19th NEXT Workshop
(Katsura campus, Kyoto Univ., Japan, Aug. 29-30, 2013)

Beta limit of MHD equilibrium with pressure anisotropy in magnetospheric configuration

M. Furukawa : Grad. Sch. Eng., Tottori Univ.

鳥取大学
Tottori University

Brief Summary

- Effects of pressure anisotropy on the properties of magnetospheric, axisymmetric toroidal equilibrium with only poloidal magnetic field is investigated.
- Beta limit increases as the pressure anisotropy becomes weak
- A very high beta, isotropic pressure equilibrium is found where the plasma separates into two torus

Background and Motivation

- Plasma confinement experiment in magnetospheric configuration (internal coil system) has been conducted

  [Z. Yoshida et al., Plasma Fusion Res.**1**, 008 (2006).]

- Previous studies:
  - Numerical study by kinetic calculation of pressure, with magnetic field line connected to the central planet

    [C. Z. Cheng, J. Geophys. Res. **97**, 1497 (1992).]
  - Analytic solution in point dipole magnetic field

    [S. I. Krasheninnikov and P. J. Catto, Phys. Plasmas **7**, 626 (2000).]
  - Numerical studies by using CGL pressure tensor, with internal ring current

    [S. Iizuka, M. Furukawa, Plasma Conference 2011 (2011 Autumn Meeting, Physical Society of Japan) 25F09.]
    [M. Furukawa, 68th Annual Meeting, Physical Society of Japan, 26aEA-9.]

- We study properties of MHD equilibrium with anisotropic pressure in a simplified situation (e.g., a filament approximation of coil current), especially by focusing on diamagnetic current

Extended Grad-Shafranov equation (1)

- Static MHD equilibrium equation using pressure tensor is given by

$$\begin{cases} \boldsymbol{J} \times \boldsymbol{B} = \nabla \cdot \mathsf{P} \\ \mu_0 \boldsymbol{J} = \nabla \times \boldsymbol{B} \\ \nabla \cdot \boldsymbol{B} = 0 \end{cases}$$

- We adopt the CGL pressure tensor in this study:

$$\mathsf{P} = p_{\|} \boldsymbol{b}\boldsymbol{b} + p_{\perp}(\mathsf{I} - \boldsymbol{b}\boldsymbol{b})$$

[G. F. Chew, M. L. Goldberger and F. E. Low, Proc. R. Soc. London Ser. A **236**, 112 (1956).]

$\boldsymbol{b}$: unit vector along magnetic field

- For axisymmetric toroidal plasma, magnetic field can be expressed as

$$\boldsymbol{B} = I\nabla\phi + \nabla\psi \times \nabla\phi$$

- in cylindrical coordinates (R, ϕ, Z)

- We obtain, from the force balance in toroidal direction,

$$I^*(\psi) := \sigma I \qquad\qquad \sigma := 1 + \frac{\mu_0(p_{\perp} - p_{\|})}{B^2}$$

Extended Grad-Shafranov equation (2)

- If we assume $p_{\|}(\psi, B)$, we obtain, from the force balance along the magnetic field,

$$\left.\frac{\partial p_{\|}}{\partial B}\right|_{\psi} + \frac{p_{\perp} - p_{\|}}{B} = 0$$

[H. Grad, Phys. Fluids **10**, 137 (1967).]

- Now, we obtain, from the force balance in minor-radius direction,

$$\Delta^* \psi = -\mu_0 \frac{R^2}{\sigma} \left.\frac{\partial p_{\|}}{\partial \psi}\right|_{B} - \frac{1}{\sigma^2} I^* I^{*\prime}(\psi) - \frac{1}{\sigma} \nabla\psi \cdot \sigma$$

$$\Delta^* := R^2 \nabla \cdot \left(\frac{1}{R^2} \nabla \right) \qquad \left(\sigma := 1 + \frac{\mu_0 (p_{\perp} - p_{\|})}{B^2} \right)$$

- Boundary condition adopted here is $\psi = 0$ at infinity

- $p_{\perp}(\psi, B)$ is obtained by assuming a functional form of $p_{\|}(\psi, B)$

- $I^* \equiv 0$ is used assuming only poloidal magnetic field

Assumptions for pressure (1)

