Alpha particle transport in the presence of driftwave turbulence

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August 29 - 30, 2013

The 19th Numerical Experiment of Tokamak Workshop

Kyoto University Katsura Campus

Introduction : Alpha particle behavior is investigated with a turbulent background

- Gyrokinetic simulation (PIC and Vlasov) are employed to investigate energetic particle transport. [a] While the ITG turbulence signals are self-consistent in those work, the energetic particle transport is not self-consistent. The energetic particle transport is from an orbit following.

- We model the electromagnetic fluctuation in the form of ballooning type drift-wave eigenmode structure

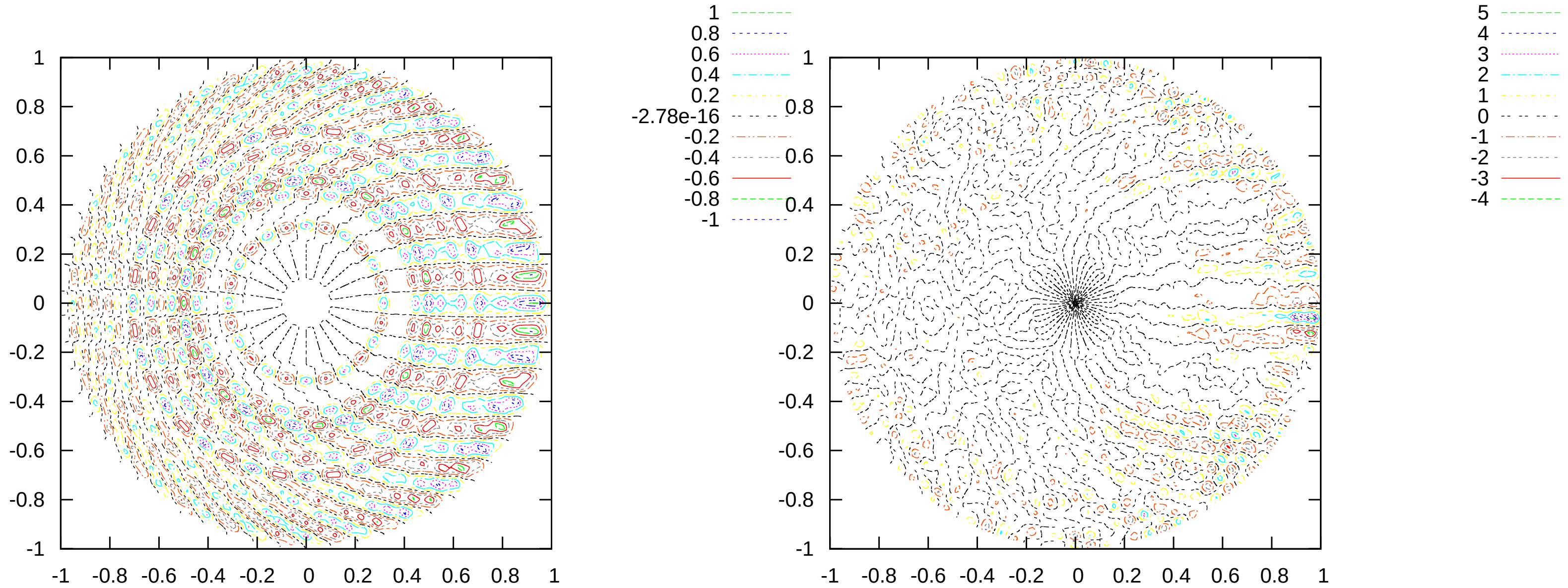
$$\Phi(r, \theta, \zeta) = \sum_{m/n} \Phi_{m/n}(r) \exp\left[-(r - r_{m/n})^2/\delta^2\right] \exp i(m\theta - n\zeta + \omega_\star t)$$

allows us to see one to one correlation between the transport and the free parameters. The latter Weber type "(Hermite polynomials $\times$ Gaussian) " function is the eigen-solution of the ballooning equation.[b]

[a] e.g. C. Estrada-Milla Phys. Plasmas 13, 112303 (2006), W. Zhang, Z. Lin, and L. Chen, Phys. Rev. Lett. 101, 095001 (2008). T. Hauff et al. Phys. Rev. Lett. 102, 075004 (2009).

[b] J. W. Connor, R. J. Hastie, and J. B. Taylor, Proc. R. Soc. London A **365**, 1 (1979).

The linear and nonlinear toroidal driftwave structure are plotted on a poloidal plane



Toroidal drift eigenmode structure is applied to the analysis.[a] **(Left)** $\Phi(r,\theta)$ with $n = 10$ only and **(Right)** $\Phi(r,\theta)$ with $10 \leq n \leq 20$ profiles plotted on a poloidal plane. Note the ballooning structure on a bad curvature side. The turbulent background with finite $\omega_\star$. Parameters $q = 1 + 2r^2/a^2$ $(0 \leq r \leq a)$, $10 \leq n \leq 20$, $\omega_\star/\Omega_{ci} \sim 0.01$.

