Thermodynamic properties of plasmas in magnetic reconnection

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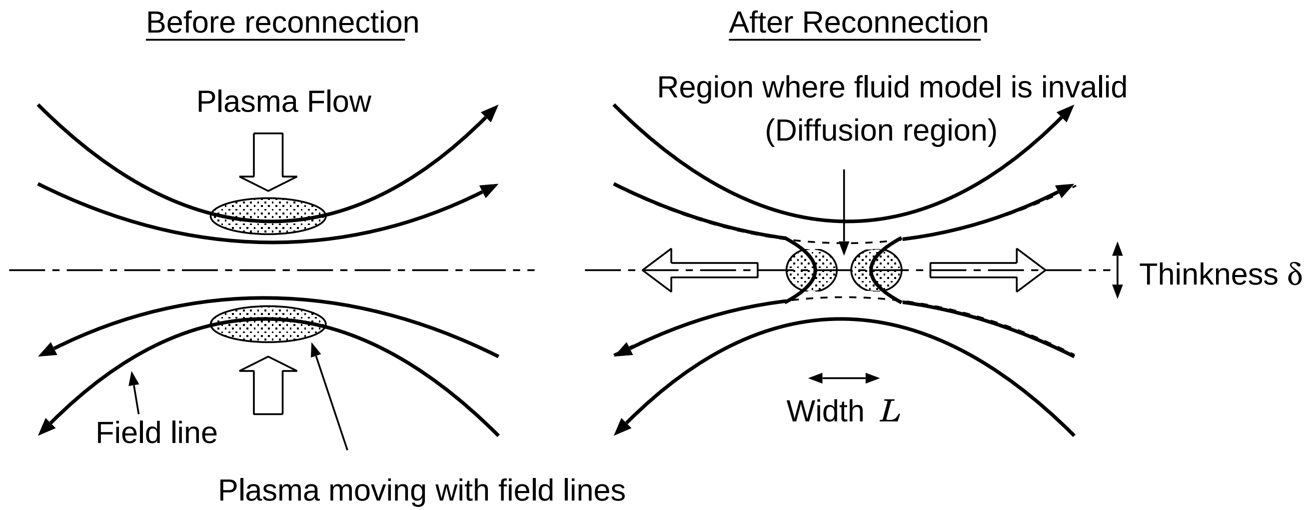
Introduction

Magnetic reconnection: What it looks like?



- Observed in many astrophysical and laboratory plasmas
- Topological re-organization of mag. field lines
- Energy conversion mechanism: Magnetic energy → Bulk flow, Heating, Energetic particles (cosmic rays?)

Challenge to magnetic reconnection

Simple explanation based on MHD theory may fail because of huge scale separation: SP employed resistivitiy ($1/S$) as a field-line breaking mechanism and obtained too slow time scale ($\sim S^{1/2}$) where S huge!

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B}) + \frac{1}{S}\nabla^2 \boldsymbol{B} \tag{1}$$

To explain macro scale events

- Current sheet, Shocks
- Explosive event
- Energy conversion mechanism [heating, energetic particles, jet]
- Transport

We need micro scale physics

- Collision, Resistivity
- Electron dynamics (inertia, pressure tensor)
- Instabilities

Challenge to magnetic reconnection

How to bridge huge scale separation?

- Phenomenological models:
  Fractal reconnection: Tajima and Tanuma (2001)
  Multiple chaos region: Numata and Yoshida (2002)
- Interaction with turbulence
  Plasmoid instability: Loureiro *et al.* (2007).
  Statistics of plasmoid: Uzdensky *et al.* (2010), Drake *et al.*, Bhattacharjee *et al.*.
  Interaction with turbulence: Lazarian and Vishniac (1999), Ishizawa and Nakajima (2010).
  Microtearing (high-m drift tearing) instability:
- Advanced numerical simulations
  Multiscale modeling
  Reduced kinetic model – Gyrokinetics

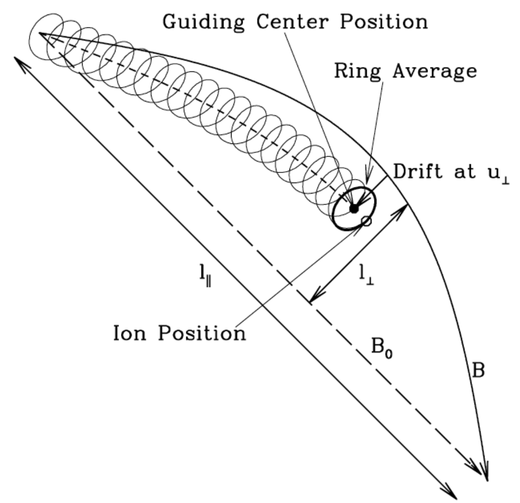
Today's topic: What we can do with collisions

- We consider magnetic reconnection problem using a kinetic model.
- In most of environments, plasmas are considered to be collisionless. However, collisions are still important.
- Only collisions can smooth out velocity space structures of distribution functions, and can lead the system to thermodynamic equilibrium.
- With appropriate treatment of collisions, we can handle smooth transition from collisional (fluid-like) to collisionless regime (kinetic).
- Energy partitioning and plasma heating can be correctly understood with collisions.

Gyrokinetics and `AstroGK`

Gyrokinetics

- Reduced kinetic model – 5 dimensional phase space
- Mean magnetic field (B_0) allows to separate out fast cyclotron motion
- Fast-MHD waves, the cyclotron resonance are ordered out; finite Larmor radius effects, Landau damping are kept
- Linear formalism by Rutherford & Frieman (1968), Taylor & Hastie (1968), and extended to nonlinear regime by Friemann & Chen (1982).
- To study turbulence in laboratory plasmas driven by microinstabilities, e.g. the ion and electron temperature gradient instabilities.
- Useful for astrophys. plasmas as well as fusion plasmas



$$\epsilon \sim \frac{k_\parallel}{k_\perp} \sim \frac{\omega}{\Omega_i} \quad (2)$$

Distribution function f and electromagnetic fields are expanded ϵ:

$$f = f_0 + \delta f_1 + \mathcal{O}(\epsilon^2), \quad (3)$$

$$\boldsymbol{B} = B_0 \hat{\boldsymbol{z}} + \nabla \times \boldsymbol{A}, \quad (4)$$

$$\boldsymbol{E} = -\nabla\phi + \frac{\partial \boldsymbol{A}}{\partial t} \quad (5)$$