- Assuming

$$\begin{cases} p_{\parallel}(\psi, B) = n(\psi, B) T_{\parallel}(\psi) \\ p_{\perp}(\psi, B) = n(\psi, B) T_{\perp}(\psi) \end{cases} \xrightarrow{\text{substitute}} \left.\frac{\partial p_{\parallel}}{\partial B}\right|_{\psi} + \frac{p_{\perp} - p_{\parallel}}{B} = 0$$

- and substitute into the force balance along magnetic field, we obtain

$$n(\psi, B) = \bar{n}(\psi) \left(\frac{B}{B_{\mathrm{n}}(\psi)} \right)^{1-\lambda(\psi)} \qquad \begin{cases} \lambda(\psi) := \dfrac{T_{\perp}(\psi)}{T_{\parallel}(\psi)} \\ B_{\mathrm{n}}(\psi) \text{ : integration constant} \end{cases}$$

- Therefore,

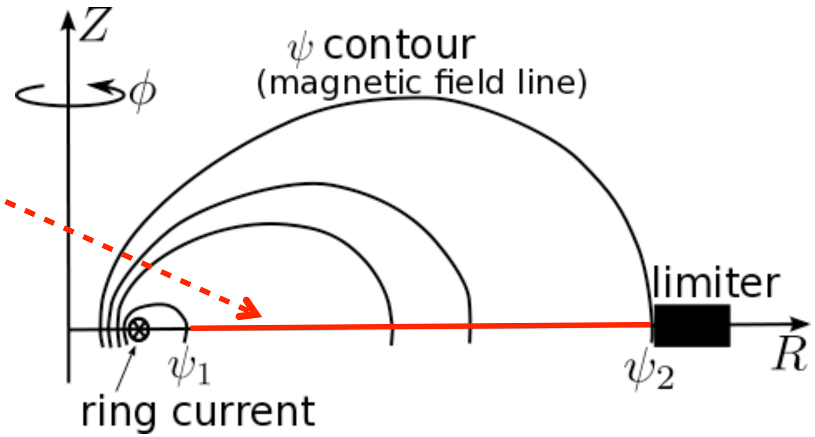
$$\begin{cases} p_{\perp}(\psi, B) = \bar{p}(\psi) \left(\dfrac{B}{B_{\mathrm{n}}(\psi)} \right)^{1-\lambda(\psi)} \\ p_{\parallel}(\psi, B) = \dfrac{1}{\lambda(\psi)} p_{\perp}(\psi, B) \end{cases}$$

- where,

$$\bar{p}(\psi) := \bar{n}(\psi) T_{\perp}(\psi)$$

Assumptions for pressure (2)

- We further assume the following:
  - λ is constant
  - $B_{\rm n}(\psi)$ is taken to be the same
  - as B on the outer midplane



- Profile is assumed as

$$\bar{p}(\psi) \propto (\psi - \psi_1)(\psi - \psi_2)$$

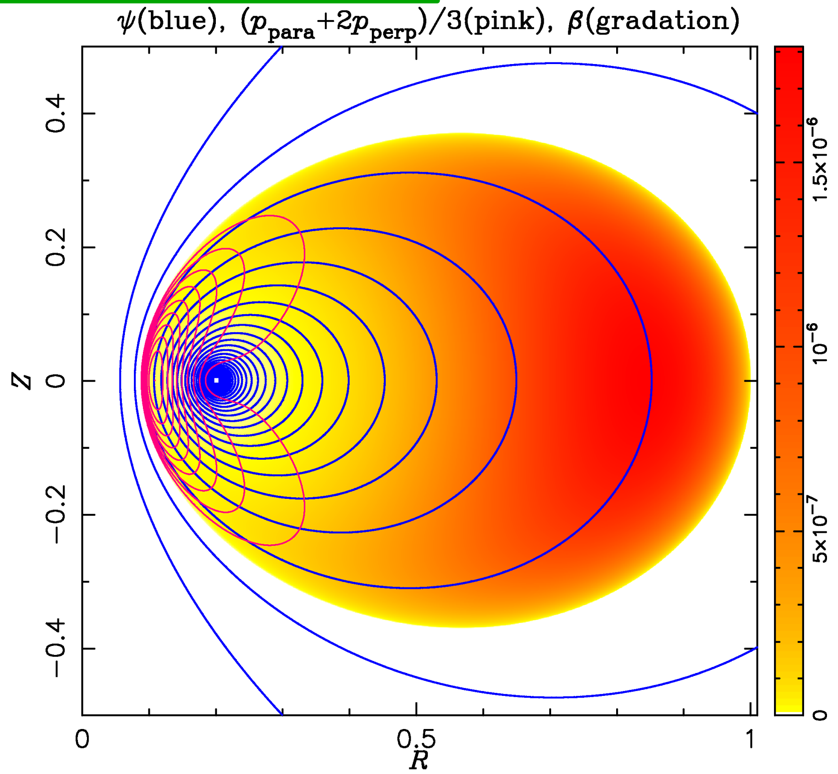
- When we compare equilibria by changing λ , we keep

