Vlasov simulation of Langmuir solitons and high energy electron formation

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Outline

- Reproduction of Vlasov simulation in Cheng and Knorr '76 which is based on the splitting scheme.[a]
- Non-Maxwellian distribution function effects on Landau damping. The kappa distribution function is taken as an example.[b]
- Formation of high energy electron tails in the presence of Langmuir soliton. [c]

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[a] C.Z.Cheng and G. Knorr, J. Comput. Phys. **22**, 330 (1976).

[b] D. Summers and R.M.Thorne, Phys. Fluids, B **3**, 1835 (1991).

[c] G.J.Morales and Y.C.Lee, Phys. Rev. Lett. **33**, 1534 (1974); High energy tail is formed but the bulk electrons do not change.

Electrostatic Vlasov simulation is employed

- Let us consider 1d 1v phase space. A Vlasov-Poisson system normalized by "λ_d and ω_e " reads

$$\partial_t f + v\partial_x f - E\partial_v f = 0$$

$$\partial_x E = 1 - \int f dv$$

- To time advance Vlasov equ numerically, we employ splitting scheme.

$$f^* (x, v) = f^n (x - v\Delta t/2, v)$$

$$f^{**} (x, v) = f^* (x, v + E(x)\Delta t)$$

and finally,

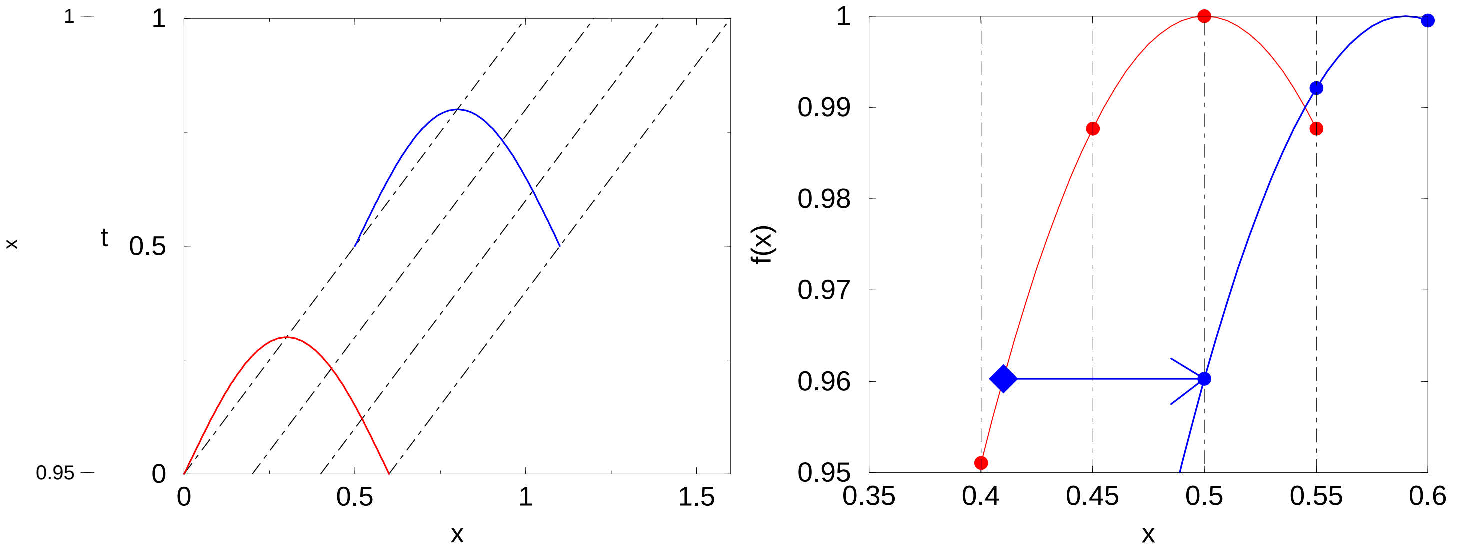
$$f^{n+1} (x, v) = f^{**} (x - v\Delta t/2, v)$$

which is equivalent to leap-frog in PIC simulation, possessing a symplectic (a phase volume conserving) nature.[a]

[a] C.Z.Cheng and G.Knorr, J. Comput. Phys. **22**, 330 (1976).

Splitting scheme is based on the method of characteristics

- It <u>is not</u> a finite difference method.
  We **trace** the distribution function along the characteristic curves.



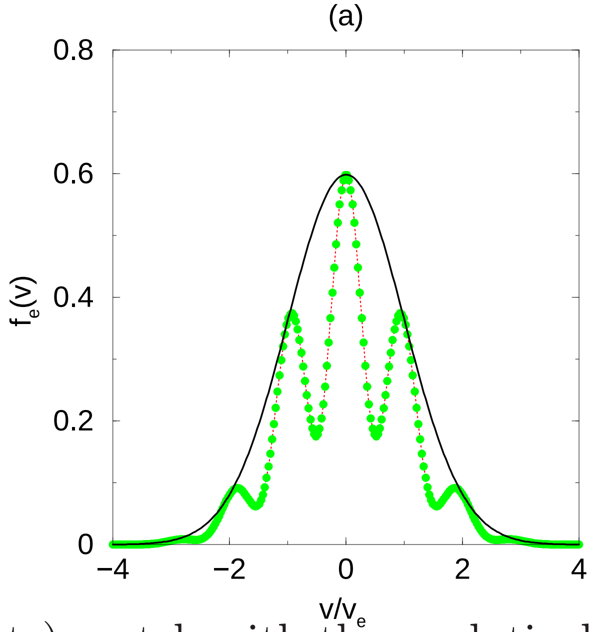
- In doing this, if the reference points along the characteristic curves "$x - v\Delta t$" are exactly on the mesh points, the method is quite trivial.
  $\rightarrow$ In general, the reference points ($\diamond$) are located in between mesh points.
  $\rightarrow$ Need an interpolation technique.[a]

[a] C.Z.Cheng and G.Knorr, J. Comput. Phys. **22**, 330 (1976).