[a] C.Z.Cheng, Phys. Fluids **25**, 1020 (1982).

The guiding center equation is solved in three dimensional geometry

- The guiding center equation by Euler-Lagrange equation is given by operating $\times\hat{b}$ and $\cdot\hat{b}$:

$$\dot{\mathbf{X}} = v_{\parallel}\frac{\mathbf{B}^{\star}}{B_{\parallel}^{\star}} + \frac{1}{qB_{\parallel}^{\star}}\mathbf{b}\times\left(\mu\nabla B - q\mathbf{E}^{\star}\right)$$

$$\dot{v_{\parallel}} = -\frac{\mathbf{B}^{\star}}{mB_{\parallel}^{\star}}\cdot\left(\mu\nabla B - q\mathbf{E}^{\star}\right)$$

- The magnetic field at an equilibrium is given by

$$\mathbf{B}_{cov} = G\left(\psi\right)\nabla\zeta + I\left(\psi\right)\nabla\theta + \delta\left(\psi,\theta,\zeta\right)\nabla\psi$$

$$\mathbf{B}_{clebsch} = \nabla\psi\times\nabla\theta + \nabla\zeta\times\nabla\chi = \nabla\psi\times\nabla(\theta - \zeta/q)$$

- Normalization by minor radius a, cyclotron frequency Ω_c of the test particle (α or lower energy ions), magnetic field strength at the axis B_0, and $a^2\Omega B_0$ for the electrostatic potential.

- Each component of the guiding center equation for $(r, \theta, \zeta$, and $v_\parallel)$ is given by (hereafter quantities are normalized quantities)

$$\frac{dr}{dt} = -\varepsilon\mu \sin(\theta) - \sum_n \sum_m \left(\frac{-m}{r}\right) \Phi_{m/n} e^{\frac{-(r-r_{m/n})^2}{d^2}} \sin(m\theta - n\zeta + \delta_n + \omega_{\star,n} t),$$

$$\frac{d\theta}{dt} = \frac{\varepsilon v_\parallel}{q(r)} - (\varepsilon\mu/r)\cos(\theta),$$

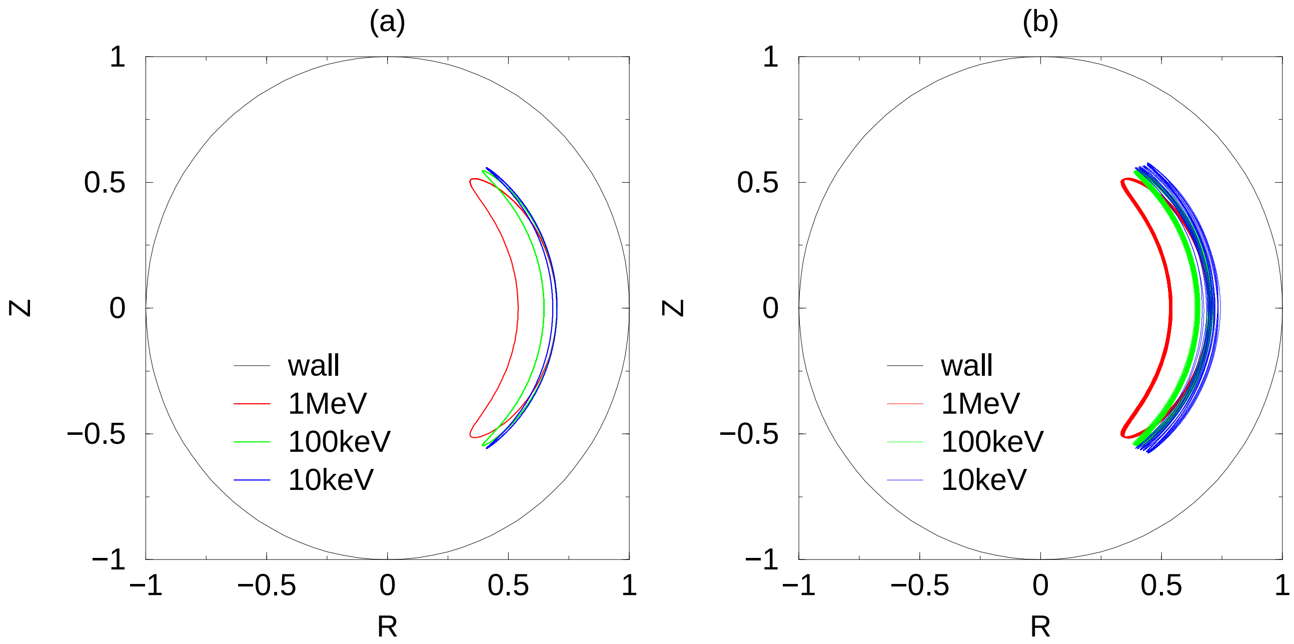
$$\frac{d\zeta}{dt} = \varepsilon v_\parallel,$$

$$\begin{aligned} \frac{dv_\parallel}{dt} &= -\frac{\varepsilon^2 \mu \sin(\theta)}{q(r) r} \\ &+ \sum_n \sum_m [m/q(r) - n] \, \Phi_{m/n} e^{\frac{-(r-r_{m/n})^2}{d^2}} \sin(m\theta - n\zeta + \delta_n + \omega_{\star,n} t). \end{aligned}$$

Here the inverse aspect ratio is given by $\varepsilon = a/R_0$.

