mathrm{s}}[\boldsymbol{k}|\boldsymbol{p},\boldsymbol{q}]$$

Collisional dissipations

$$T_{\mathrm{s}} D_{\mathrm{s}k} = T_{\mathrm{s}} \left\langle \mathrm{Re} \int d\boldsymbol{v} \frac{h^*_{\mathrm{s}k}}{F_{M\mathrm{s}}} \mathcal{C}_{\mathrm{s}}(h_{\mathrm{s}k}) \right\rangle$$

High frequency (Ω_H) modes via kinetic electrons

- Time-evolution of ITG-TEM modes



- Electron parallel transit vs. Ω_H-modes

Frequency of Ω_H-modes : $\Omega_H = \sqrt{\frac{m_i}{m_e}}\frac{k_\parallel}{k_\perp}\Omega_i$

$$\frac{\bar{\Omega}_{H(\max)}}{\bar{\omega}_{te(\max)}} = \frac{1}{\sqrt{\tau_e}\bar{k}_{y(\min)}\bar{v}_{(\max)}} = 4 > 1$$

for $\bar{k}_{y(\min)} = 0.05,\ \tau_e = 1,\ \bar{v}_{(\max)} = 5,$
$\Delta t = 0.001(L_n/v_{ti})$

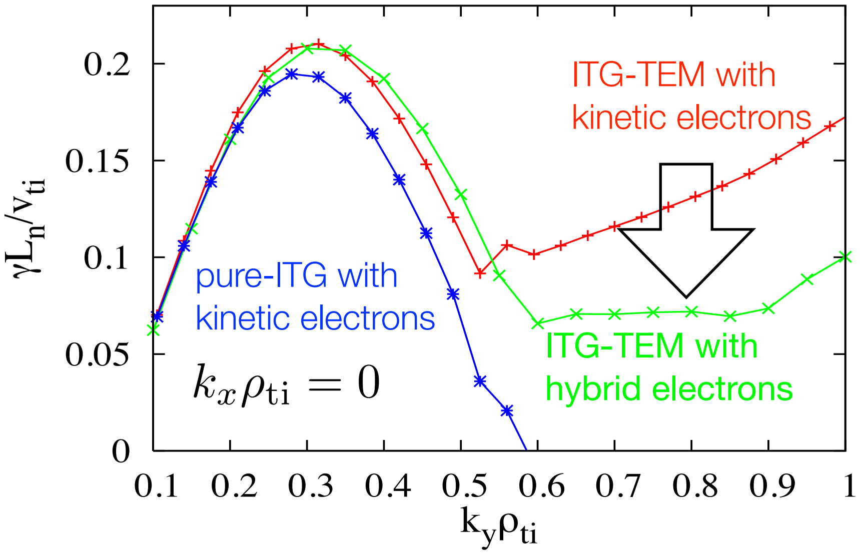
$\rightarrow$ $(CFL)_{//} = 0.47 < 1$
$(CFL)_H = 1.8773 > 1$

- High frequency oscillations appear in the time evolution with kinetic electrons (solid lines) while Ω_H-modes are suppressed in the case with hybrid electrons(dashed lines).

- In most cases, time-step is bounded by CFL for Ω_H-modes, which also depends the box size($k_{y(\min)}$), rather than electron transit frequency.

Collisionless linear ITG-TEM(-ETG) modes

- ITG-TEM modes (also ETGs)