Gyrokinetics equation

The distribution function of particles is given by $f = \left(1 - \frac{q\phi}{T_0}\right) f_0 + h$, where $f_0 = n_0/(\sqrt{\pi} v_{\mathrm{th}})^3 \exp(-v^2/v_{\mathrm{th}}^2)$ is the Maxwellian, and the thermal velocity is given by $v_{\mathrm{th}} = \sqrt{2T_0/m}$. The equations to solve are the gyrokinetic equation for $h = h(\boldsymbol{R}, V_\perp, V_\parallel)$,

$$\frac{\partial h}{\partial t} + V_\parallel \frac{\partial h}{\partial Z} + \frac{1}{B_0}\{\langle\chi\rangle_{\boldsymbol{R}}, h\} - \langle C(h)\rangle_{\boldsymbol{R}} = q\frac{f_0}{T_0}\frac{\partial\langle\chi\rangle_{\boldsymbol{R}}}{\partial t}, \qquad (6)$$

where $\chi = \phi - \boldsymbol{v}\cdot\boldsymbol{A}$, and gyro-center coordinate $(\boldsymbol{R}_s, \boldsymbol{V}_s)$ is defined by

$$\boldsymbol{R}_s = \boldsymbol{r} + \frac{\boldsymbol{v}\times\hat{\boldsymbol{z}}}{\Omega_s}, \quad \boldsymbol{V}_s = \boldsymbol{v}. \qquad (7)$$

The angle bracket $\langle\cdot\rangle_{\boldsymbol{R}_s}$ denotes the gyro-average at fixed gyro-center coordinate $\boldsymbol{R}_s$:

$$\langle F(\boldsymbol{r})\rangle_{\boldsymbol{R}_s} = \frac{1}{2\pi}\oint F\left(\boldsymbol{R_s} + \frac{\boldsymbol{V}_s\times\hat{\boldsymbol{z}}}{\Omega_s}\right) \mathrm{d}\Theta_s. \qquad (8)$$

Gyrokinetics Field Equatiions

Gyrokinetic field equations for $\phi(\boldsymbol{r})$, $A_\parallel(\boldsymbol{r})$, and $\delta B_\parallel(\boldsymbol{r})$,

$$\sum_s \left[-\frac{q_s^2 n_{0s}\phi}{T_{0s}} + q_s \int \langle h_s \rangle_{\boldsymbol{r}} \mathrm{d}\boldsymbol{v} \right] = 0, \tag{9}$$

$$\nabla_\perp^2 A_\parallel = -\mu_0 \sum_s q_s \int \langle h_s \rangle_{\boldsymbol{r}} v_\parallel \mathrm{d}\boldsymbol{v} \tag{10}$$

$$B_0 \nabla_\perp \delta B_\parallel = -\mu_0 \nabla_\perp \cdot \sum_s \int \langle m_s \boldsymbol{v}_\perp \boldsymbol{v}_\perp h_s \rangle_{\boldsymbol{r}} \mathrm{d}\boldsymbol{v}. \tag{11}$$

Note that the velocity moments are defined as usual:

$$\delta n = \int \left(-\frac{q\phi}{T_0} f_0 + h \right) \mathrm{d}\boldsymbol{v}, \tag{12}$$

$$n_0 \delta u_\parallel = \int \left(-\frac{q\phi}{T_0} f_0 + h \right) v_\parallel \mathrm{d}\boldsymbol{v}, \tag{13}$$

$$\delta \overleftrightarrow{P}_{\perp\perp} = \int \left(-\frac{q\phi}{T_0} f_0 + h \right) m \boldsymbol{v}_\perp \boldsymbol{v}_\perp \mathrm{d}\boldsymbol{v}. \tag{14}$$

Reduced-MHD: fluid counterpart

Applying the same ordering to MHD we obtain reduced-MHD.
Alfvén wave:

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} = v_{\mathrm{A}} \frac{\partial \phi}{\partial z}, \tag{15}$$

$$\frac{\mathrm{d}\nabla_\perp^2 \phi}{\mathrm{d}t} = v_{\mathrm{A}} \hat{b} \cdot \nabla \nabla_\perp^2 \psi \tag{16}$$

Slow wave:

$$\frac{\mathrm{d}}{\mathrm{d}t} \frac{\delta B_\parallel}{B_0} = \frac{1}{1 + v_{\mathrm{A}}^2 / c_{\mathrm{s}}^2} \hat{b} \cdot \nabla u_\parallel, \tag{17}$$

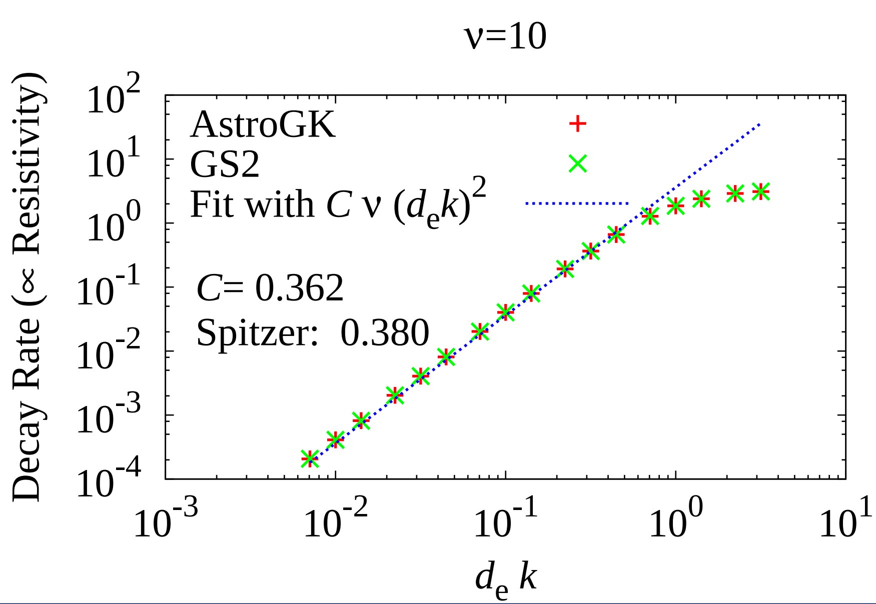
$$\frac{\mathrm{d}u_\parallel}{\mathrm{d}t} = v_{\mathrm{A}}^2 \hat{b} \cdot \nabla \frac{\delta B_\parallel}{B_0}. \tag{18}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \{\phi, \cdot\}, \quad \hat{b} \cdot \nabla = \frac{\partial}{\partial z} + \frac{1}{v_{\mathrm{A}}} \{\psi, \cdot\} \tag{19}$$

Collisions

Recently, linearized collision operators for gyrokinetic simulations, which satisfies physical requirements are established and implemented in `AstroGK` [Abel *et al*, 2008; Barnes *et al*, 2009]