Larger the energy smaller the radial transport

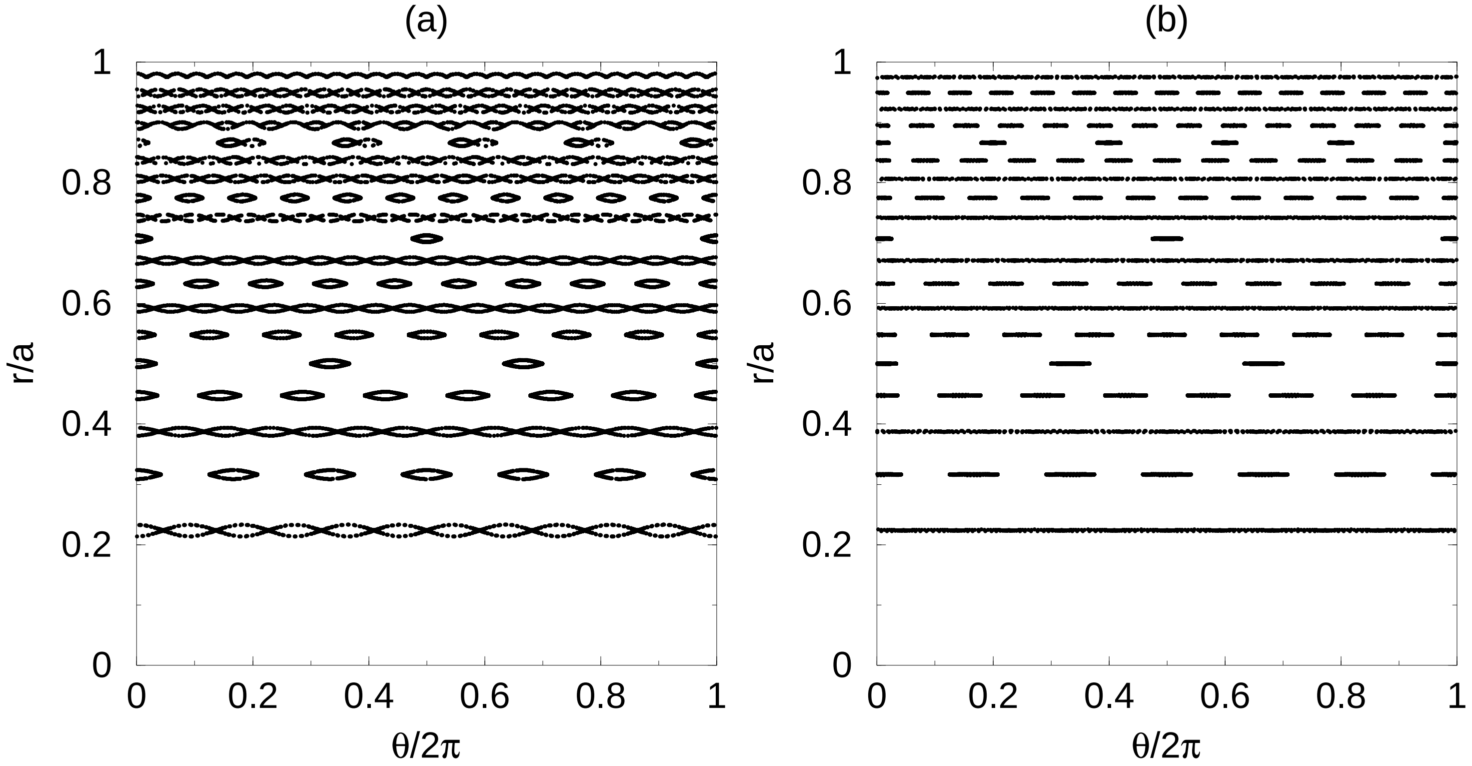
- Particle transport in the presence of time dependent turbulent signal.



- Larger the energy (higher the bounce frequency) smaller the radial transport (J conserves for high energy particles).

Passing particles form electric islands at each mode rational surface

- Particle transport in the presence of linear $n = 10$ eigenmodes for the demonstration purpose (a) 1.5 keV ions and (B) alpha particles.



- The island width is larger for lower energy particles.

