Local gyrokinetic turbulence simulations with realistic tokamak geometries towards ITER and JT-60SA

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• Introduction
• Current status of local gyrokinetic Vlasov code GKV
• Interface between MEUDAS (2D equilibrium solver) and GKV
• Application to realistic plasma shapes on JT-60SA
  • Linear ITG mode stability
  • Residual zonal flow levels
  • Nonlinear zonal flow generations
• Summary
- Turbulent transport properties can be strongly affected by the plasma shaping effects, through the change of the linear frequencies, mode-structures, and trapped/passing boundary.

Impact of the negative triangularity on stabilization of TEMs: global-GK analyses for TCV shape. Camenen et al., NF2007

Enhanced residual-ZF levels by plasma elongation: Analytic analyses for Solovev’s like equilibrium. Xiao and Catto, PoP2006

- Implementation of realistic shaped MHD-equilibrium to gyrokinetic code is useful for analyzing plasma shape effects, and for validation/prediction study against experiments.

--- In this study, tokamak equilibria calculated by a free-boundary 2D Grad-Shafranov solver MEUDAS are incorporated into a local fluxtube gyrokinetic code GKV.

--- Micro-stability and zonal flow generation in several equilibria on JT-60SA are investigated.
Current status of GKV code

- Local fluxtube 5D gyrokinetic solver, originally developed by [T. -H. Watanabe, NF2006]
  ---> Solving the evolution of delta-f in 5D phase space
  ---> Eulerian (or Vlasov) solver: spectral in 2D k-space, Finite-Difference in 3D (z, v∥, μ)-space
  ---> Electro-static, Electro-magnetic, Arbitrary numbers of species
  ---> Helical geometries from VMEC, Tokamak geometries from MEUDAS(this talk)
  ---> Entropy balance/transfer diagnostics


- GKV code is now ready for turbulence simulations of burning plasmas composed of D, T, e, He-ash, C, W, etc.
Advanced parallelization towards PETA/EXA computing

- 5D domain-decompositions with MPI and Thread parallelization with OPENMP

**Decomposition in \((z, v_\parallel, \mu)\) + Decomposition in k-space (kx or ky) + Species**

- Parallel 2D FFT (including data-transpose) for the nonlinear ExB advection term is the most time-consuming.

--- Segmented 3D rank-allocation technique (for MESH/TORUS network)

--- Communication/Computation Overlap technique (for FFT and/or FD)

[S. Maeyama, JSST2012 Y. Idomura, SC2012]
Fantastic scaling over 0.6M cores on K computer

Problem size for Multi-scale turbulence simulations (e.g. ITG-ETG)
\[
\begin{align*}
N_x &= 1024, \quad N_y = 1024, \quad N_z = 128, \quad N_v = 64, \quad N_\mu = 32 \\
P_k &= 8-64, \quad P_z = 16, \quad P_v = 8, \quad P_\mu = 4, \quad 8 \text{ threads}
\end{align*}
\]
\[\sim 274 \text{ billion grids} \quad > \quad 600 \text{ thousand cores}\]

Effects of Rank-Mapping and Overlap

Fantastic scalability over 0.6M cores has been achieved with 99.99994% efficiency.

\[\text{Multi-scale/Multi-species turbulence simulation is now accessible with GKV.}\]
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• Summary
Gyrokinetic equation in flux coordinates

- Gyrokinetic and Poisson equations: electro-static limit, multi-species

\[
\begin{align*}
\left\{ \begin{array}{l}
\frac{\partial}{\partial t} + v_\parallel \cdot \nabla + ik_\perp \cdot \mathbf{v}_{ds} - \frac{\mu}{m_s} b \cdot \nabla B \frac{\partial}{\partial v_\parallel} \end{array} \right. \\
\delta f_{sk_\perp} + \mathcal{N}(\delta f_{sk_\perp}, \delta \psi_{sk_\perp}) \\
= F_{Ms} \left[ ik_\perp \cdot \mathbf{v} \cdot T_s - ik_\perp \cdot \mathbf{v}_{ds} - v_\parallel \cdot \nabla \frac{e_s \delta \psi_{sk_\perp}}{T_s} \right] + C_s(h_{sk_\perp}) \\
k_\perp^2 \delta \phi_{k_\perp} = 4\pi \sum_s e_s \left[ \int d\mathbf{v} J_{0s} \delta f_{sk_\perp} - n_{s0} \frac{e_s \delta \phi_{k_\perp}}{T_s} (1 - \Gamma_{0s}) \right]
\end{align*}
\]