Preliminary studies with the kinetic model: Free streaming case $(E = 0)$

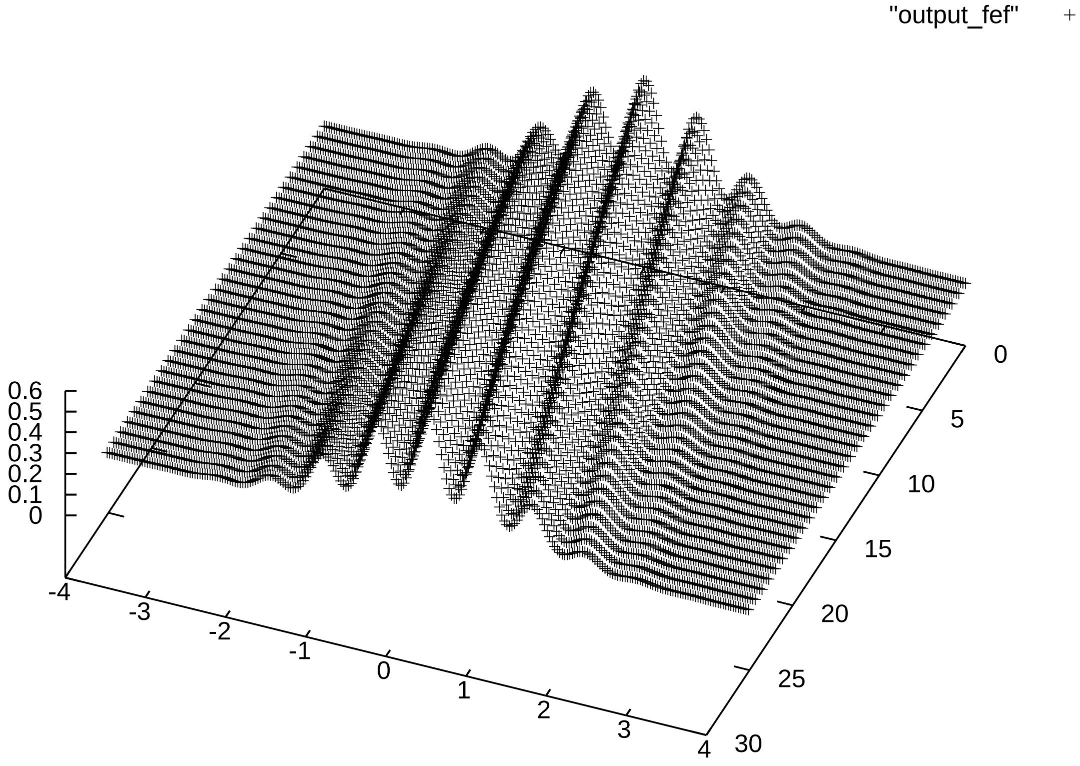
- Vlasov becomes $\partial_t f + v\partial_x f = 0$, the analytical solution can be given by $f_k(v,t) = f_k(v,0)e^{-ikvt}$.[a]
- For example, if we take an initial condition $f_k(v,0) = e^{-v^2/2}$,

$$\rightarrow f(x,v,t) = (e^{-v^2/2}e^{-ikvt})e^{ikx} = e^{-v^2/2}e^{ik(x-vt)}$$



- Simulation results (dots) match with the analytical solution (dashed).

[a] Case-Van-Kampen mode; D.R.Nicholson, *Introduction to Plasma Theory* (1983), p.120.


"output_fef"
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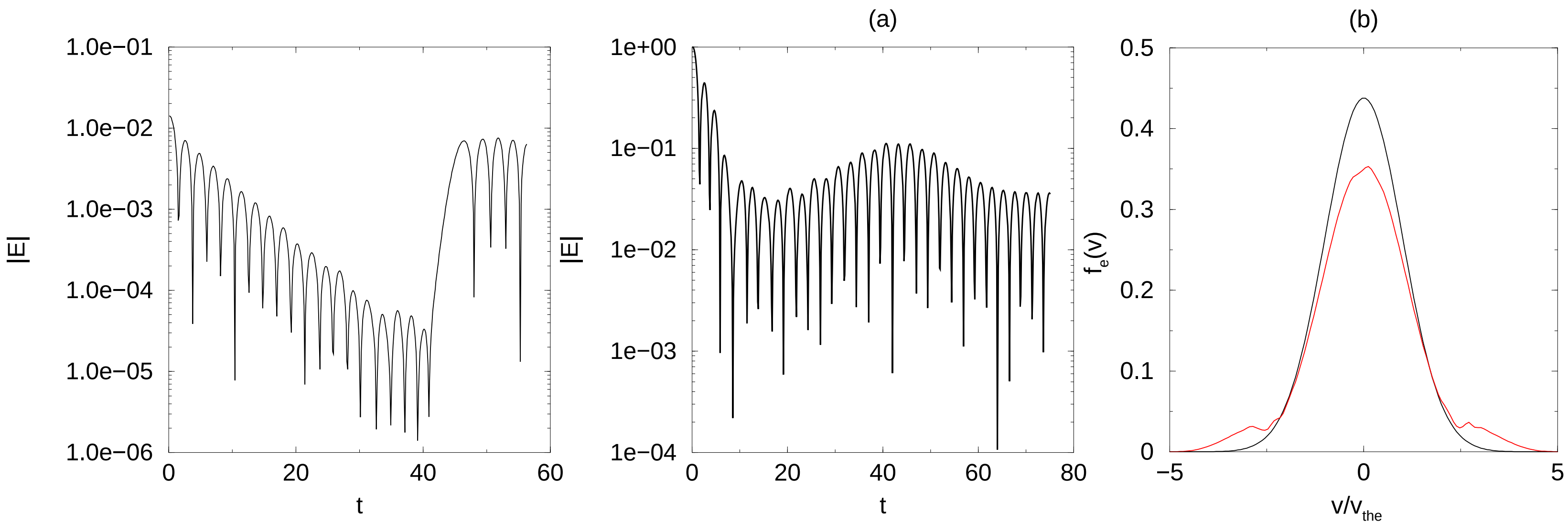
Linear and non-linear Landau damping of Cheng and Knorr '76 are reproduced

- Electric field is dynamically evolved (solving Poisson self-consistently). **(Left)** Linear damping **(Middle)** Non-linear **(Right)** Profile flattening.