- To see the parallel velocity dependence of the island widths, let us start from guiding center equation in flux coordinates for the $(r, \theta, \zeta, v_\parallel)$ components:

$$\frac{dr}{dt} = \sum_n \sum_m \left(\frac{-m}{r}\right) \Phi_{m/n} \sin\left(m\theta - n\zeta + \omega_{\star,n} t\right),$$

$$\frac{d\theta}{dt} = \frac{\varepsilon v_\parallel}{q(r)},$$

$$\frac{d\zeta}{dt} = \varepsilon v_\parallel,$$

and $dv_\parallel/dt = 0$. The toroidal angle ζ behaves as a time like coordinate. By expanding the safety factor in the vicinity of mode rational surface $r_{m/n}$,

$$\frac{dX}{dt} = \left(\frac{-m}{r_{m/n}}\right)\left(\frac{\Phi_{m/n}}{\varepsilon v_\parallel}\right) \sin\left(Y + \Omega\zeta\right),$$

$$\frac{dY}{dt} = \frac{-m s_{m/n}}{q_{m/n}} X,$$

where $X = r - r_{m/n}$, $Y = m\theta - n\zeta$, $\Omega = (\omega_{\star,n}/\varepsilon v_\parallel)$, and $s_{m/n} = (dq/dr)_{m/n}/q_{m/n}$ is the shear parameter.

- This will give rise to a Hamiltonian of the form

$$H\left(X,Y,\zeta\right)=\left(\frac{ms_{m/n}}{q_{m/n}}\right)\frac{X^2}{2}-\left(\frac{m}{r_{m/n}}\right)\left(\frac{\Phi_{m/n}}{\varepsilon v_{\|}}\right)\cos\left(Y+\Omega_{\star}\zeta\right)$$

  whose separatrix equation is given by

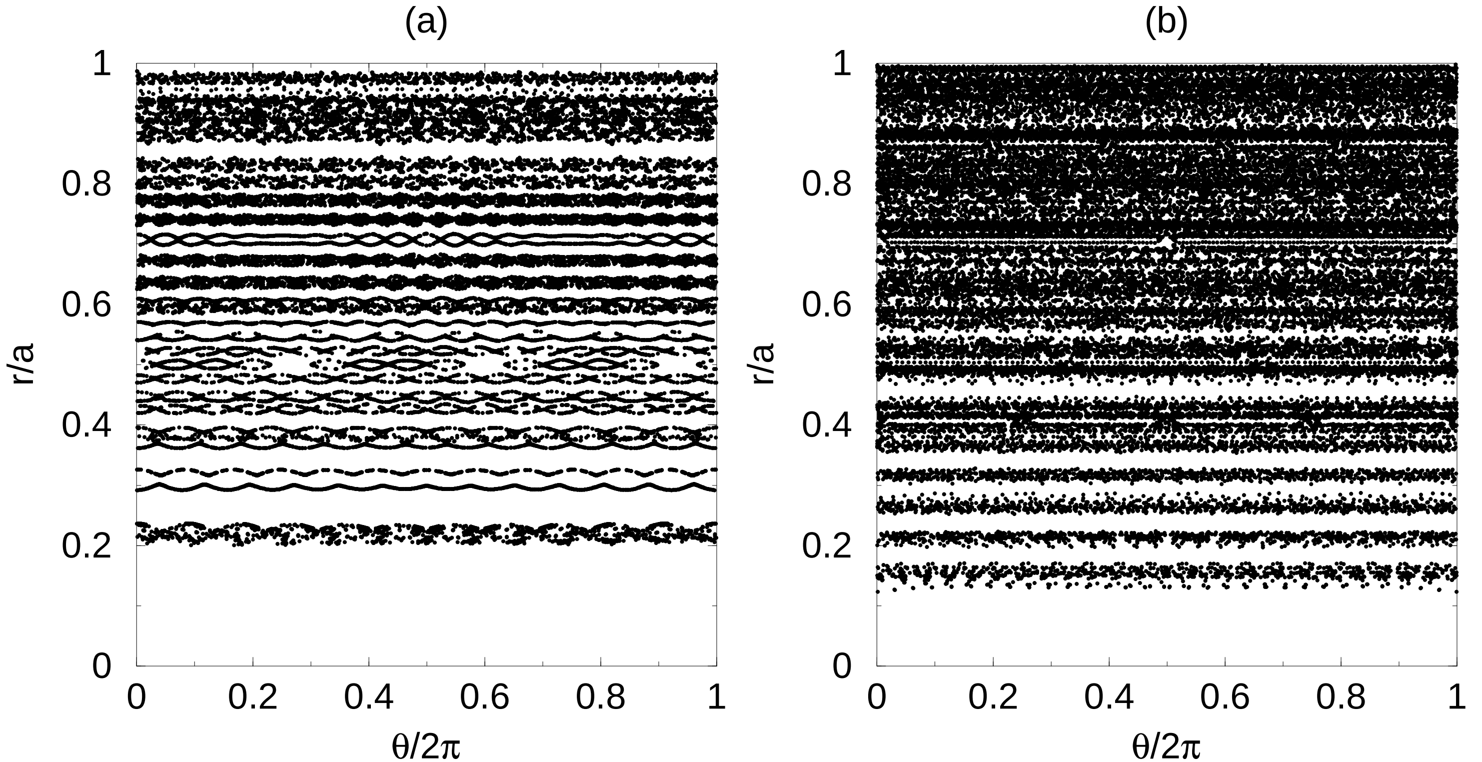
$$X=\pm 2\left(\frac{\Phi_{m/n}}{\varepsilon v_{\|}}\right)^{1/2}\left(\frac{q_{m/n}}{r_{m/n}s_{m/n}}\right)^{1/2}\cos\left(\frac{Y+\Omega_{\star}\zeta}{2}\right)$$