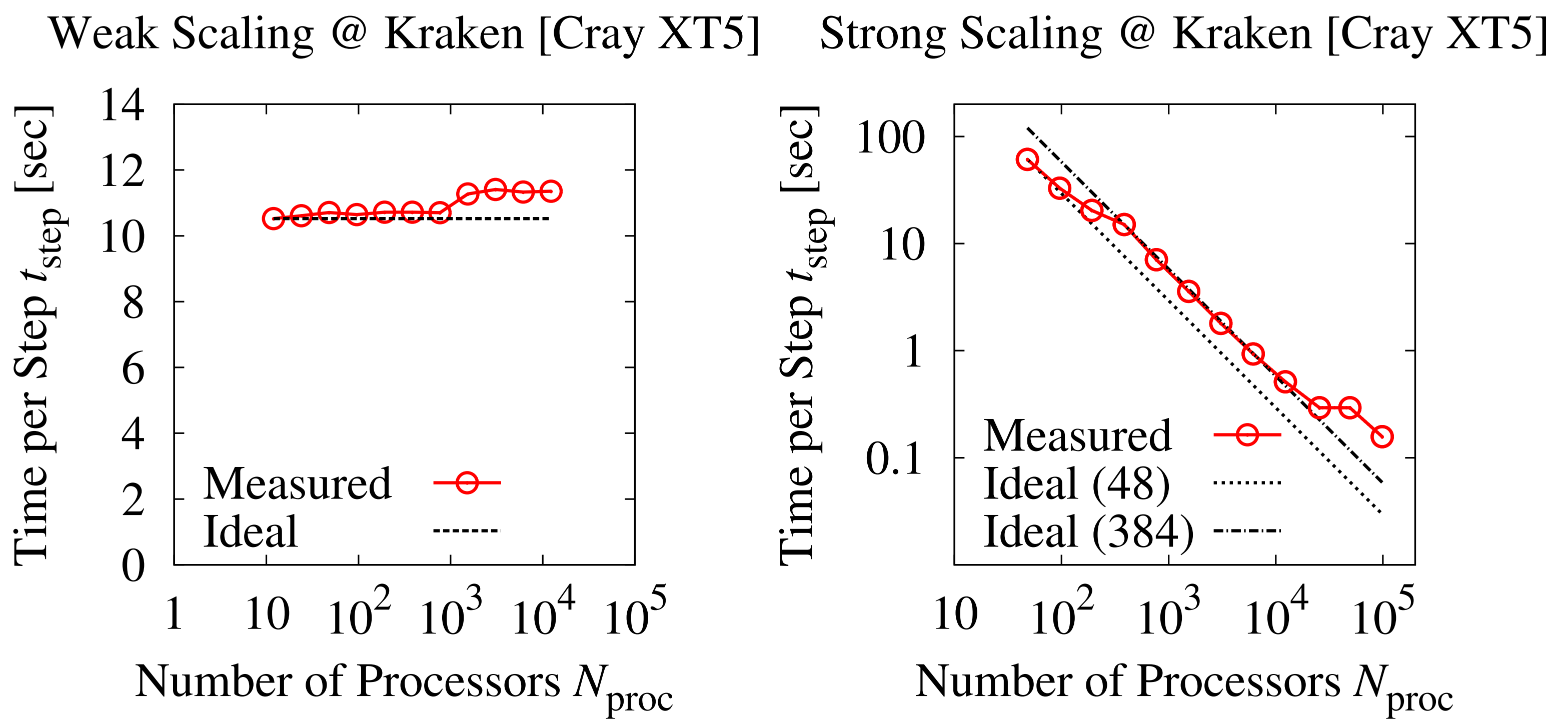
The operators are the pitch-angle scattering (Lorentz), the energy diffusion, and moments conserving corrections to those operators for like-particle collisions. Electron-ion collisions consists of pitch angle scattering by background ions and ion drag are also included.



AstroGK: Astrophysical Gyrokinetics Code

- Derived from GS2
- Eulerian continuum, local flux tube, δf, electromagnetic code.
- Publicly available at https://sourceforge.net/projects/gyrokinetics/.
- Numata *et al.*, J. Comput. Phys. **229**, 9347 (2010); Corrigendum, ibid **245**, 493 (2013).
- Pitch-angle scattering (Lorentz), energy diffusion, and moments conserving corrections for like-particle collisions. Electron-ion collisions provides resistivity.
- Fourier spectral in x and y, finite difference in z, Gaussian quadrature for velocity space integration, finite difference on non-uniform grids in velocity space.
- Implicit Euler for linear terms, 3rd order Adams-Bashforth for nonlinear terms.
- Pure MPI parallelized. (x, y, λ, E, s)

AstroGK: Scalings



Tearing Instability

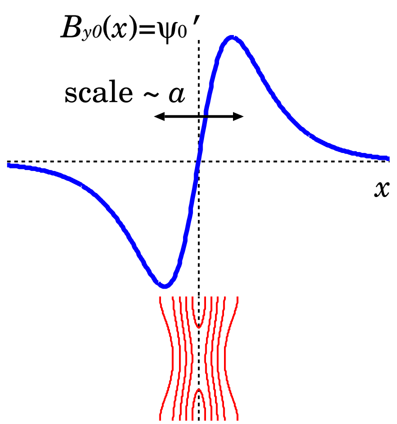
Tearing instability

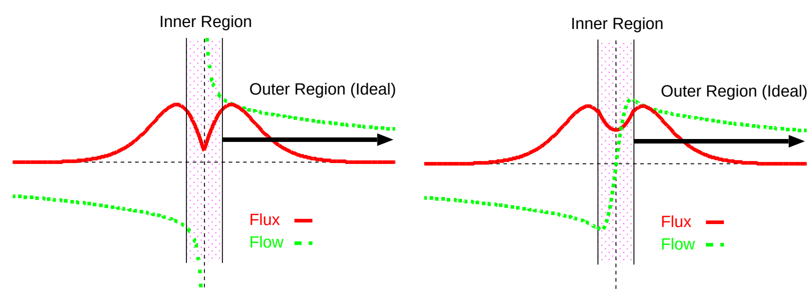
- Tearing instability is a resistive instability of current sheet configuration
- Spontaneous onset of reconnection process
- Linear stage
- Standard boundary layer or singular perturbation problem

Without resistivity and inertia, solution is singular

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} - \left(k_y^2 + \frac{\psi_0'}{\psi_0'''}\right)\psi = 0 \qquad (20)$$