$$p = \frac{1}{3}(2p_\perp + p_\parallel)$$

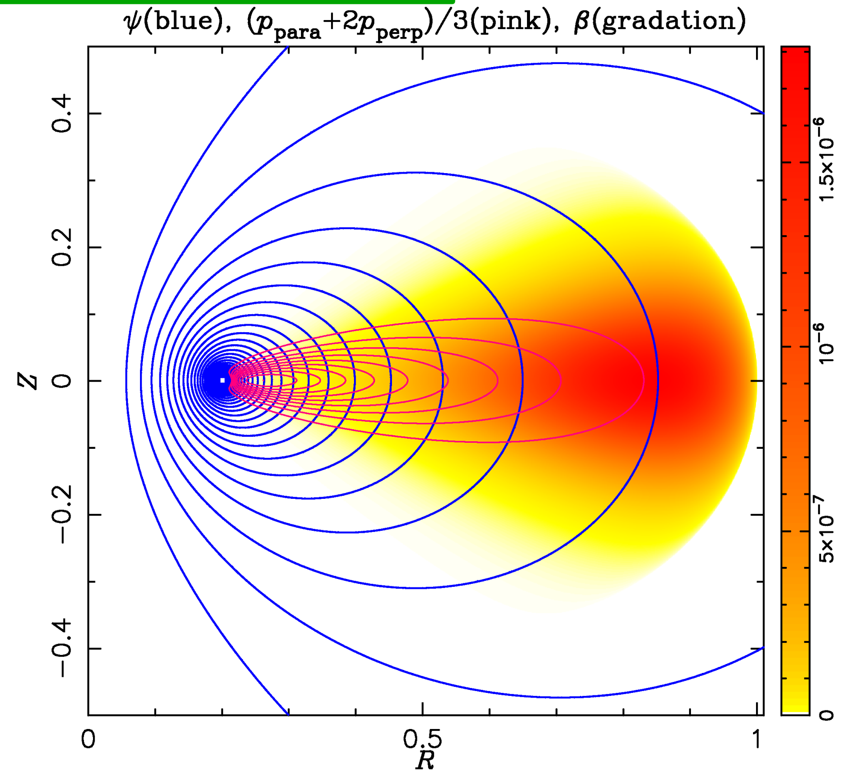
- unchanged on the outer midplane

Low beta ($\beta_0 = 10^{-10}$)