  which demonstrates clear $\sim v_{\|}^{-1/2}$ dependence on the island width. Island width of the simulation matches exactly. Net $E\times B$ drift is larger for the slower particles which spends longer time at the same phase of the driftwave.

- Finite drift wave frequency ($\omega_{\star}$ effect) gives rise to the shift of the resonant location.

Passing particles become stochastic in the presence of multiple toroidal mode numbers

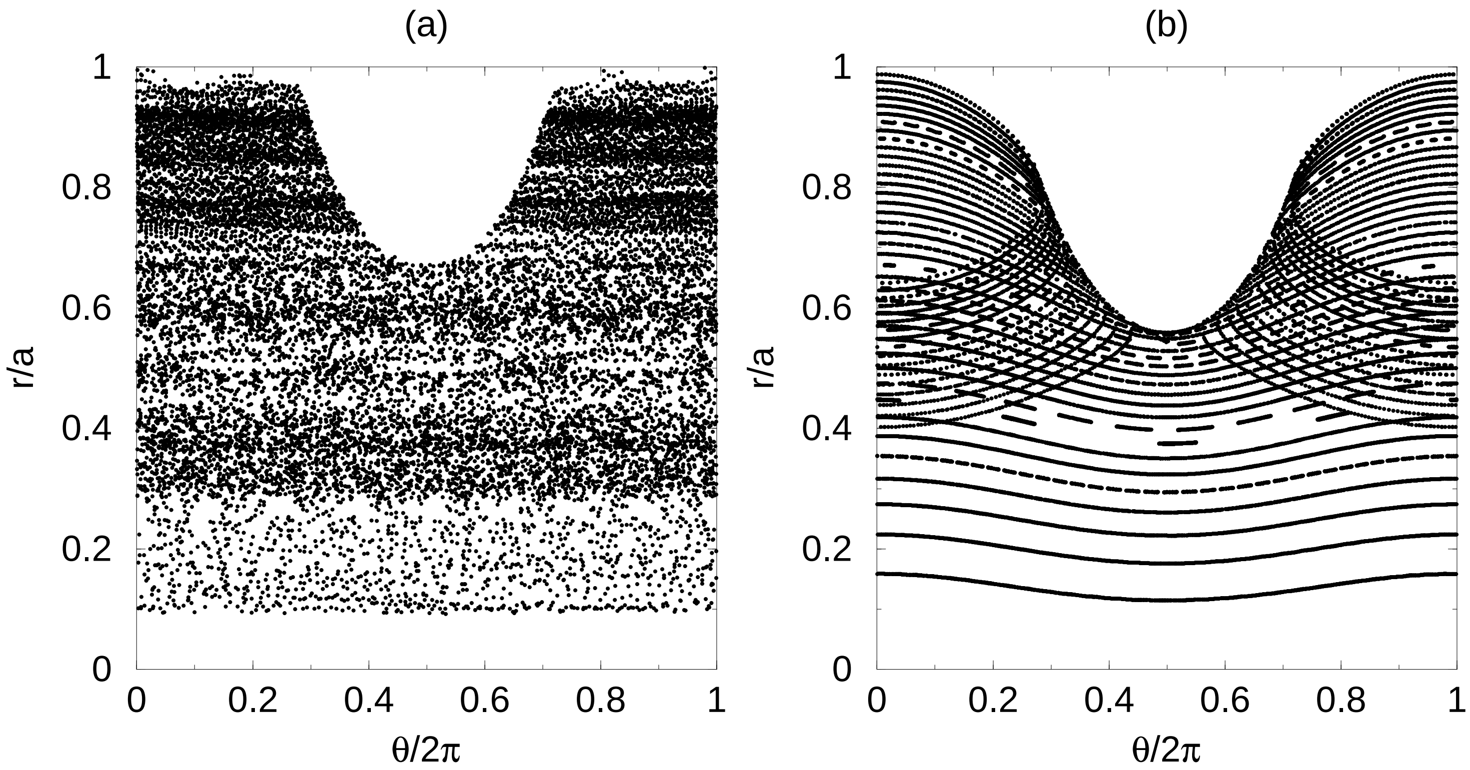
- Particle transport in the presence of (a) $n = 10$ and $n = 11$ and (b) $10 \leq n \leq 50$. Both for the $1.5keV$ ions.