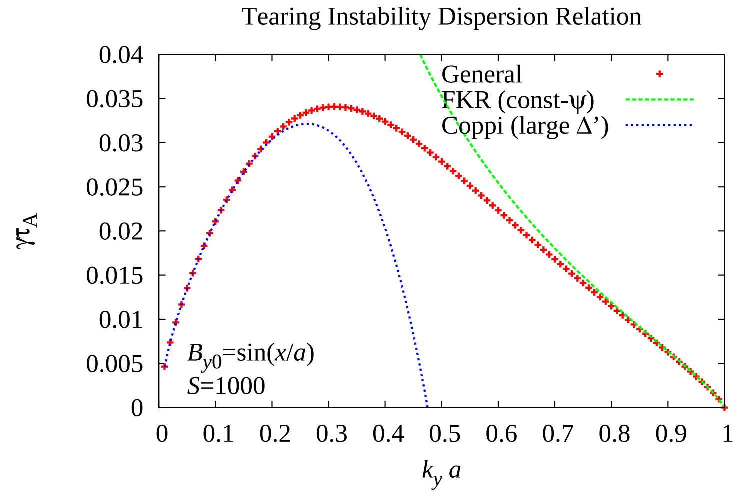
BG current profile

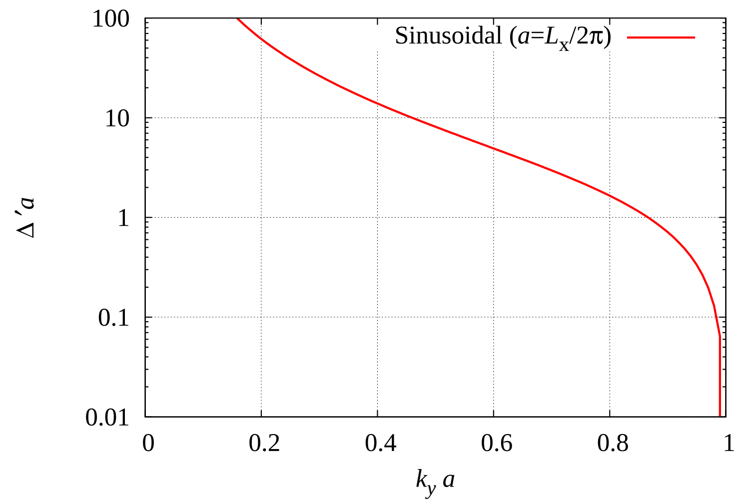




Dispersion relation

Typical form of dispersion relation of tearing instability.
Left: growth rate, Right: Stability Index Δ'





Large Δ' $\quad \gamma\tau_{\mathrm{A}} \sim S^{-1/3}$
Small Δ' $\quad \gamma\tau_{\mathrm{A}} \sim S^{-3/5}$

Problem Setup

- Uniform background ($\nabla n_0 = \nabla T_0 = \nabla B_{0z} = 0$), two-dimensional ($\partial/\partial z = 0$)
- Parameters: $\sigma \equiv m_\mathrm{e}/m_\mathrm{i}$, $\tau \equiv T_{0\mathrm{i}}/T_{0\mathrm{e}}$, β_e, ρ_Se/a:
  $\rho_\mathrm{i} = \sqrt{2\tau}\rho_\mathrm{Se}$, $\rho_\mathrm{e} = \sqrt{2\sigma}\rho_\mathrm{Se}$, $d_\mathrm{i} = \sqrt{2/\beta_\mathrm{e}}\rho_\mathrm{Se}$, $d_\mathrm{e} = \sqrt{2\sigma/\beta_\mathrm{e}}\rho_\mathrm{Se}$.
- Equilibrium magnetic field: $\boldsymbol{B}_0 = \left(\partial A_\parallel^\mathrm{eq}/\partial x\right)\hat{y} + B_{0z}\hat{z}$

$$A_\parallel^\mathrm{eq}(x) \sim \cosh^{-2}\left(\frac{x - L_x/2}{a}\right) \tag{21}$$

Maximum of B_y defines the Alfvén time:

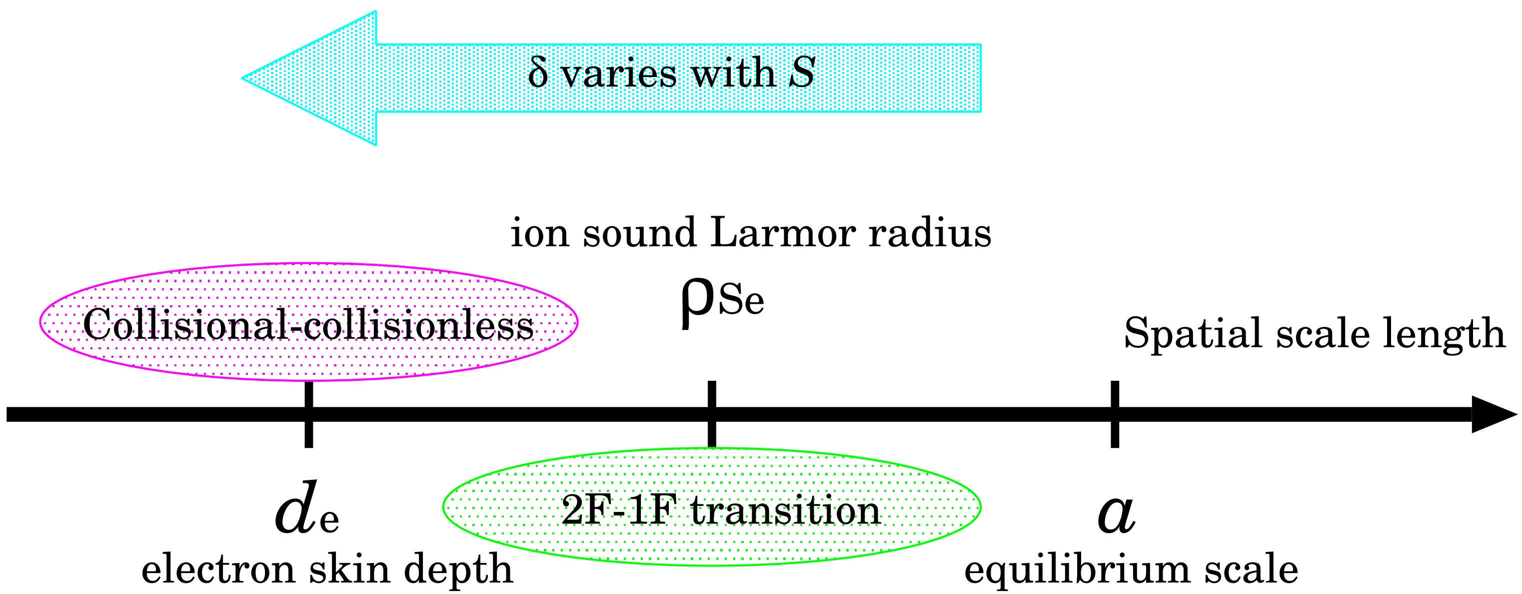
$$\tau_\mathrm{A} = a/\left(B_y^\mathrm{max}/\sqrt{\mu_0 n_0 m_\mathrm{i}}\right) \tag{22}$$

- Perturbation: $k_y a = 2\pi a/L_y = 0.8$, $\Delta' a = 23.2$.

$$\tilde{A}_\parallel \sim \cos(2\pi/L_y y) \tag{23}$$

Problem setup

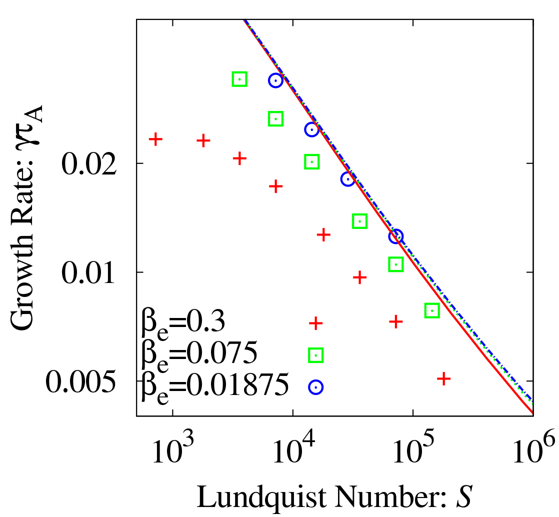
Collisionality ν_e is scanned to vary current layer width δ. As ν_e is decreased, the current layer width becomes narrower, and the ion and electron kinetic scales become important.