Cyclone base case parameters with

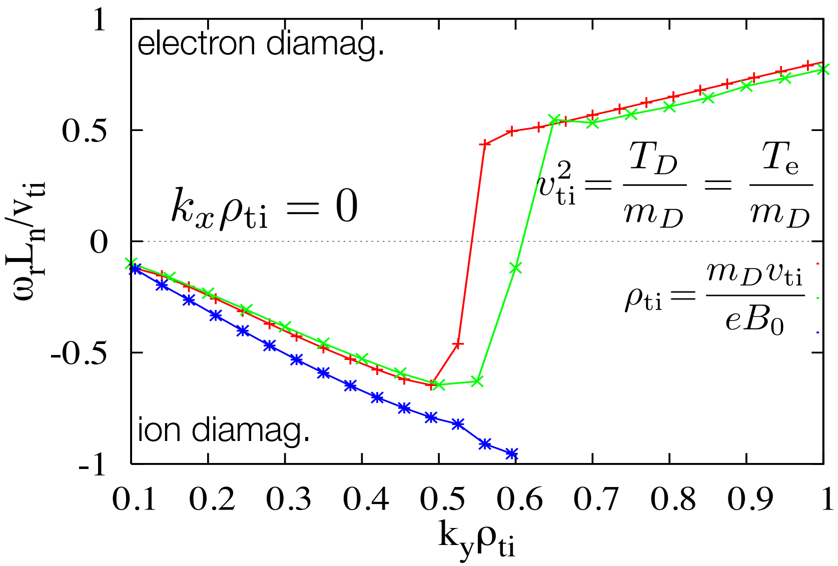
$R_0/L_n = 2.22$

$\eta_{\rm i} = 3.114$

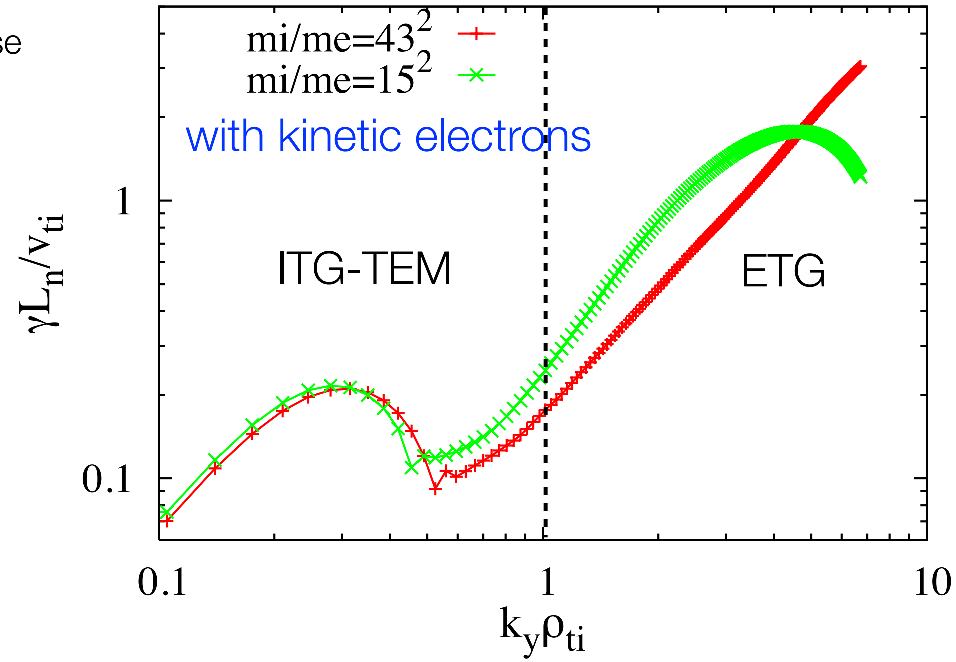
$\eta_{\rm e} = 3.114$

$\eta_{\rm e} \to 0$ (for pure-ITG)