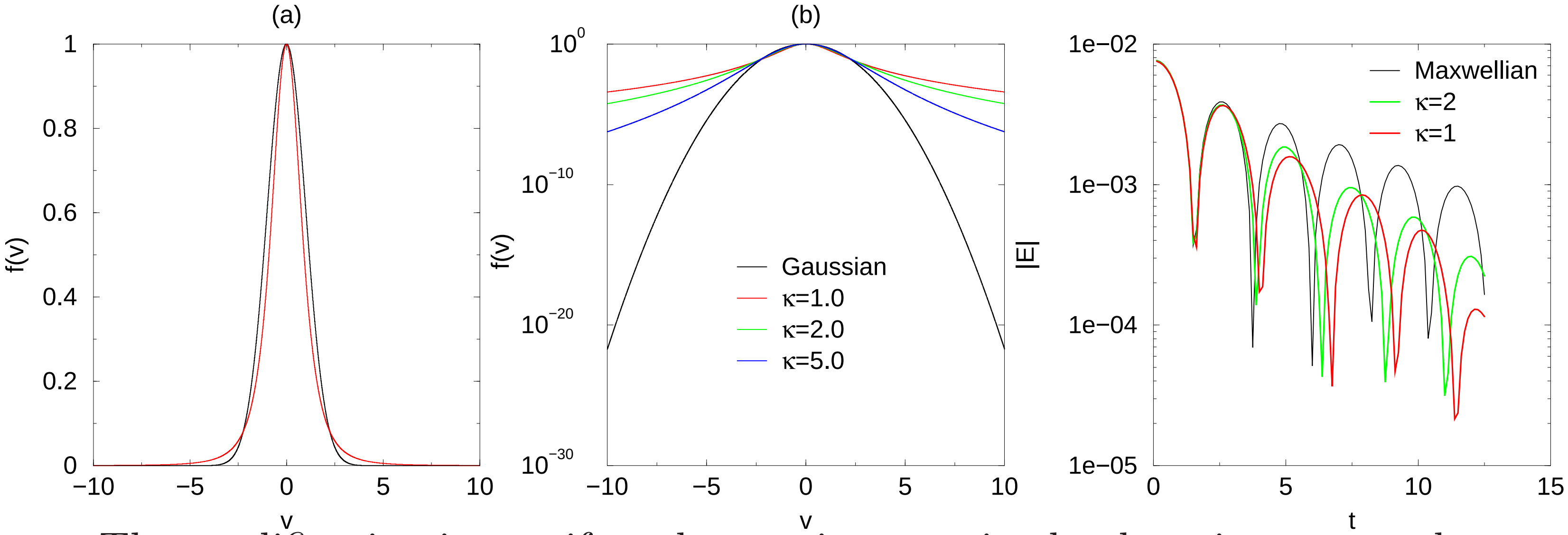
- Recurrence occurs ($t = 2\pi/k_x\Delta v \sim 48$) due to finite number of mesh points.

- Parameters: cut-off velocities $v_{max} = 4.0v_{the}$ (linear) / $v_{max} = 8.0v_{the}$ (nl), and $0 \leq x/\lambda_e \leq 4\pi$. For linear, $\omega = 1.41$ and $\gamma = -0.155$ match with the theory. Initial condition $f(x, v, 0) = [1 + A\cos(kx)]e^{-v^2/2}$ with $k = 0.5$.

We employ Kappa distribution function

- A "κ distribution function" is given by (note the exponent "$-\kappa - 1$")

$$f_v\left(v\right) \propto \left[1 + \left(v^2/2\kappa\right)\right]^{-\kappa-\mathbf{1}}.$$



- The modification is manifested as an increase in the damping rate and a decrease in the real frequency.[a] As a reminder, Langmuir wave dispersion relation (at $\gamma \gg \omega$) is given by [b]

$$\omega = \omega_e^2\left[1 + (3/2)k^2\lambda_e^2\right] + i\left(\pi\omega_e^3/2k^2\right)\partial_v f\left(v\right)|_{v=\omega_r/k}$$

[a]Initial condition $f(x, v, 0) = [1 + A\cos\left(kx\right)]f_\kappa(v)$ with $k = 0.5$.

[b]D.R.Nicholson, *Introduction to Plasma Theory* (1983), p.82.

Initial value simulation is compared with the analytical dispersion relation using "$Z_\kappa(\xi)$"

- Summers[a] employs a κ-function with the exponent "$-\kappa$"

$$f_v\left(v\right) = \frac{n_0}{\sqrt{\pi} v_e} \frac{\sqrt{\kappa}}{\sqrt{2\kappa - 3}} \frac{\Gamma(\kappa)}{\Gamma(\kappa - 1/2)\kappa^{3/2}} \left(1 + \frac{v^2}{2\kappa - 3}\right)^{-\kappa}.$$

- A normalized dispersion relation is given by

$$\xi Z^*_\kappa(\xi) + 1 - \frac{1}{2\kappa} + \frac{\kappa - 3/2}{\kappa}\bar{k}^2 = 0$$