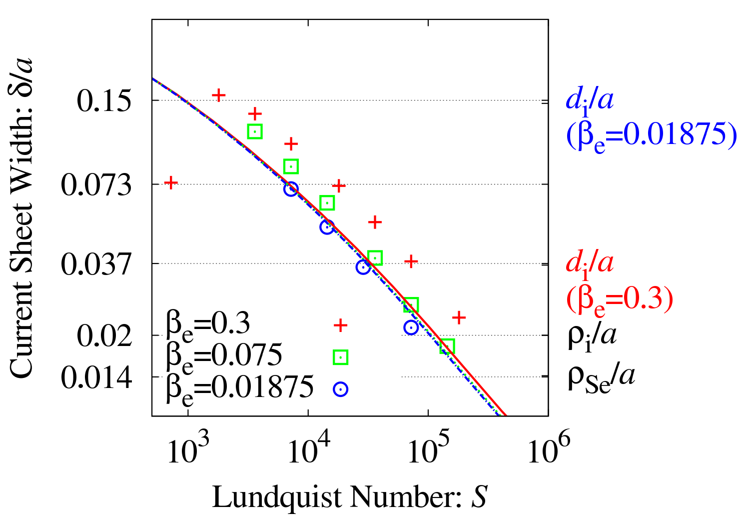


Case 1 electron scale $\ll$ ion scale $\sim \delta \ll$ equili. scale
Case 2 electron scale $\sim \delta \ll$ ion scale $\lesssim$ equili. scale

MHD to two-fluid MHD

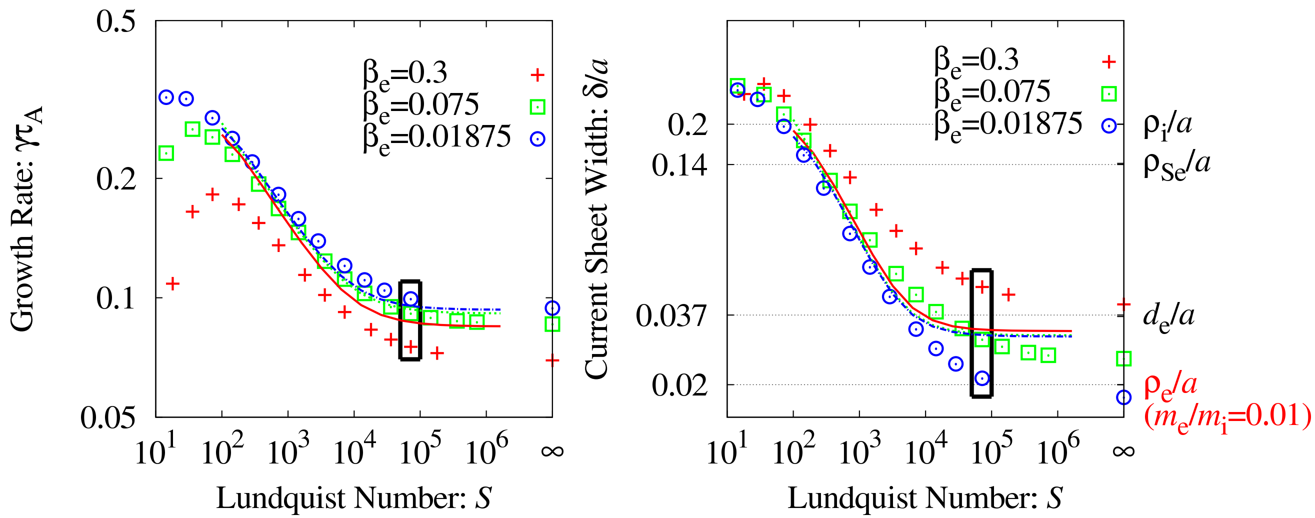
- Growth rate and current layer width scaling against Lundquist number S are obtained from GK simulation. [dots]
- Scalings are compared with reduced two-fluid model by Fitzpatrick (Fitzpatrick, 2010). [lines]
- GK and 2F results agree well only for low-β_e.
- 2F model assumes $\beta_\mathrm{e} \ll \sqrt{m_\mathrm{e}/m_\mathrm{i}}$, which is marginally satisfied for $\beta_\mathrm{e} = 0.01875$ case.
- Eqn. of state used in 2F model may not be valid.