$T_{\rm e}/T_{\rm i} = 1$



- Transition toward ETG modes

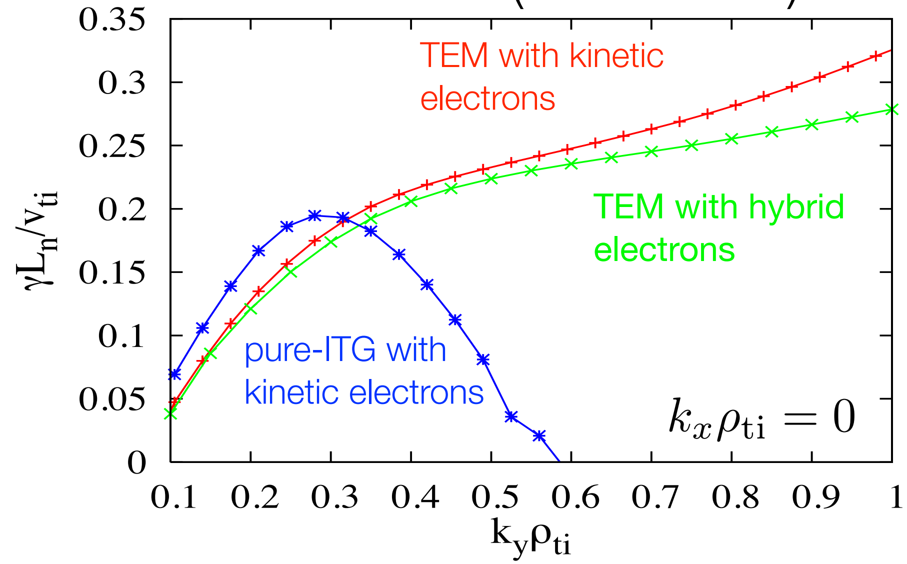


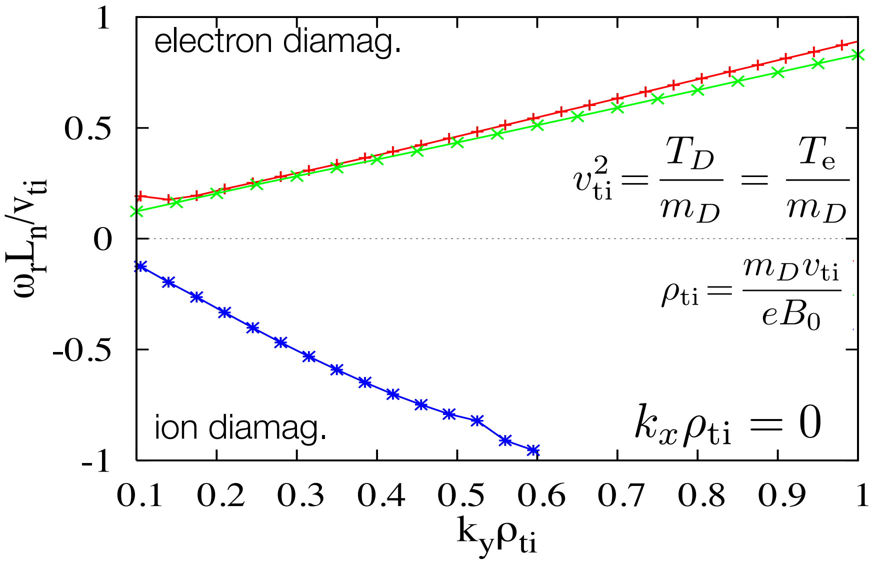
- Since ETG modes are also unstable for the Cyclone base case parameters, ITG-TEM modes are connected to the ETG modes for $k_y\rho_{\rm ti} > 1$.

- Hybrid electrons suppresses the contribution of low-*k* ETG modes for $k_y\rho_{\rm ti} > 0.6$.

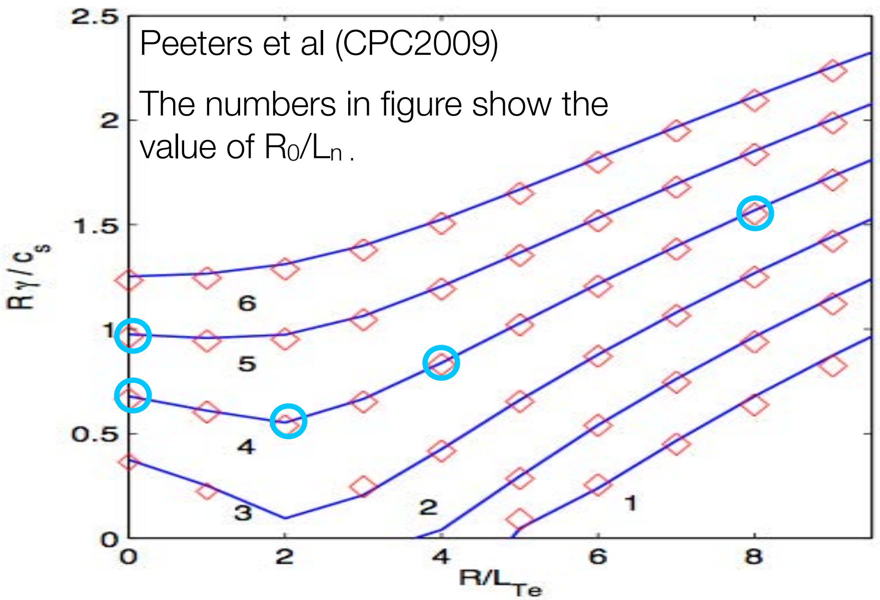
Collisionless linear TEM(-ETG) modes

- TEM modes (also ETGs)