$\lambda = 0.1$
$\beta_{\mathrm{local}}^{\max} = 1.81 \times 10^{-6}$



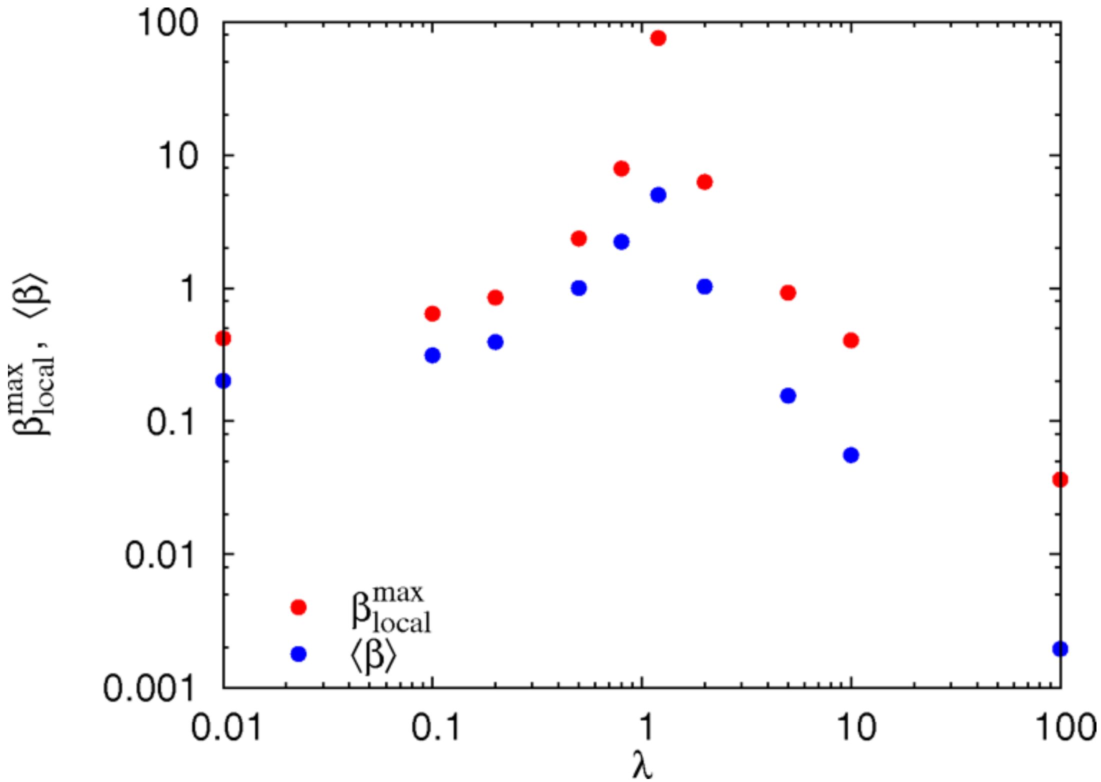
$\lambda = 10$
$\beta_{\mathrm{local}}^{\max} = 1.81 \times 10^{-6}$



- Because of $\left.\frac{\partial p_\parallel}{\partial B}\right|_\psi + \frac{p_\perp - p_\parallel}{B} = 0$, on the same ψ (magnetic field line),
  - $\lambda < 1$: $p_\parallel$, and thus $p_\perp$ also, becomes large at larger B (inner region of ring current)
  - $\lambda > 1$: $p_\parallel$, and thus $p_\perp$ also, becomes large at smaller B (outer region of ring current)

Equilibrium beta limit

- Numerically obtained beta limit is plotted



$\beta_{\rm local}^{\rm max}$: local maximum beta

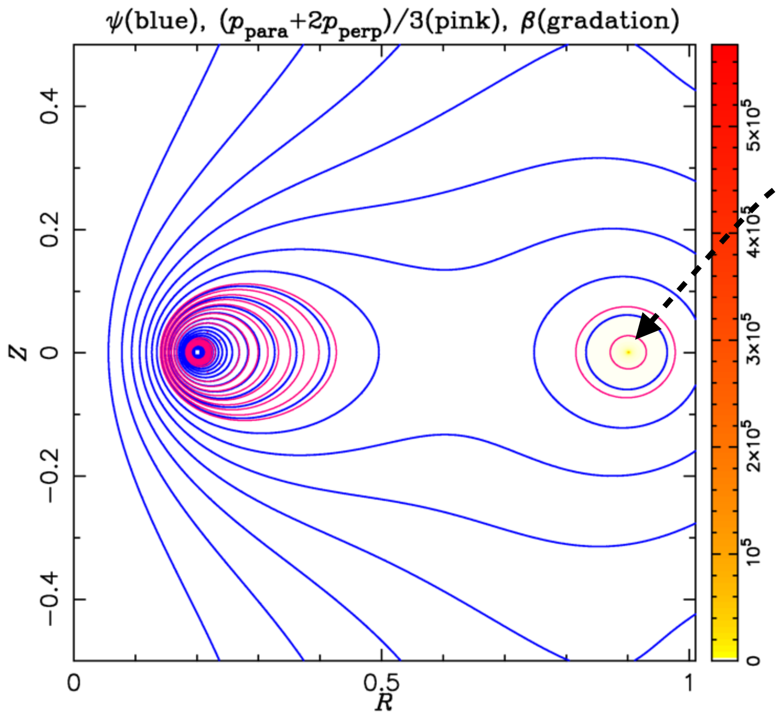
$\langle\beta\rangle$: volume averaged beta

- β limit seems to be maximum at $\lambda = 1$, and decreases more rapidly as λ increases

- Difference between $\beta_{\rm local}^{\rm max}$ and $\langle\beta\rangle$ is larger at $\lambda > 1$

High beta, two torus equilibrium at $\lambda = 1$

- At $\lambda = 1$, a very high beta equilibrium is obtained, where the plasma separates into two torus



$$\langle\beta\rangle = 23.2$$