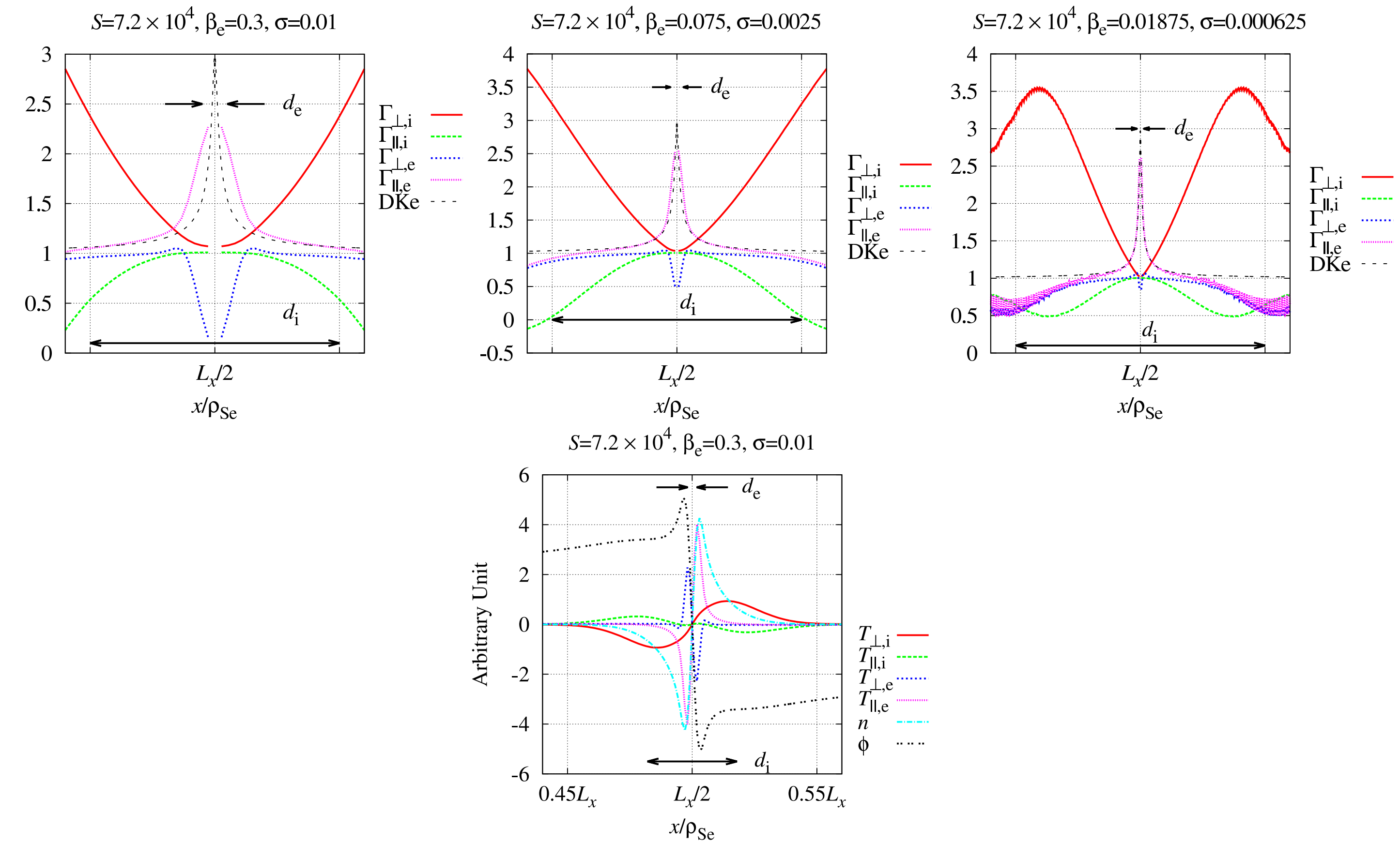


Collisional–collisionless transition

- Collisional-collisionless transitional regime is reproduced: For large S, electron inertia mediates reconnection instead of collisions.



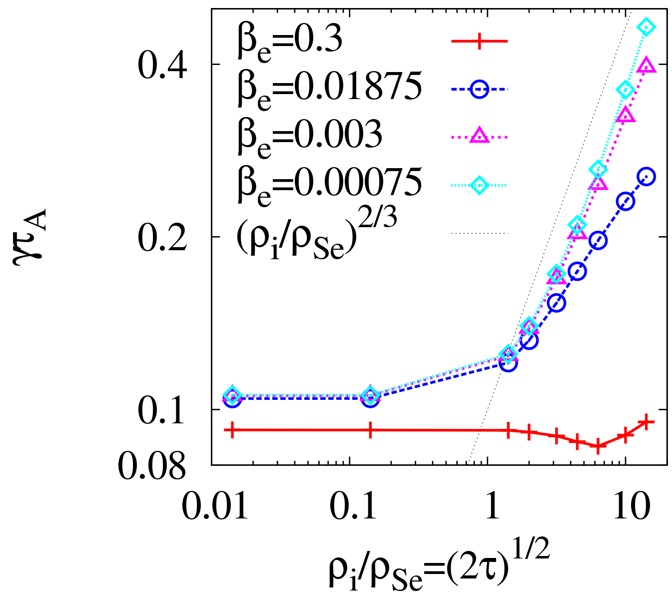
Temperature fluctuations



Eigenfunctions and polytropic indices: $(\delta p = T_0 \delta n + n_0 \delta T = \Gamma T_0 \delta n)$.

Ion temp. dependence: kinetic Alfvén wave

- Ion temperature ($\tau \equiv T_{0\mathrm{i}}/T_{0\mathrm{e}}$) dependence.
- Theoretical prediction, $\gamma\tau_{\mathrm{A}} \sim \tau^{1/3}$, because of the transition of Alfvén wave to kinetic Alfvén wave.
- For higher β_{e}, sound wave couples to Alfvén wave. A compressible effect play a role, and $\tau^{1/3}$ dependence is no longer seen.



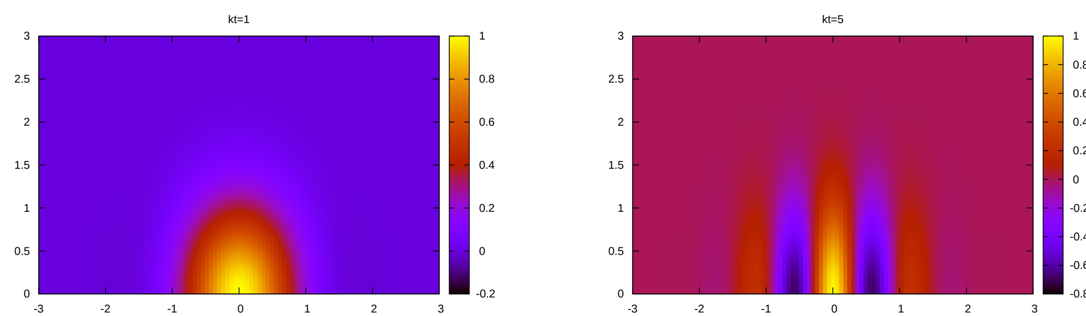
Phase mixing and heating

Landau damping (parallel phase mixing)

Linearized kinetic equation:

$$\frac{\partial \tilde{f}}{\partial t} + v\frac{\partial \tilde{f}}{\partial z} = \delta(t=0) f_0(z,v) \tag{24}$$

Solution is given by $\tilde{f}_k = f_{\mathrm{M}} e^{\mathrm{i}k(z-vt)}$ for $f_0 \propto f_{\mathrm{M}}$. The moment will decay as $n_1 \propto e^{-k^2 v_{\mathrm{th}}^2 t^2/2}$



Progressively oscillatory structure in v-space develops, which is susceptible to collisional dissipation

Nonlinear FLR effects (perp. phase mixing)

Dorland & Hammett (1993)
$v_\perp$ dependence comes in via the gyroaveraging operation which is given in Fourier space by

$$\langle \phi \rangle = \sum_{\boldsymbol{k}_\perp} J_0(k_\perp v_\perp / \Omega) \phi_{\boldsymbol{k}_\perp}, \tag{25}$$

And, GK equation gives

$$\frac{\partial \tilde{f}}{\partial t} + J_0 v_E \frac{\partial \tilde{f}}{\partial z} = \delta(t=0) f_0(z, v) \tag{26}$$

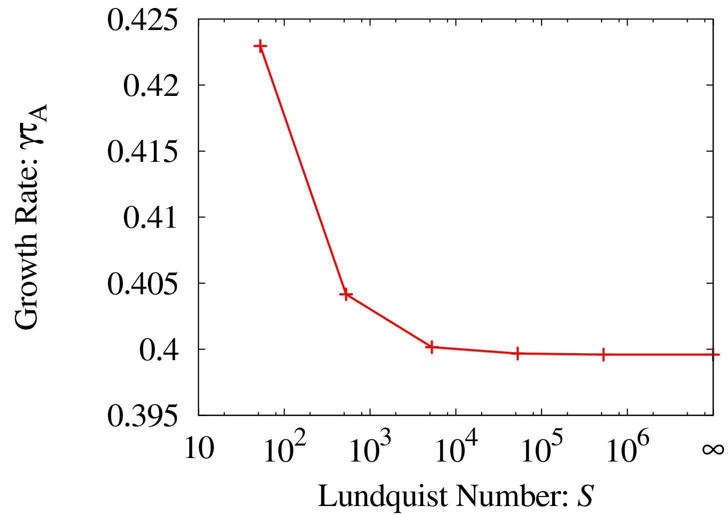
This $v_\perp$ dependence also gives damping,

$$n_1 \propto \frac{1}{1 - \mathrm{i} k_x b v_E t/2} \quad b = k_y^2 v_{\mathrm{th}}^2 / \Omega^2 \tag{27}$$

but it has long tail $\propto 1/t$.