- R_0/L_n & R_0/L_T scan for pure-TEM modes

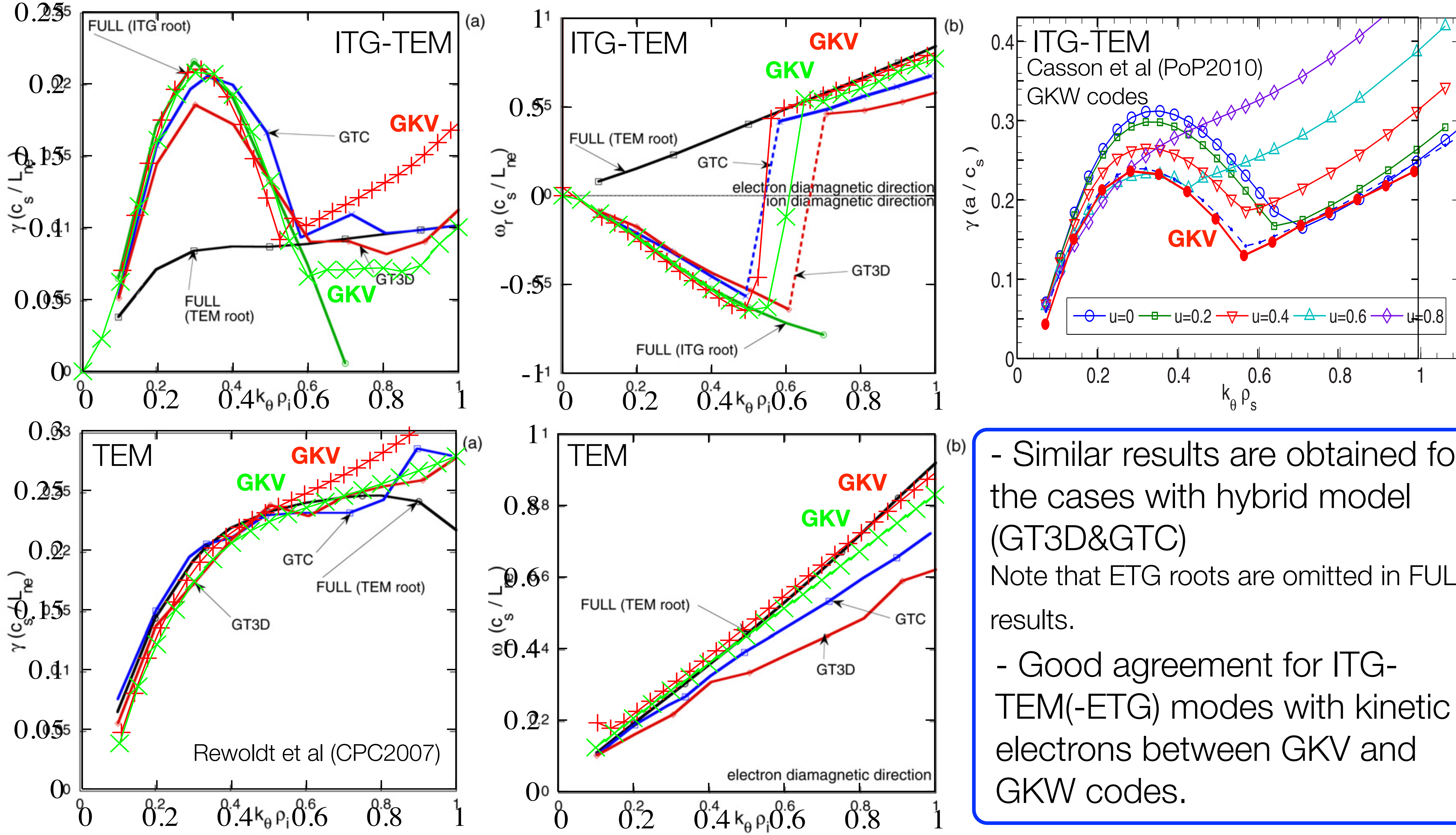


s = 1.07,
q = 1.57,
epsilon = 0.177,
Te/Ti = 3,
R/LTi = 0
mi/me = 3600

line: GS2
diamond: GKW
○: GKV

- TEM modes are caused by trapped particles so that the decrease of growth rate for $k_y\rho_{ti} > 0.6$ is not significant even in the hybrid case.
- R_0/L_n & R_0/L_T scan shows a threshold of the onset of pure-TEM at $R_0/L_n = 2,\ R_0/L_{T_e} = 3.8$.