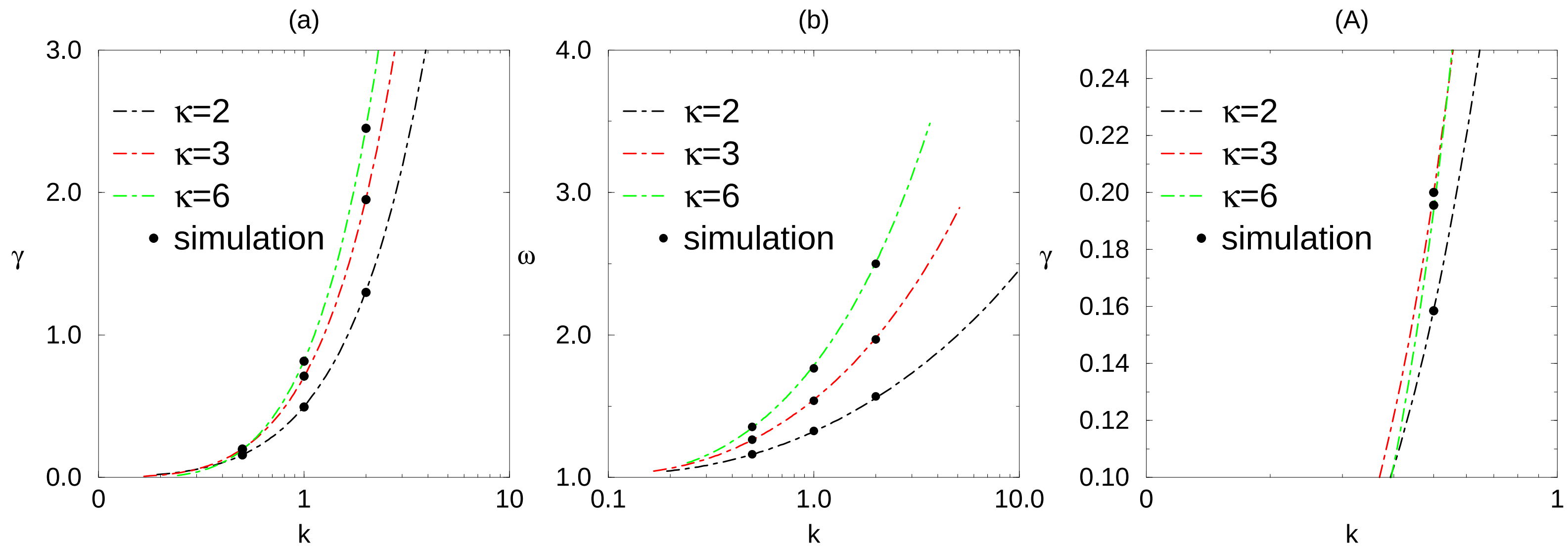
where $\xi = x + iy$ (x and y both real), and the modified dispersion function Z^*_κ is given by (analogous to Fried and Conte 1961)

$$Z^*_\kappa\left(\xi\right) = \frac{1}{\sqrt{\pi}} \frac{\Gamma(\kappa)}{\Gamma(\kappa - 1/2)\kappa^{3/2}} \int_{-\infty}^{\infty} \frac{\left(1 + s^2/\kappa\right)^{-\kappa-1}}{s - \xi} ds,$$

To find the root, we fix y values and solve the imaginary part of the dispersion relation, $Im\left[\xi Z^*_\kappa(\xi)\right] = 0$, to obtain x. When both x and y are given, we solve the real part for $\bar{k}$.

[a]D. Summers and R.M.Thorne, Phys. Fluids, B **3**, 1835 (1991).

- Taking exactly the same form of Summers 91, Vlasov simulation is conducted for $\kappa = 2$, $\kappa = 3$, and $\kappa = 6$.



Initial value simulation results (dots) compare favorably with the analytical results (dashed curves).[b]

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[b]Y.Chen, Y.Nishimura, and C.Z.Cheng "Kappa distribution function effects on Landau damping in electrostatic Vlasov simulation", Terr. Atmos. Ocean. Sci. 24, 273 (2013).

Zakharov Eq. is revisited for Langmuir soliton studies

- Nonlinear Schrödinger equation is derived by V.E.Zakharov by a fluid approach.[a] For low frequency E (envelope for the high frequency part)

$$i\partial_t E + \partial_x^2 E = nE$$

- Ion density equation in the presence of ponderomotive force

$$\partial_t^2 n - \partial_x^2 n = \partial_x^2 |E^2|$$

- By letting $n = -|E|^2$, we obtain a nonlinear Schrödinger equation.

$$i\partial_t E + \partial_x^2 E + |E^2|E = 0$$

- More general solution has the form of a soliton [here, ${E_0}^2 = 2{K_0}^2 \left(1 - V_g^2\right)$]

$$E(x,t) = E_0 \cdot sech\left[K_0 \left(x - V_g t\right)\right] e^{-i\left[K_1 x - \left({K_1}^2 - {K_0}^2\right)t\right]}$$

$$n(x,t) = -2{K_0}^2 \cdot sech^2\left[K_0 \left(x - V_g t\right)\right].$$

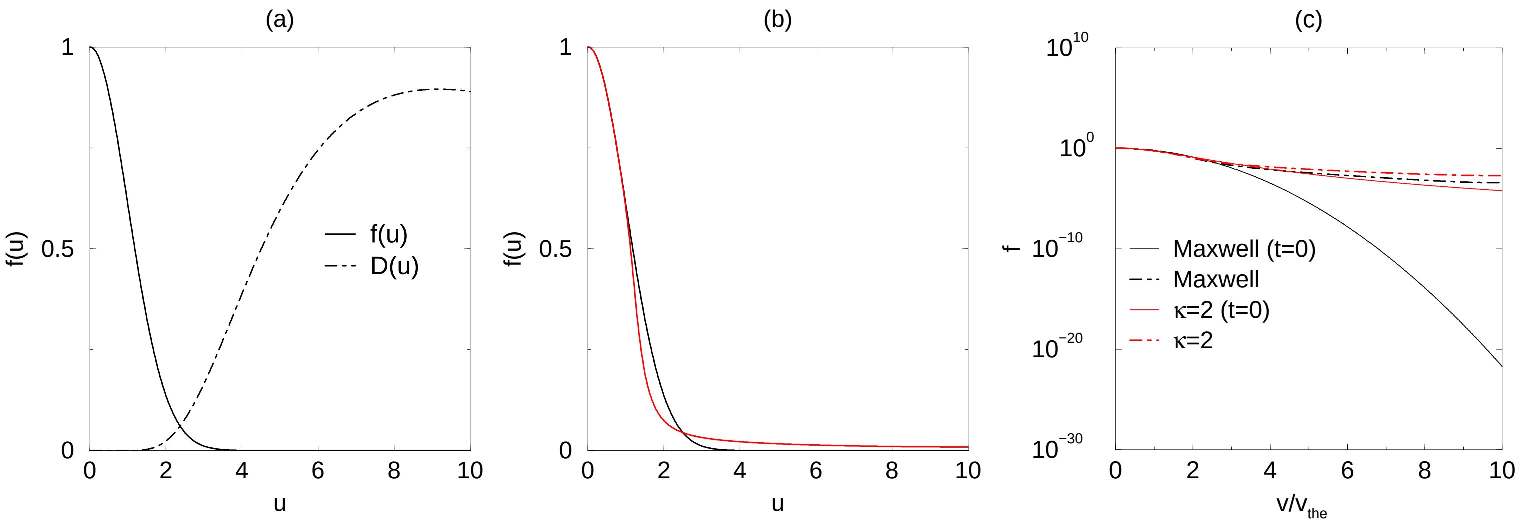
[a] V.E.Zakharov, Sov. Phys. JETP **35**, 908 (1972). Now parameterized by K_0 and K_1 only. We set $K_1 = 0$ in this study.