- Fluctube coordinates \((x, y, z)\) defined by general SFL flux coordinates \((\rho, \theta, \zeta)\):

coordinates: \(x = c_x (\rho - \rho_0), y = c_y [q(\rho)\theta - \zeta], z = \theta\)

magnetic field: \(\mathbf{B} = c_b \nabla x \times \nabla y, c_b = \dot{\Psi} / c_x c_y\)

Jacobian: \(\sqrt{g_{xyz}} = (\nabla x \times \nabla y \cdot \nabla z)^{-1} = (c_x c_y)^{-1} \sqrt{g_{\rho \theta \zeta}}\)

Derivatives: \(\frac{\partial}{\partial x} = \frac{1}{c_x} \frac{\partial}{\partial \rho} + \frac{\dot{q}(\rho)\theta}{c_x} \frac{\partial}{\partial \zeta}, \quad \frac{\partial}{\partial y} = -\frac{1}{c_y} \frac{\partial}{\partial \zeta}, \quad \frac{\partial}{\partial z} = \frac{\partial}{\partial \theta} + q(\rho) \frac{\partial}{\partial \zeta}\)

safety factor and magnetic shear: \(q(\rho) = \frac{d\Phi}{d\Psi}, \quad \dot{s}(\rho) = \frac{\rho}{q} \frac{dq}{d\rho}\)
Gyrokinetic equation in flux coordinates

- Metric tensor

\[
\begin{align*}
   g^{xx} &= \nabla x \cdot \nabla x = c_x^2 g^{\rho \rho} \\
   g^{xy} &= \nabla x \cdot \nabla y = c_x c_y [\dot{q} \theta g^{\rho \rho} + q g^{\rho \theta} - g^{\rho \zeta}] \\
   g^{xz} &= \nabla x \cdot \nabla z = c_x g^{\rho \theta} \\
   g^{yy} &= \nabla y \cdot \nabla y = c_y^2 [(\dot{q} \theta)^2 g^{\rho \rho} + q^2 g^{\theta \theta} + g^{\zeta \zeta} + 2q \dot{q} \theta g^{\rho \theta} - 2q g^{\theta \zeta} - 2 \dot{q} \theta g^{\rho \zeta}] \\
   g^{yz} &= \nabla y \cdot \nabla z = c_y [\dot{q} \theta g^{\rho \theta} + q g^{\theta \theta} - g^{\theta \zeta}] \\
   g^{zz} &= \nabla z \cdot \nabla z = g^{\theta \theta}
\end{align*}
\]

- Advection operators in general forms for arbitrary flux coordinate systems:

\[
\begin{align*}
   b \cdot \nabla &= \frac{c_b}{B \sqrt{g_{xy}^2}} \frac{\partial}{\partial z}, \quad k^2_\perp = k_x^2 g^{xx} + 2k_x k_y g^{xy} + k_y^2 g^{yy}, \quad \frac{1}{L_{n_s}} = -\frac{d \ln n_s}{dx}, \quad \frac{1}{L_{T_s}} = -\frac{d \ln T_s}{dx} \\
   k_\perp \cdot v_{ds} &= \frac{m_s v_{||}^2 + \mu B}{e_s c_b} [\mathcal{K}_x k_x + \mathcal{K}_y k_y], \quad k_\perp \cdot v_{*s} = -\frac{T_s}{e_s c_b} \left[ \frac{1}{L_{n_s}} + \frac{1}{L_{T_s}} \left( \frac{m_s v_{||}^2}{2T_s} + \frac{\mu B}{T_s} - \frac{3}{2} \right) \right] k_y \\
   \mathcal{K}_x &= \frac{g^{xz} g^{yy} - g^{xx} g^{yz}}{B^2/c_b^2} \frac{\partial \ln B}{\partial z} - \frac{\partial \ln B}{\partial y}, \quad \mathcal{K}_y = \frac{g^{xz} g^{yy} - g^{xy} g^{yz}}{B^2/c_b^2} \frac{\partial \ln B}{\partial z} + \frac{\partial \ln B}{\partial x}
\end{align*}
\]