Benchmark tests with the other codes

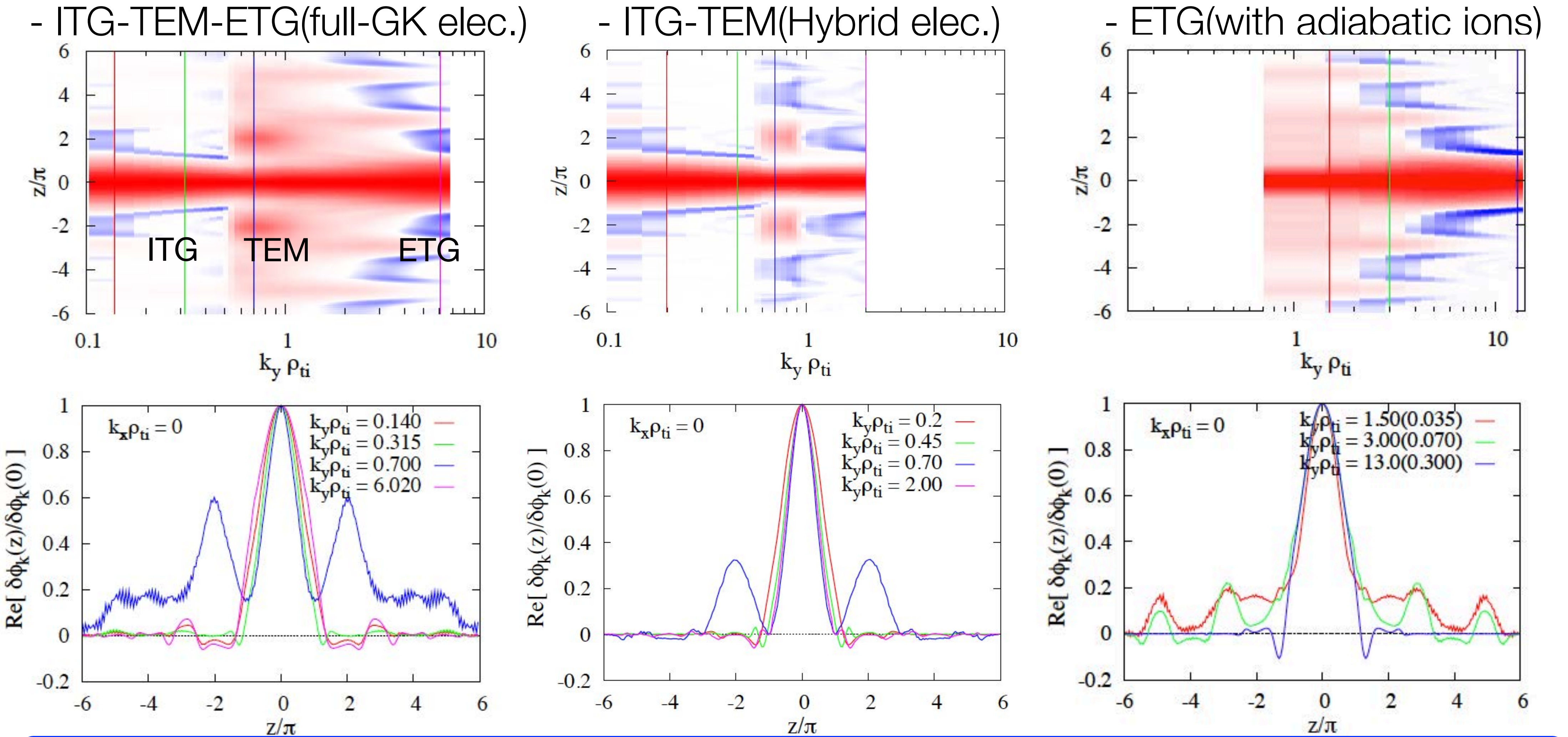


- Similar results are obtained for the cases with hybrid model (GT3D&GTC)

Note that ETG roots are omitted in FULL results.

- Good agreement for ITG-TEM(-ETG) modes with kinetic electrons between GKV and GKW codes.

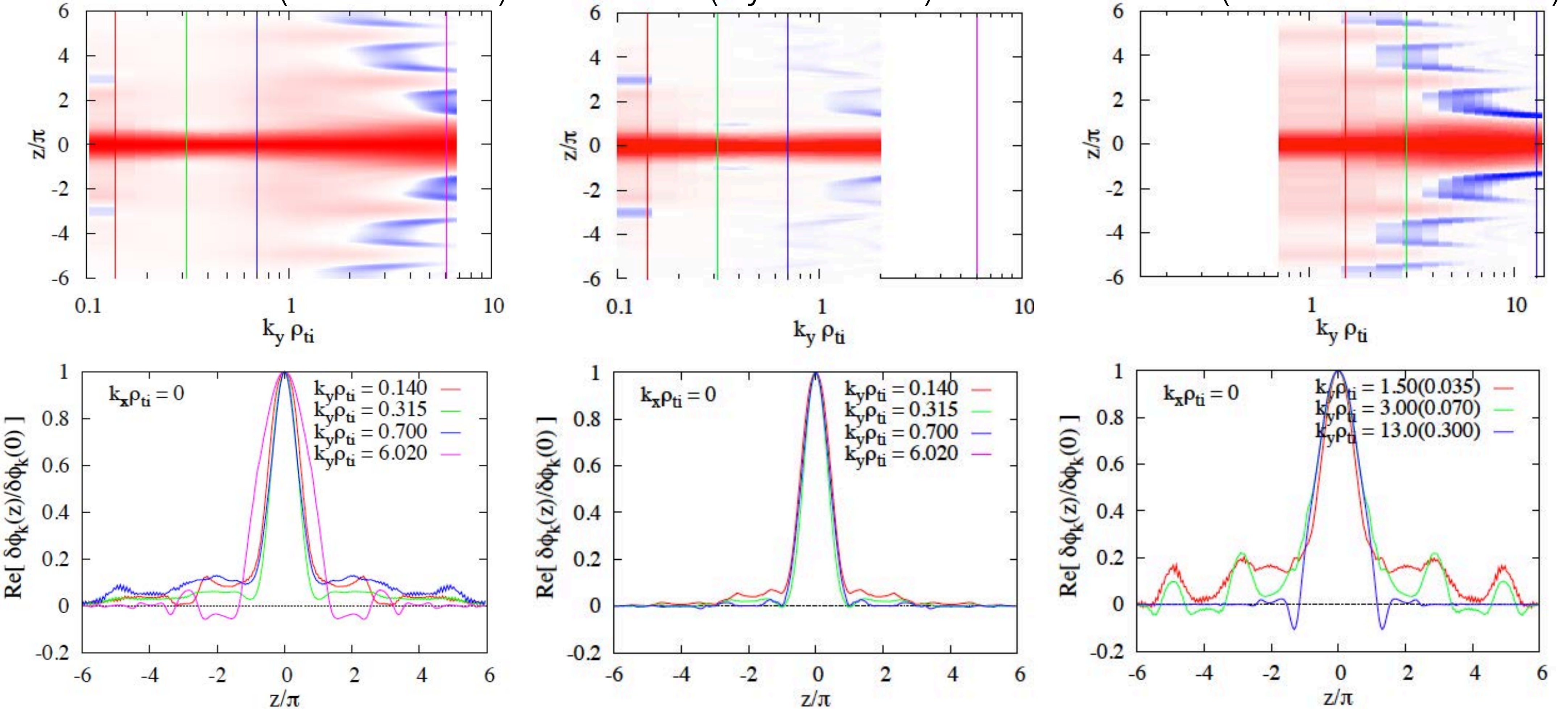
Parallel mode structures in ITG-TEM-ETG modes