Fokker-Planck solution demonstrates high energy tail

- Fokker-Planck equation is solved in the velocity space[a]

$$\partial_t f(u) = \partial_u \left[D(u,w)\, \partial_u f(u) \right]$$

with $D(u,w) = \pi(w/u) sech^2(\pi w/2u)$.[b]



- High energy tail formation. On the other hand, the bulk electrons do not change since they participate in the formation of density cavities which are required to support localized electric fields.

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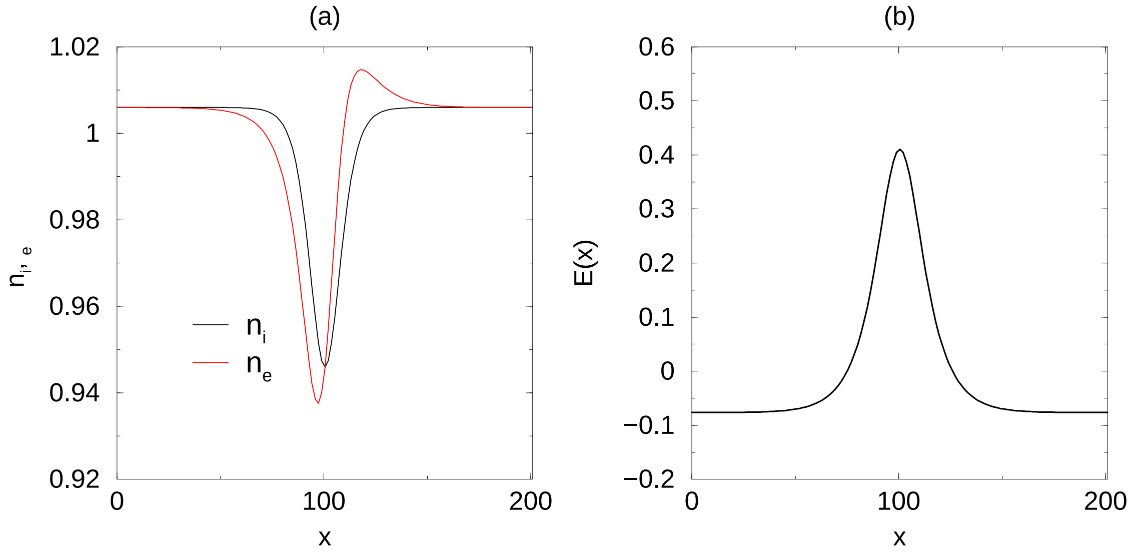
[a] G.J.Morales and Y.C.Lee, Phys. Rev. Lett. **33**, 1534 (1974).

[b] Represents the soliton shape of the electric field envelope *of the configuration space.*

In the Vlasov simulation we incorporate Zakharov solutions as initial conditions

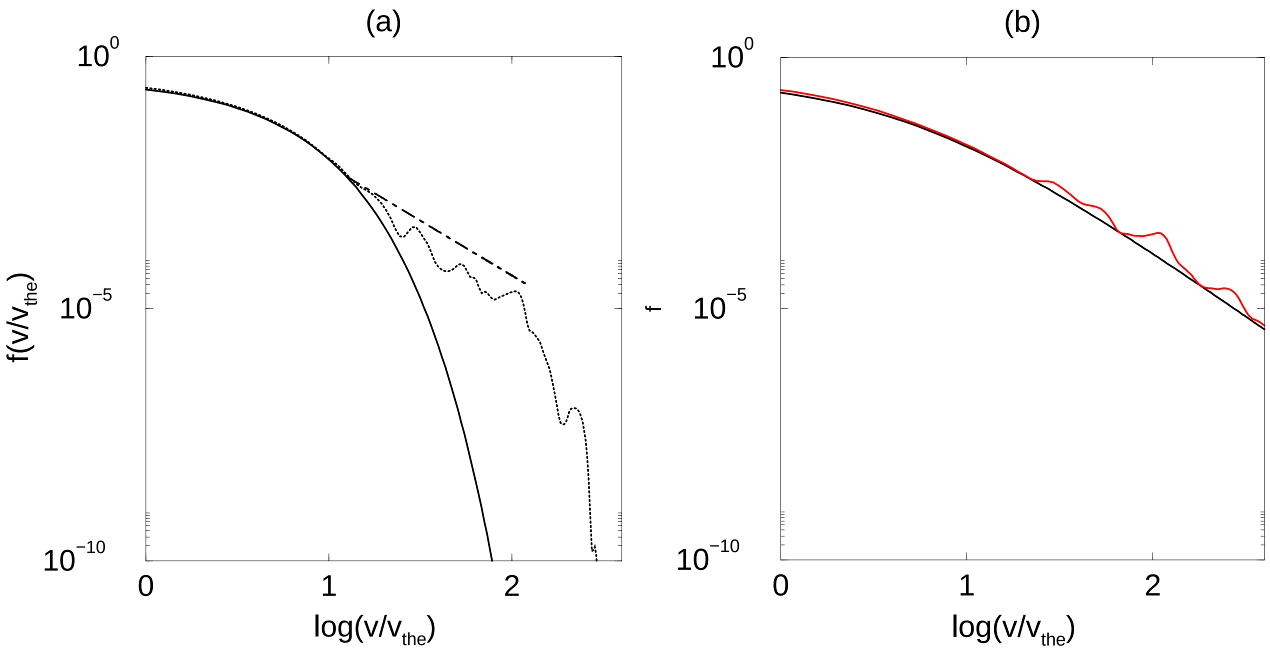
- As initial condition, we take $E(x,t) = E_0 \cdot sech\,(K_0 x)$ and $n_i(x,t) = -2{K_0}^2 \cdot sech^2\,(K_0 x)$.[a] Inverting Poisson equation, $n_e = n_i - \partial_x E$.
- Both the electrons and ions are time advanced. The initial f_e and f_i are given by $(n_{i,e})\times$ (Maxwellian: $e^{-v^2/2v_{e,i}^2}$) (or κ functions).