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Only B-intensity, and its derivatives, and the contra-variant metric components are necessary to calculate these operators.
Construction of SFL flux coordinates from MEUDAS

Data flow

MEUDAS (developed in JAEA)
2D Free-boundary Grad-Shafranov solver

\[ \Psi = \Psi(R, Z) \]

IGS [Matsuyama]
Flux coordinate Interface
Interpolation, tracing flux surfaces
Constructing Axisymmetric, Hamada, and Boozer coordinates

GKV
Local fluxtube code
Calculating metric components and advection operators
Turbulence simulation

\[ P = P(\Psi), \ I = I(\Psi) \]

The radial label:
\[ \rho = \sqrt{\Phi/\Phi_{\text{edge}}}, \quad \rho = \sqrt{\Psi/\Psi_{\text{edge}}} \]

The minor radius:
\[ a = \sqrt{V_{\text{edge}}/2\pi^2 R_{\text{ax}}} \]
\[ a = \sqrt{2\Phi_{\text{edge}}/B_{\text{ax}}} \]
\[ a = \sqrt{2\Psi_{\text{edge}}/B_{\text{ax}}} \]

\[ R = R[\rho, \theta], \ Z = Z[\rho, \theta], \ \phi = \zeta - q(\rho)G[\rho, \theta] \]
\[ B = B[\rho, \theta], \ dB/d\rho, \ dB/d\theta \]

Co-variant metric components are calculated by
\[ g_{ij} = \frac{\partial R}{\partial u_i} \frac{\partial R}{\partial u_j} + \frac{\partial Z}{\partial u_i} \frac{\partial Z}{\partial u_j} + R^2 \frac{\partial \phi}{\partial u_i} \frac{\partial \phi}{\partial u_j} \]
\[ u_i = \{\rho, \theta, \zeta\} \]

, then converted to contra-variant components.
Verification of flux coordinates with solovev equilibrium

Analytic expression of solovev equilibrium with finite elongation, triangularity, and up-down asymmetry:

\[
\frac{\Psi(\bar{R}, \bar{Z})}{\Psi_{\text{edge}}} = \frac{1}{\epsilon_a^2} \left[ \frac{(1 - d)\bar{R}^2 \bar{Z}^2 + d\bar{Z}^2}{E^2} + \frac{1}{4} (\bar{R}^2 - 1)^2 \right] + \frac{\nu_{\text{asym}}}{\epsilon_a} (\bar{R}^2 - 1) \bar{Z}, \quad \bar{R} = R/R_{ax}, \quad \bar{Z} = Z/R_{ax}
\]

**Boozer coord. with** \( \rho = (\Psi/\Psi_{\text{edge}})^{\frac{1}{2}} \)

Less deformation of constant \( \theta \)-surface near the edge for Hamada coord. (especially, out-board side)

Original 2D equilibrium data with

- \( E = 1.6, \ d = 0.05, \)
- \( \epsilon_a = 0.25, \ \nu_{\text{asym}} = -0.3 \)
Verification of flux coordinates with solovev equilibrium

- Consistency is verified by the following analytical expressions:

\[ \frac{\partial R[\rho, \theta]}{\partial \theta} = -\frac{\sqrt{g}}{R} \frac{\partial \rho(R, Z)}{\partial Z} \]

\[ g^{\rho \rho} = \left\| \frac{\partial \rho}{\partial R} \nabla R + \frac{\partial \rho}{\partial Z} \nabla Z \right\|^2 \]

Relative error to analytical expressions

All the constructed coordinate systems show sufficient accuracy (up to \( \sim 10^{-8} \)) for turbulent transport analyses.
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Application to realistic plasmas on JT-60SA