- Trapped electrons broaden the mode-width in the parallel direction(with hybrid model)
- But, passing electrons broaden more(with kinetic model), and this is associated with a contribution of long-wavelength modes of ETGs.

Parallel mode structures in TEM-ETG modes

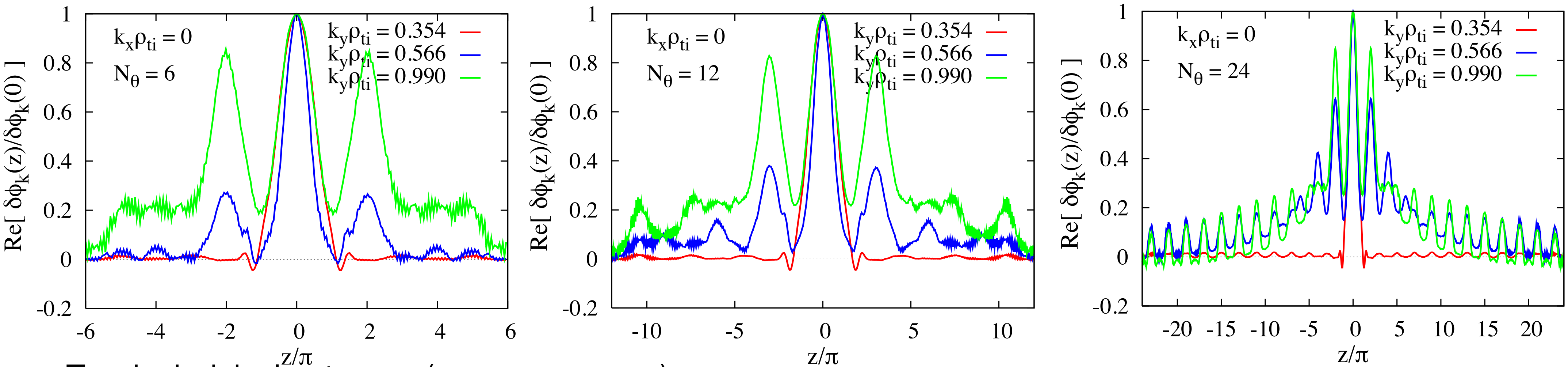
- TEM-ETG(full-GK elec.) - TEM(Hybrid elec.) - ETG(with adiabatic ions)



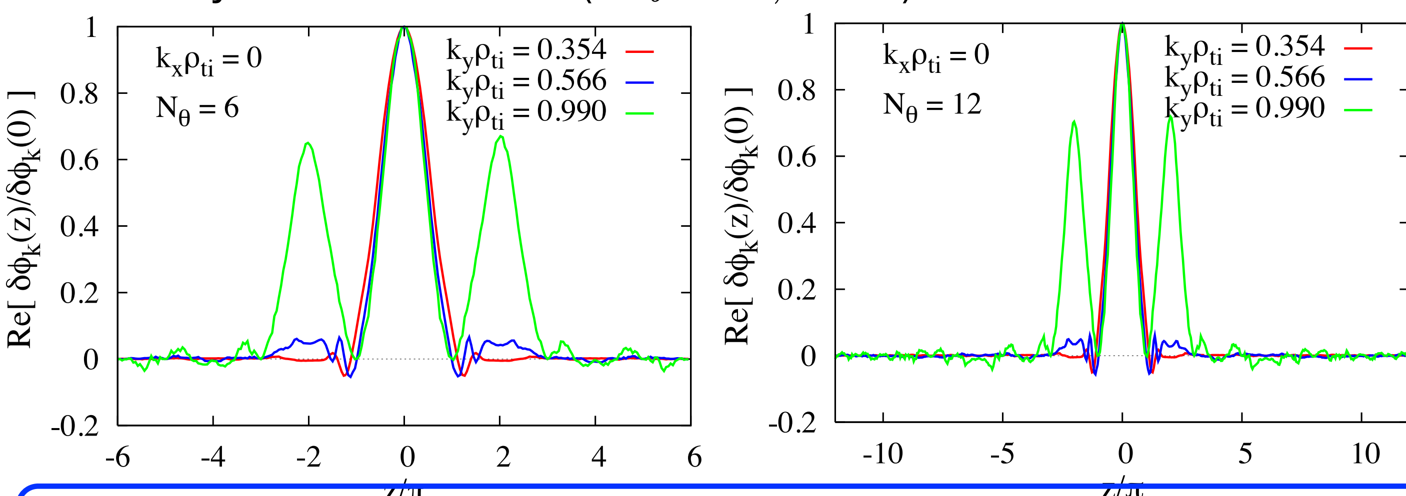
- Passing electrons still broaden the mode-width in the parallel direction, but it indicates almost ballooning type structures.