Simulation Setup

- Same as the previous linear tearing case
- Parameters: $m_\mathrm{i}/m_\mathrm{e} = 100$, $T_{0\mathrm{i}}/T_{0\mathrm{e}} = 1$, $\beta_\mathrm{e} = 0.01$, $\rho_\mathrm{i}/a = 0.25$, $\rho_\mathrm{i} = d_\mathrm{e} = 0.1 d_\mathrm{i} = 10\rho_\mathrm{e}$.
- $\nu_\mathrm{e}\tau_\mathrm{A}$ is scanned: $8 \times 10^{-2} \sim 8 \times 10^{-5}$ $(S = 530 \sim 530,000)$
- Linear Growth



To estimate plasma heating, we measure the collisional energy dissipation rate is

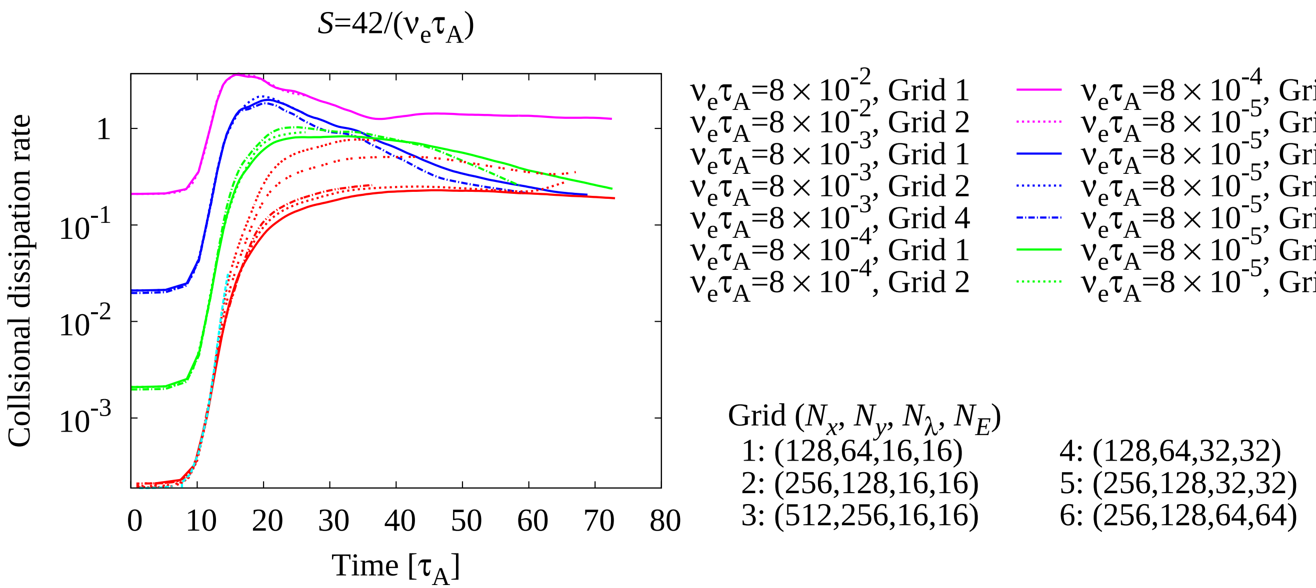
$$D_s = -\left[\int\int\left\langle \frac{T_{0s}h_s}{f_{0s}}\left(\frac{\partial h_s}{\partial t}\right)_{\mathrm{coll}}\right\rangle_{\boldsymbol{r}}\right]\mathrm{d}\boldsymbol{r}\mathrm{d}\boldsymbol{v} > 0. \qquad (28)$$

Without collisions, the gyrokinetic energy conserves the generalized energy consisting of the particle parts E_s^{p} and the magnetic field part $E_{\perp,\parallel}^{\mathrm{m}}$

$$W = \sum_s E_s^{\mathrm{p}} + E_\perp^{\mathrm{m}} + E_\parallel^{\mathrm{m}} = \int\left[\sum_s\int\frac{T_{0s}\delta f_s^2}{2f_{0s}}\mathrm{d}\boldsymbol{v} + \frac{\left|\delta\boldsymbol{B}\right|^2}{2\mu_0}\right]\mathrm{d}\boldsymbol{r}. \quad (29)$$

The generalized energy is dissipated by collisions as $\mathrm{d}W/\mathrm{d}t = -\sum_s D_s$. The collisional dissipation increases the entropy δS_s ($E_s^{\mathrm{p}} = T_{0s}\delta S_{\mathrm{s}}$), and is turned into the thermal energy of background plasma (heating).

Electron heating diagnostics



- Collisional dissipation rate for different collisionality
- Collisional dissipation rate remains finite as $\nu_e \to 0$
- Large number of grids in velocity space is necessary for weakly collisional case

Time Evolution

Reconnection rate measured by the electric field at the X point. The peak reconnection reaches ~ 0.2, achieving the fast reconnection.

Collisionally dissipated energy is about 1% of the initial magnetic energy after dynamical phase $(t/\tau_A = 25)$. The energy dissipation starts to grow rapidly when the maximum reconnection rate is achieved. It stays long after the dynamical stage, and an appreciable amount is lost in the later time.