[a] N.R.Pereira, R.N.Sudan, J. Denavit Phys. Fluids **20**, 271 (1977). We start from stationary soliton case by setting $V_g = 0$.

Formation of high energy tail in Vlasov simulation is demonstrated

- (a) Maxwelllian and (b) $\kappa = 2.0$ distribution taken as initial conditions.



- Parameters employed are $n_v = 256$, $n_x = 128$, cut-off velocity $12.0v_{the}$ and $12.0v_{thi}$ and $0 \le x/\lambda_e \le 64\pi$ for (a). Followed up to $5000{\omega_e}^{-1}$. Dashed line for v^{-4}. (b) For $1000\omega_e^{-1}$, $16.0v_{the}$ and $16.0v_{thi}$ for $\kappa = 2$. [a]

[a] C. H. Li, J. K. Chao, and C.Z.Cheng, Phys. Plasmas **2**, 4195 (1995).

We obtain island separatrix equation in the phase space

- Given the electric field of the form $\sin(kx - \omega t)$, the equation of motion reads

$$dx/dt = v$$
$$dv/dt = \bar{E}_k \sin(kx - \omega t)$$

- By going to the moving frame [letting $X = x - (\omega/k)t$ and $V = v - \omega/k$],

$$dX/dt = V$$
$$dV/dt = \bar{E}_k \sin(kX).$$

  For the corresponding Hamiltonian $H = V^2/2 - (\bar{E}_0/k)\cos(kX)$, we obtain the separatrix equation $V = \pm 2\sqrt{\bar{E}_k/k}\cos(kX/2)$ for each Fourier mode.

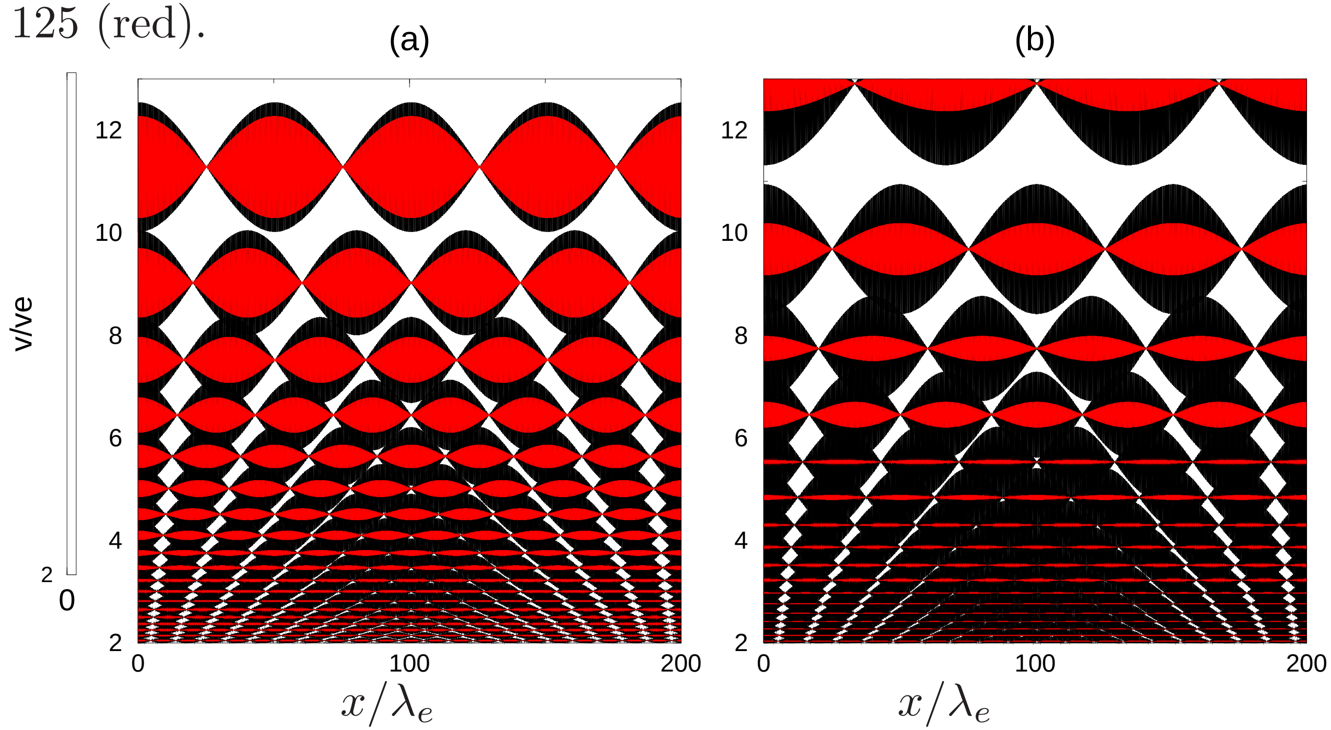
- Extracting E_k values from the numerical Vlasov simulation, we can identify the separatrix widths and locations

$$v = \omega/k \pm 2\sqrt{\bar{E}_k/k}\cos(kX/2),$$

  and can apply the island overlapping criterion.