- Island chains are densely formed at mode rational surfaces, easily overlap.

Adiabatic invariant persists for trapped particles

- Poincare plot of trapped particles for (a) $1.5keV$ and (b) alpha particles.



- "J"does not easily break. Seemingly diffusive but can return after long but finite period. Secondary island formation can be found in the action-angle space when the precession frequency becomes comparable to ω_b.

- The normalized guiding center Hamiltonian is given by

$$\begin{aligned} H\left(v_{\parallel}, \theta, t\right) &= \frac{v_{\parallel}^2}{2} + \mu B(r,\theta) + \Phi(r,\theta,\zeta) \\ &= \frac{v_{\parallel}^2}{2} + \mu B_0\left[1 - \varepsilon r \cos\left(\theta\right)\right] + \sum_{n,m} \Phi_{m/n}(r)\cos\left(m\theta - n\zeta + \omega_{\star,n} t\right). \end{aligned}$$

- Moving to $J - \Theta$ space ($J = \int v_{\parallel} dl$) allows us to examine generation of secondary islands. The Hamiltonian is separated into

$$H\left(J, \Theta, t\right) = H_0\left(J\right) + H_1\left(J, \Theta, t\right),$$

where $\zeta = \Omega_{\zeta} t + A \sin \omega_b t$ (ω_b is the banana's bounce frequency). By a Fourier expansion in the new angle Θ,

$$H_1\left(J, \Theta, t\right) = \sum_{m,n} \sum_{l} a_{mnl}(J) \begin{pmatrix} \cos \\ \sin \end{pmatrix} \left(l\Theta - n\Omega_{\zeta} t\right)$$

$$a_{mnl} = \frac{1}{\pi} \int_{-\pi}^{\pi} \begin{pmatrix} \cos \\ \sin \end{pmatrix} \left(m\theta\left(\Theta\right)\right) \begin{pmatrix} \cos \\ \sin \end{pmatrix} \left(l\Theta\right) d\Theta.$$

whereas, the equation of motion is given by

$$\frac{dJ}{dt} = \sum_{m,n}\sum_{l} l a_{ml}(J) \begin{pmatrix} \sin \\ -\cos \end{pmatrix} [l\Theta - n\Omega_\zeta t]$$

$$\frac{d\Theta}{dt} = \omega_b\,(J)\,.$$

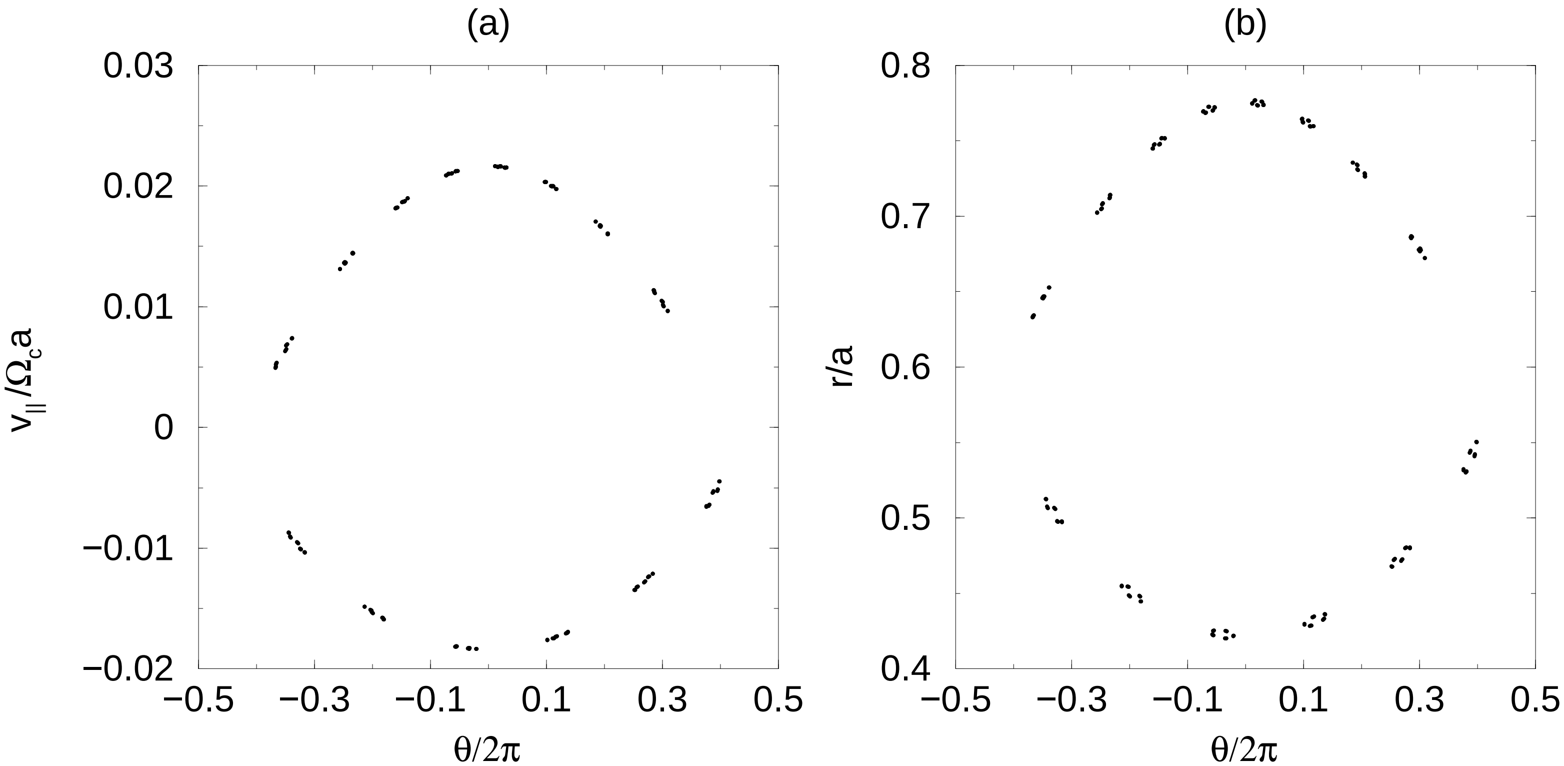
Substitutiong $\Theta = \Omega t$ into dJ/dt equation, the resonant condition is given by