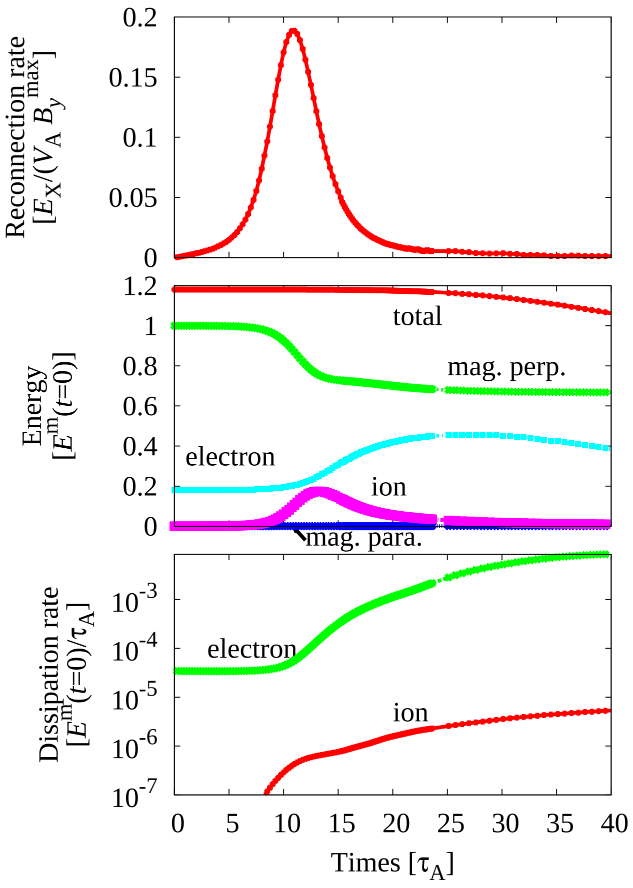
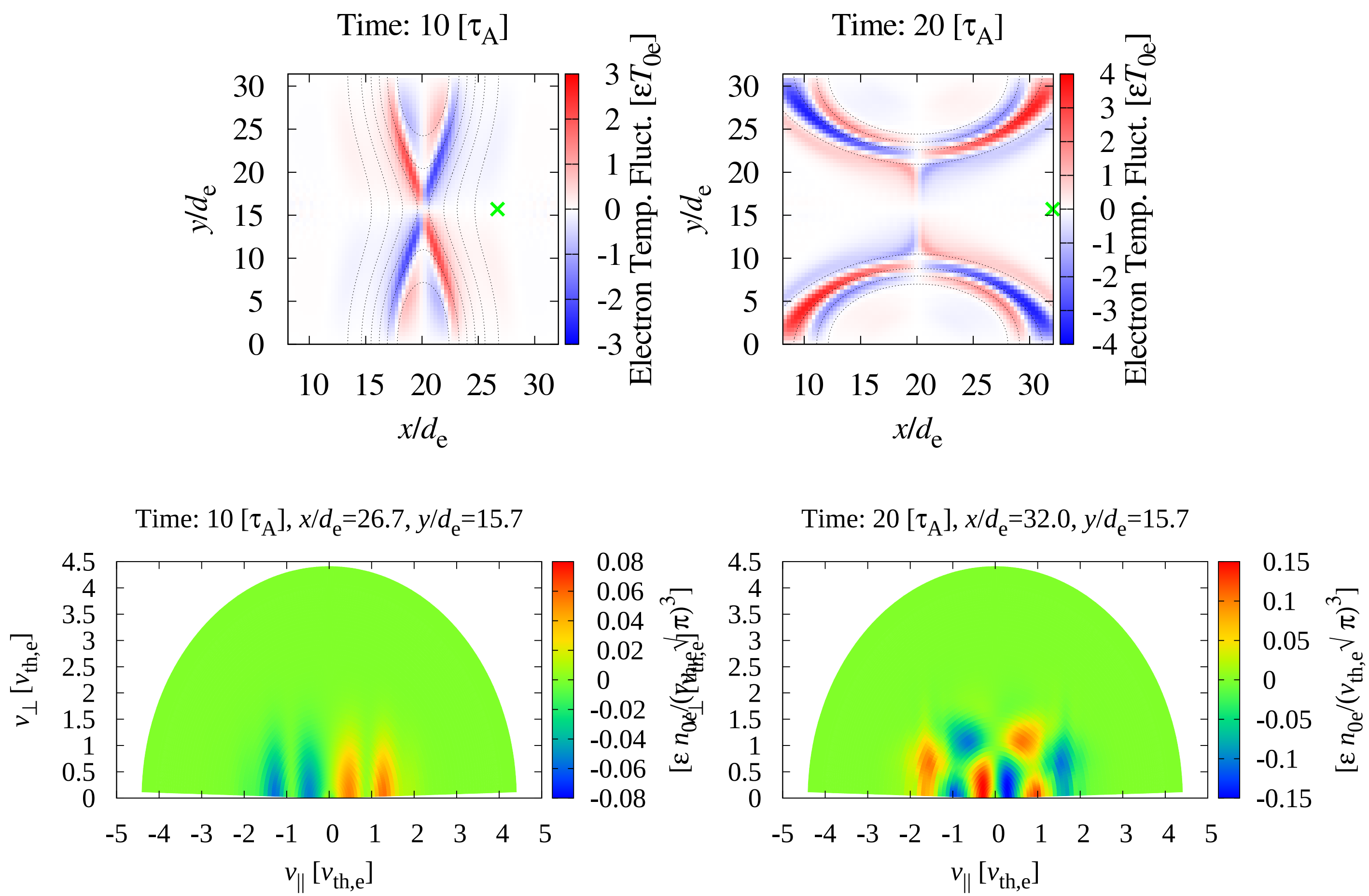


Figure 1: Time evolution of reconnection rate, energies, and dissipation rate

Velocity Space Structure



Velocity space structures show oscillatory structures in both parallel and perpendicular directions.

High-β case

In high-β plasms, compressible fluctuations will be excited which are strongly damped collisionlessly. This may open up another dissipation channel where phase mixing of the ion distribution function ends up with ion heating.

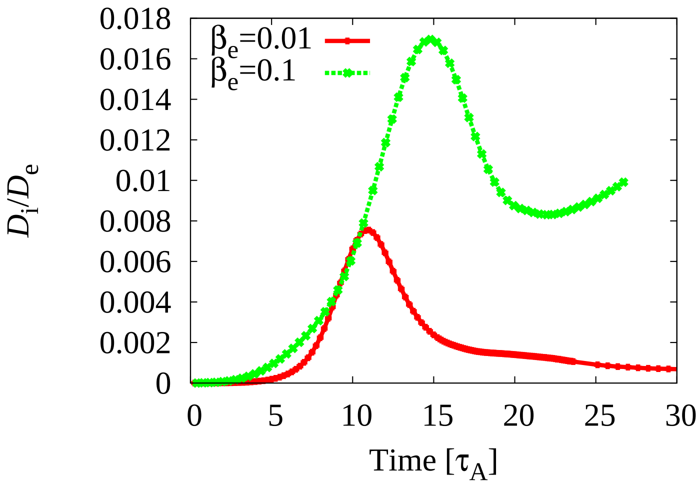


Figure 2: Ratio of ion to electron heating ratio: $D_{\rm i}/D_{\rm e}$

Heating ratio of ions to electrons increases with increasing β though it is still small. Ion heating may be relevant for much higher-β plasmas.

Summary

Summary

- Gyrokinetics is a reduced kinetic framework, which is applicable to various astrophysical problems as well as fusion plasmas.
- We have developed an electromagnetic gyrokinetics code `AstroGK`, and have verified its usefuleness.
- We have studies magnetic reconnection using AstroGK with emphasis on collision effects.
- Scaling law of linear tearing mode against collisionality is studied: Scaling law in MHD regimes, collisional-collisionless transition is successfully reproduced. For high-β plasmas, equation of state should be carefully considered because of the coupling between ion sound wave to Alfvén wave.
- In nonlinear regime, we have shown oscillatory velocity space structures enhance collisional dissipation rate (electron heating rate). Dissipation rate remains finite as $\nu_\mathrm{e} \to 0$.
- Result is consistent with simulation of reduced kinetic model for low-β. [Loureiro *et al.*]
- For high-β plassmas, ion heating may become important.

Some related works

- Heating in solar wind turbulence: Howes *et al.*
- Nonlinear phase mixing: Tatsuno *et al.*
- Comparison with PIC–guide field dependence: TenBarge, Daughton *et al.*
- Reduced kinetic model for magnetic reconnection (low-β): Zocco, Loureiro, Schekochihin.
- Micro-tearing mode analysis which drives electromagnetic microturbulence: RN, Loureiro, Zocco
- Diamagnetic stabilization of tearing mode: B. Rogers, S. Kobayashi, RN.