Chains of islands in velocity space is shown

- The electric field profiles are expanded in Fourier series. (a) Mawellian case at $t = 0$ (black) and $t = 5000$ (red). (b) $\kappa = 2$ case at $t = 0$ (black) and $t = 125$ (red).



- While the island overlapping is seen (Chirikov's criterion as a reminder) at $t = 0$, the islands at $v/v_{the} \geq 2.0$ are intact at the later phase. Remind that the damping rate is large with an existing high energy tail (κ distribution).

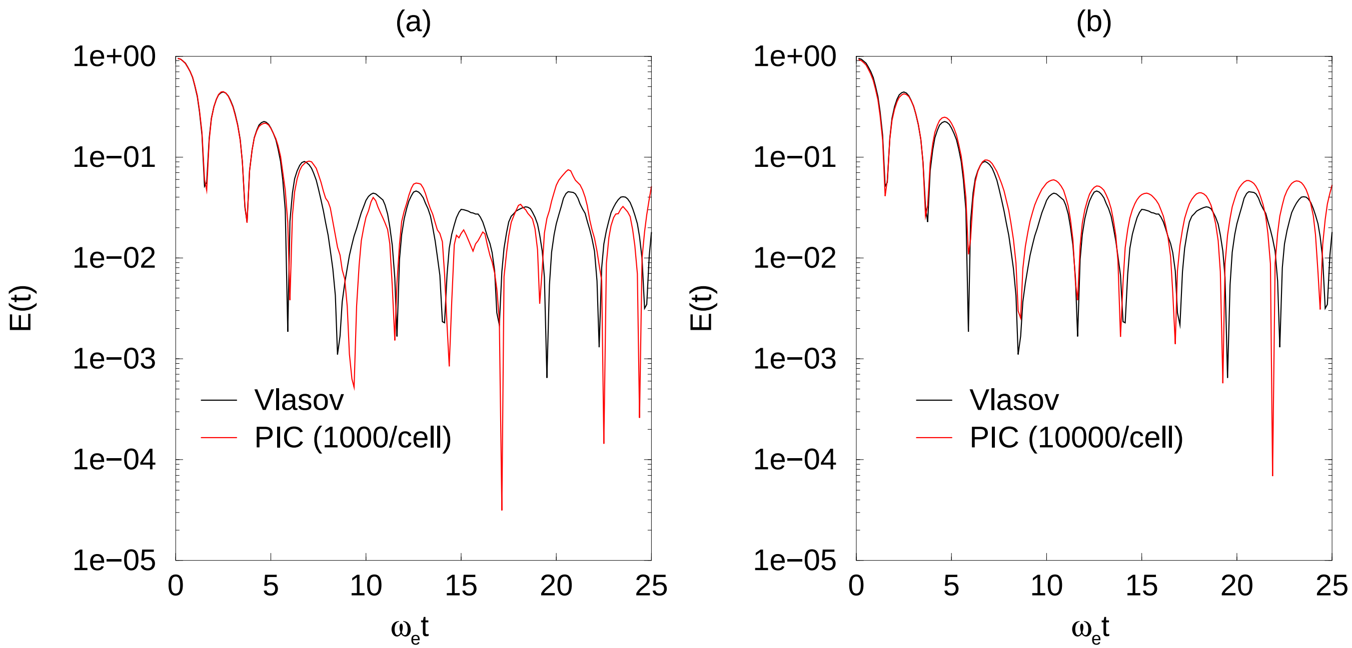
Summary and discussions

- A 1d-1v Vlasov-Poisson simulation to study Langmuir wave dynamics is developed employing the splitting scheme.[a]
- Non-Maxwellian (κ function) employed. The modification is manifested as an increase in the Landau damping rate. The analyses by the modified plasma dispersion function is reproduced.[b]
- The formation of high energy electron tails by Langmuir soliton is demonstrated.[c] On the other hand, κ functions are not evolved by the solitions.
- Origin of kappa like distribution.[d] For the high energy tail to get heated effectively, nonlinear waves must be playing an important role.

[a] C.Z.Cheng and G.Knorr, J. Comput. Phys. **22**, 330 (1976).
[b] Summers, D. and R.M.Thorne, 1991: Phys. Fluids, B **3**, 1835-1847.
[c] Advantage of Vlasov simulation over PIC in investigating high energy tail dynamics.
[d] V.M.Vasyliun, J. Geophys. Res. **73**, 2839 (1968).

Particle-in-cell simulation is employed

- *Vlasov simulation and Particle-in-Cell simulation of Langmuir solitions.*
- A linear Landau damping of Langmuir wave is compared between Vlasov and PIC.



- The number of grids are 64 within a simulation domain of $0 \leq x \leq 4\pi$. Took standard case from previous Vlasov (black curves). Numbers of particles per cell are 1000 and 10000.

δf method is based on the method of characteristics

- Since $\delta f/f_0 \sim \delta n/n_0$, this method reduces the unfavorable artificial collision.
- The linearized Vlasov equation is given by

$$\dot{\delta f} = -v\partial_x f_0 - (qE/m)\partial_v f_0.$$

We now introduce weight function $w = \delta f/g$ (g is a numerically loaded distribution function).