- Two L-mode plasmas are considered (The MHD stability has been well investigated: N. Aiba PFR2007):
  
  **ITER-like plasma with single null: “IT”**
  
  - $R_{ax} = 3.02m$, $a = 1.29m$, $V = 90.7m^3$,
  - $B_{ax} = 2.64T$, $q_{ax} = 1.82$, $q_{95} = 3.88$,
  - $I_p = 2.59MA$, $S = q_{95}I_p/B_{ax}a = 2.95$

  **Highly-Shaped plasma with quasi double null: “HS”**
  
  - $R_{ax} = 3.05m$, $a = 1.53m$, $V = 129.5m^3$,
  - $B_{ax} = 2.88T$, $q_{ax} = 1.41$, $q_{95} = 3.59$,
  - $I_p = 5.00MA$, $S = q_{95}I_p/B_{ax}a = 4.07$
Field aligned structures of $B$-intensity, $\omega_d$, and $k^2$

- Field aligned structures in shaped plasmas deviate from those in circular plasmas (outer sides).
- Stronger asymmetry in $k^2$ for ITER-like plasma than that in Highly-Shaped plasma.
- More sharp boundary, but less trapped region for shaped plasmas compared with circular ones.

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Strong impacts on ITG with kinetic elec. and TEM turbulence dynamics.
Broader ITG-spectra with more localized eigenmode appear especially in strongly shaped region (outer side).
- ITG mode growth rates in Highly-shaped case is slightly lower than those in the ITER-like cases.
Geometric dependence of residual zonal flow levels

IT: ITER-like, HS: Highly shaped

- Shaping effects in IT- and HS- plasmas lead to
  --> significant enhancement of residual zonal flow levels
  --> stronger $k_x$- (or $k_r$-) dependence compared with circular plasmas.

HS is expected to be more favorable to ITG-stability and ZF-response!
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Nonlinear simulation and entropy balance

- The entropy balance relations are well satisfied both in the non-zonal and zonal parts.

- Highly-Shaped cases show lower transport levels with higher critical temperature gradient.
Nonlinear zonal flow generation

IT: ITER-like, HS: Highly shaped

- As is expected by linear ZF-damping analyses, more efficient ZF generation is observed in the nonlinear phase of the Highly-Shaped case.

- Stronger $k_x$-dependence of $Z/T$ is also identified in the Highly-Shaped case. (Z: ZF-intensity, T: turbulence intensity)

$\rightarrow$ Qualitative features on the amplitude and $k_x$-dependence are well agreed with linear results shown in the previous slides.
Towards the transport modeling

- ITG turbulence simulations on LHD show that the ion heat diffusivity well scales with $T/Z^{1/2}$, and GK-simulation-based transport model including zonal-flow effects has been developed.

[M. Nunami PoP2012, 2013]

- Good scaling between $\chi_i$ and $T/Z^{1/2}$ is also observed in JT-60SA case.

- Turbulence spectra near the peak region are well characterized with linear spectra, $\gamma_{ITG}/k^2$.

--- Promising to construct the similar reduced transport model produced by the linear spectra and residual ZF levels. (currently underway...)
Summary

- An interface code to generate flux coordinates system from realistic plasma equilibria calculated by free-boundary 2D Grad-Shafranov solver MEUDAS is successfully implemented to a local fluxtube code GKV.

- The accuracy of the flux coordinates, i.e., Axisymmetric, Boozer, and Hamada, are verified with an analytical solovev equilibrium model.

- Linear ITG-ae stability (ae: adiabatic electrons) is investigated for two types of shaped plasmas in JT-60SA, i.e., ITER-like plasma (IT) and Highly shaped one (HS), then the difference from the concentric circular equilibrium, which is conventionally used in gyrokinetic code, has been clarified.

  ---> Highly-shaped configuration shows less ITG-driven transport than that in the standard ITER-like configuration, due to stronger generation of zonal flows.

  ---> The turbulent ion heat diffusivity well scales with $T/Z^{1/2}$, which suggests the applicability of a GK-simulation based transport model.

More detailed analyses including a multi-species/scales stability analysis, nonlinear simulations, and constructing the GK-based transport model are in progress.