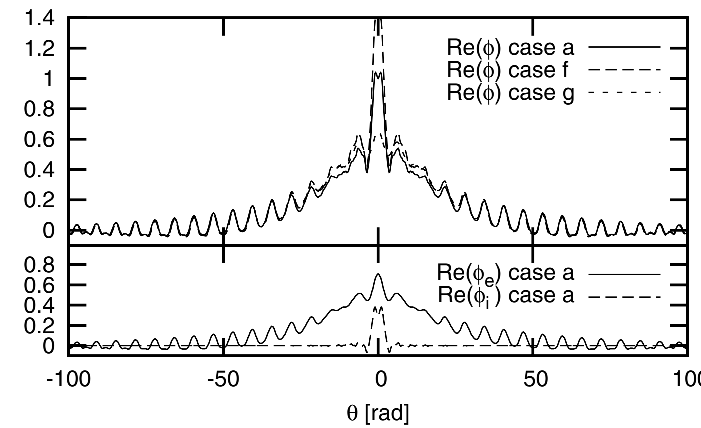
Mode-elongation in parallel direction

- For kinetic electrons ($N_\theta = 6,\ 12,\ 24$)



- For hybrid electrons ($N_\theta = 6,\ 12$)



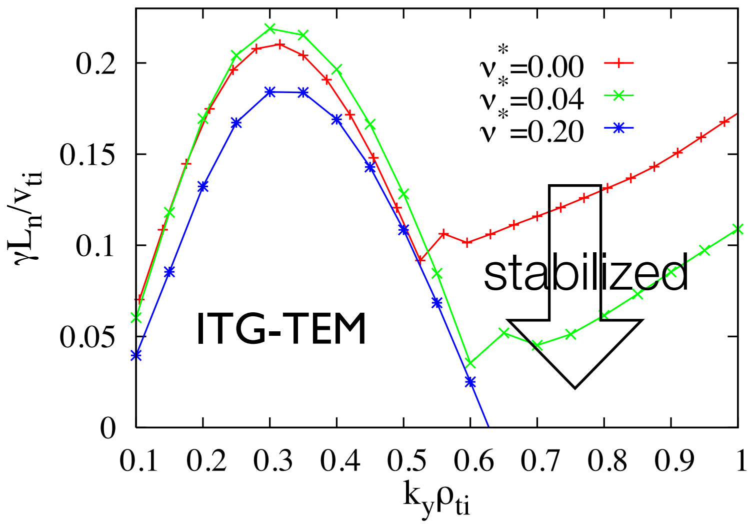


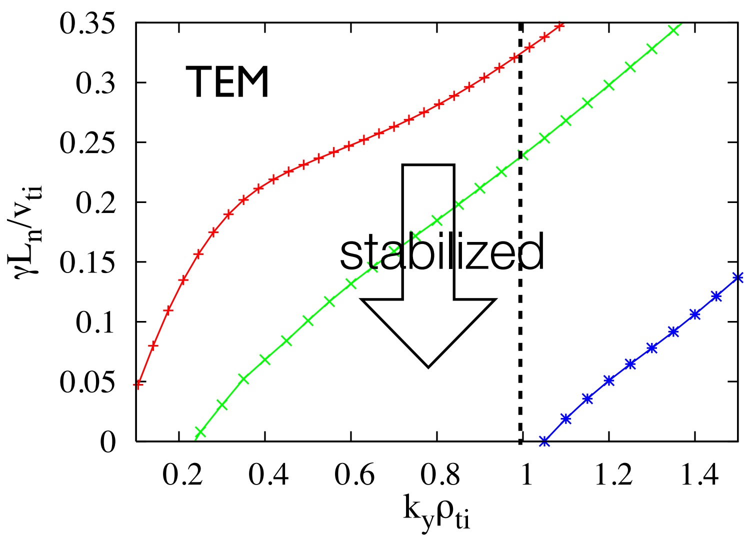
Hallatschek et al (PRL2005)

- 4 poloidal turns are enough for ITG-TEM modes with hybrid electrons, but more than 24 poloidal turns are required to fully resolve the eigenmode with kinetic electrons!