- The *marker* particles are time advanced by[a]

$$\dot{x} = v$$
$$\dot{v} = qE/m.$$

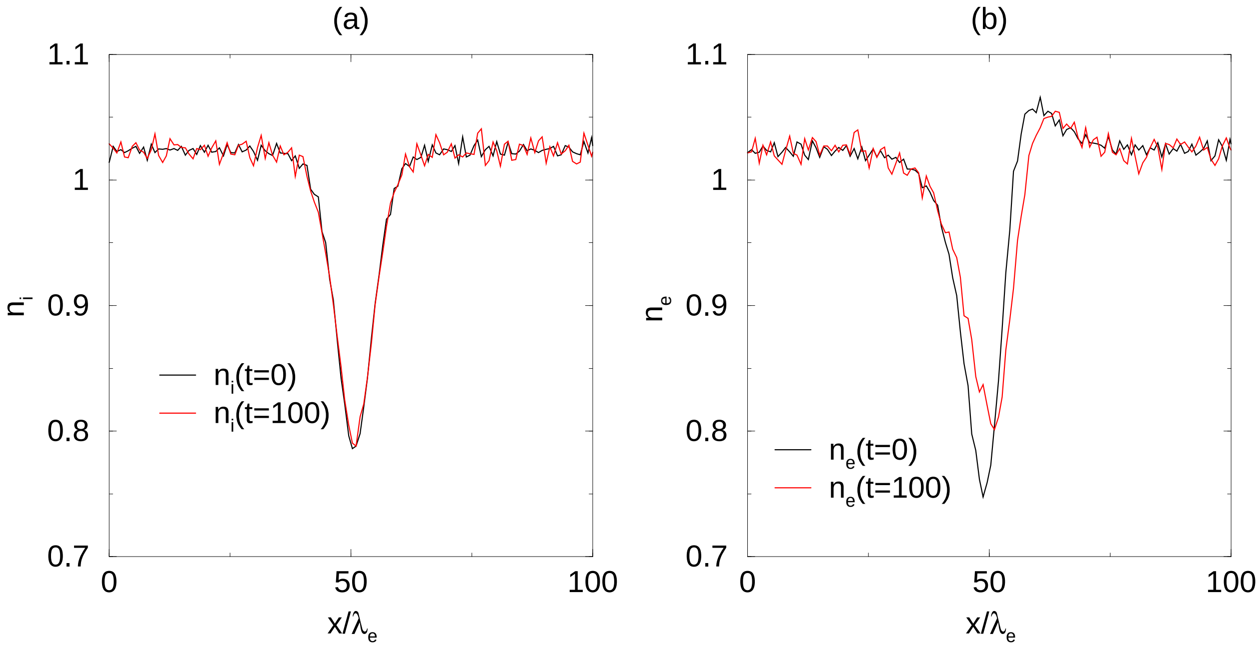
- On top, the weight equation is solved along the marker particles

$$\dot{w} = -v\frac{\partial_x f_0}{g} - (qE/m)\frac{\partial_v f_0}{g}.$$

[a]M.Kotschenreuther, Bull. Am. Phys. Soc. 34, 2107 (1988). Y. Nishimura and C.Z.Cheng, J. Plasma Fusion Research 9, 541 (2010).

Langmuir soliton simulation by particle-in-cell is initiated

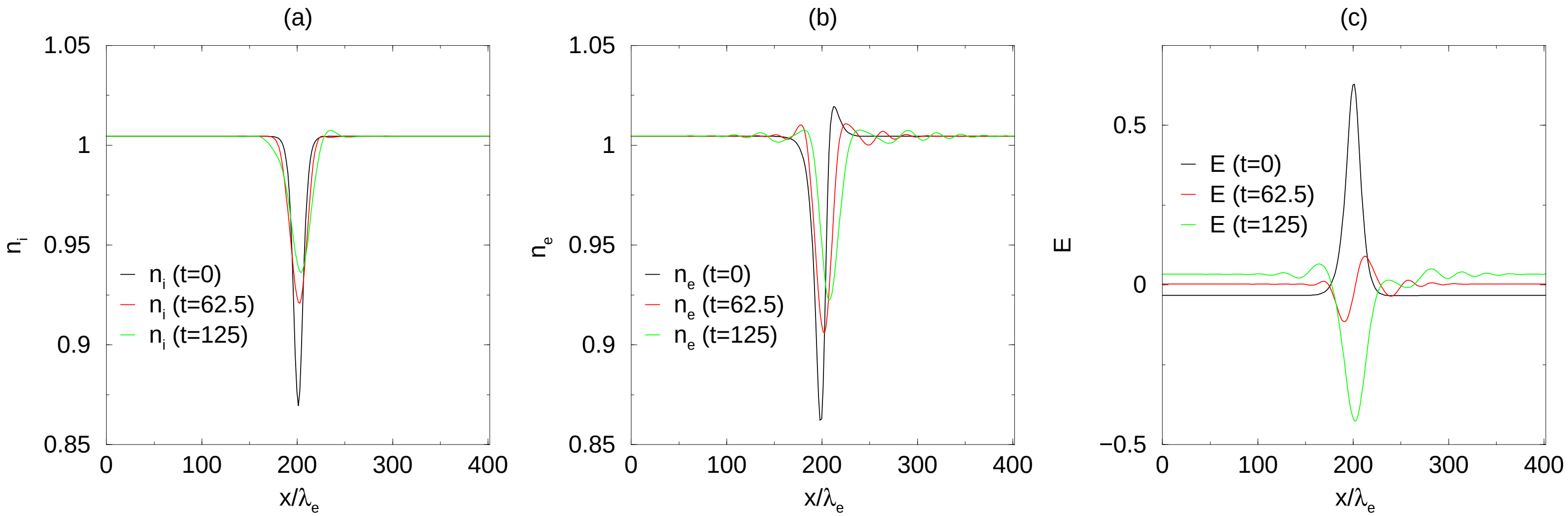
- The number of grids are 128 with a simulation domain of $0 \leq x \leq 64\pi$.



- From a numerical point of view, the Vlasov simulation in lower dimension has advantages over PIC simulation for investigating subtle effects such as a slight deviation of the equilibrium distribution function at the high energy tail.

Propagation of Langmuir soliton is regulated by ion wave-particle interaction

- Langmuir soliton propagates satably but is subject to Landau damping as the temperature ratio T_i/T_e approaches unity.



- (a) Ion density cavity (b) electron density cavity, and (c) the electric field.