$$l\omega_b - n\Omega_\zeta = 0.$$

and thus the resonant secondary Fourier modes are given by $l = n\Omega_\zeta/\omega_b$. Note that Ω_ζ is the toroidal precession frequency. By singling out a cosine term, the separatirix equation is given by

$$J = \pm 2\,(a_{mnl})^{1/2}\cos\left(\frac{l\Theta - \Omega_\zeta t}{2}\right)$$

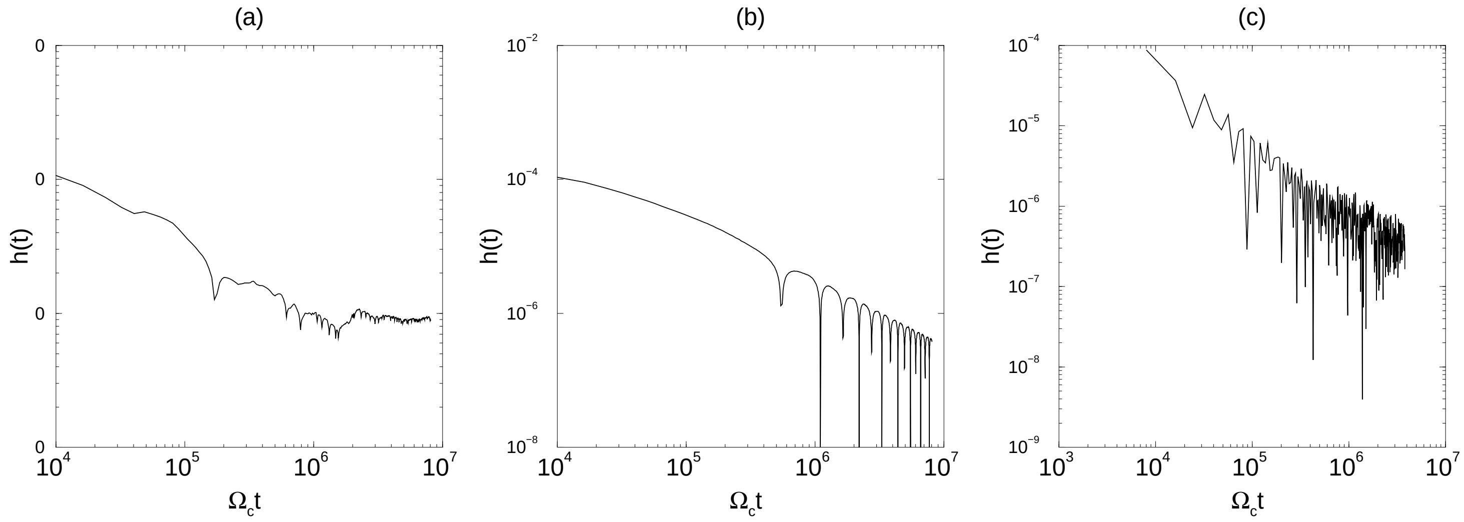
- Secondary island chains are formed in (a) the velocity space, (b) in configuration space for the alpha particle.