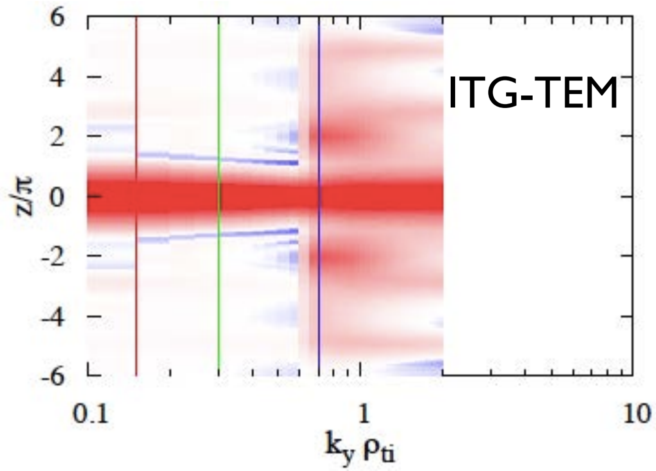
Collisional effects on ITG-TEM/TEM modes

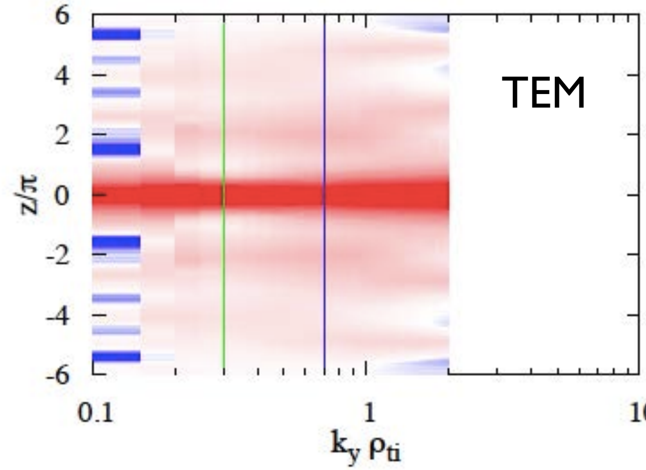
- Collisional stabilization and the effect on mode structures in ITG-TEM/TEM modes

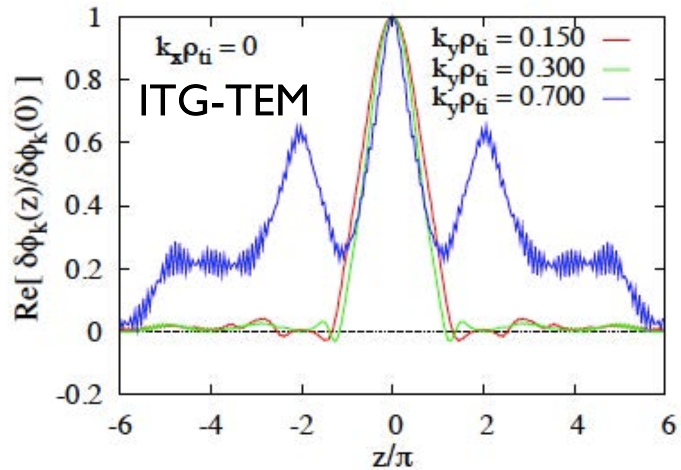


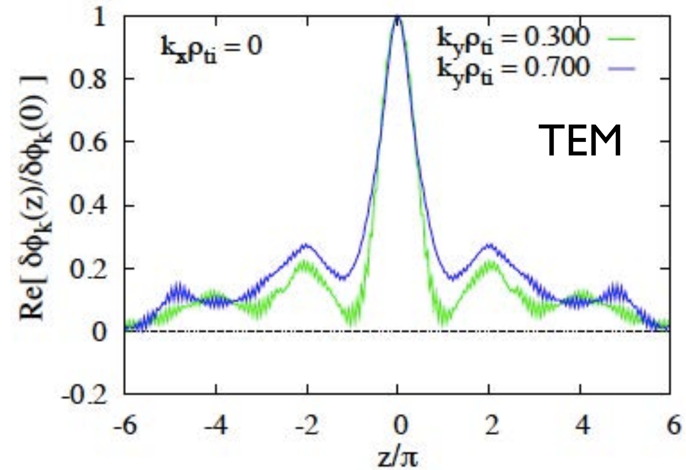


$$\nu_{*i} = \nu_{*e} = 0.04$$









- The influence of collisional effects on mode structures is weak.

Summary

- A conventional single-species model on GKV code is extended to multi-species one, and then the code-verification is confirmed by benchmark tests with ITG-TEM-ETG linear stability analyses.

- The extended version enables us to treat any number of particle species with a MPI-parallelization, to switch kinetic electron responses and hybrid model, and to switch self-consistent interactions and the passive limit for impurity ions.

- Good agreements have been confirmed for the ITG-TEM-ETG modes with kinetic/hybrid electrons among various (local/global/eigenvalue) codes.

- In the ITG-TEM, passing electrons significantly broaden the mode-width in the parallel direction while the trapped ones do not so much.

Future works

- Quasilinear analyses including impurity ions.
- TEM turbulence simulations including a convergence check for $k_{y(\max)}\rho_{\rm ti}$ and N_θ.