Whether particles are stochastic or not is examined by Lyapunov exponents

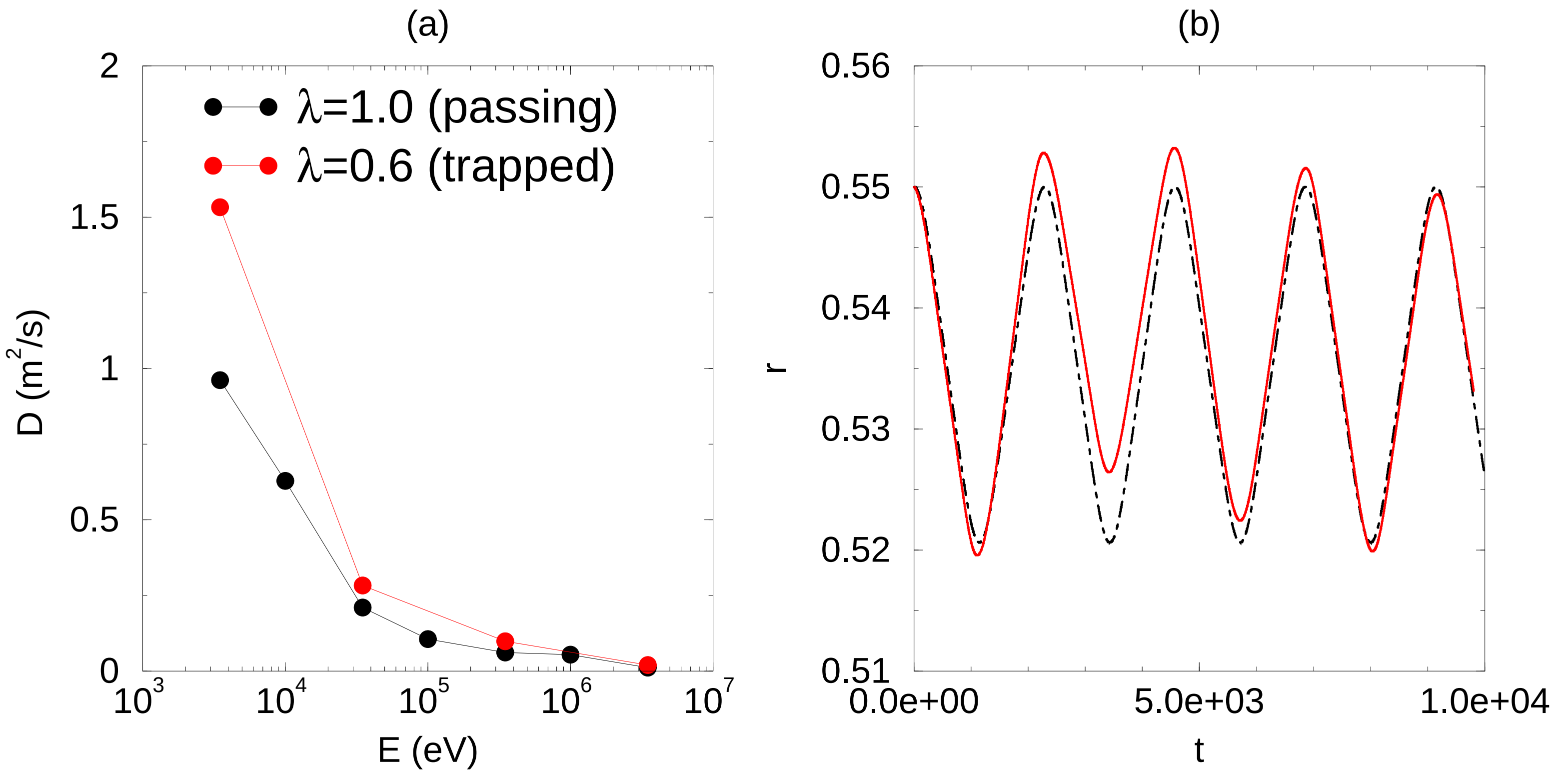
- Lyapunov exponents are estimated in the Cartesian coordinate by a mapping from flux coordinate (and then back from Cartesian to flux coordinate to push the accompanying test particles).

- (a) Stochastic (15keV passing ion at $q_{m/n} = 5/2$, (b) regular passing alphas and (c) trapped 15keV ion.



Particle diffusion is estimated

- When the banana width becomes comparable to the diffusion random step size (alpha particles for example), a direct estimation $D \sim (r - r_0)^2$ becomes troublesome. We estimate diffusion rate at a fixed $\theta = 0$ instead.



Summary and discussions

- Guiding center orbit following calculation is discussed in the presence of driftwave turbulence signal in a toroidal geometry.
- Transport is smaller for the high energy particles due to $\sim v_{\|}^{-1/2}$ dependence of the island width (net $E \times B$ drift is larger for the slower particles). Finite drift wave frequency ($\omega_{\star}$ effect) gives rise to the shift of the resonant location.
- For trapped particles, the effective $E \times B$ drift becomes extremely large and contributes to the radial transport at the banana tip (from the same mechanism as in the passing).
- Second adiabatic invariant tend to persist for trapped particles. When the precession frequency can be comparable to the bounce frequency for fast particles, secondary islands are formed in the $J - \Theta$ space and in the configuration space, correspondingly.