Three-Dimensional Numerical Analysis of interchange mode in the Large Helical Device with ion diamagnetic effects and dissipation

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Outline

1. Motivation : Understanding the stability
2. Reduced MHD
3. MIPS code results
4. Summary and future work

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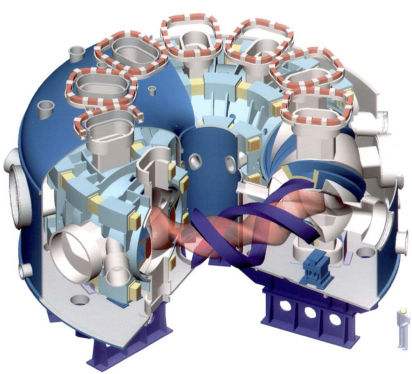
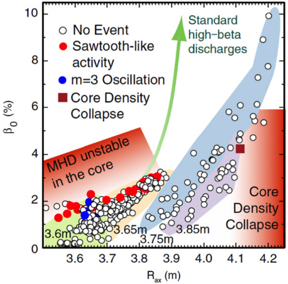
Motivation : Experimental results in the Large Helical Device



[KOMORI 2009]

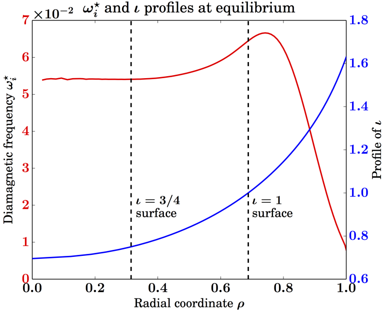


- Larger R_{ax} : Better stability
- Lower R_{ax} : Better confinement of energetic particles
- Search for the **best trade-off**

- Recently high-β plasmas unexpectedly stable at low R_{ax}
- Stability not understood in the framework of resistive MHD

Possible role of the flows

- Viscosity/heat conductivity are stabilizing [Ichiguchi 2000] but not sufficient to explain stability
- Stabilizing effect of rotation observed experimentally [Takemura 2012] (locking immediately followed by minor collapse)
- The influence of diamagnetic flows on interchange mode [Strauss 2004] has not been investigated numerically on LHD yet
- Stabilizing/destabilizing effect of diamag. flow can be due to
  - **Modification of mode structure** due to link between δp and $\delta \mathbf{v}$
  - **Stabilization by the shear** of the equilibrium flow (however small effect expected)



Outline

Reduced MHD equations with ion diamagnetic effect

- MHD model for (ψ, ϕ, p) [Strauss 1980] with addition of resistivity, viscosity, diffusion and ion diamagnetic effect :

$$\boxed{\mathbf{v}_i^\star = d_i \frac{\mathbf{B} \times \nabla p_i}{\rho B^2}, \ \omega_i^\star = \mathbf{k}_\perp \cdot \mathbf{v}_i^\star}$$

$$\begin{cases} i\omega\psi - k_\parallel \phi + \eta \nabla_\perp^2 \psi = 0 \\ i(\omega - \omega_i^\star)\nabla_\perp^2 \phi + k_\parallel \nabla_\perp^2 \psi + \frac{m}{r}\Omega' p + \nu \nabla_\perp^4 \phi = 0 \\ i\omega p + \frac{m}{r} p_0' \phi + \chi_\perp \nabla_\perp^2 p = 0 \end{cases}$$

- Ω' gives curvature contribution, essential for interchange
- Problem solved by shooting method :
  Boundary conditions $\phi(0) = \phi(1) = p(0) = p(1) = 0$
  $\psi(0) = 0, \psi'(1) = 1, \psi(1)$ free (see *e.g.* [Ueda PoP 2014])
- The frequency ω is iterated in the complex plane until the requirement $\psi(1) = 0$ is met.

Without dissipation, $\omega(\omega - \omega_i^\star) = -\gamma_I^2$

- Without dissipation, $\eta = \nu = \chi_\perp = 0$, the eigen-mode equation reduces to :

$$\omega(\omega - \omega_i^\star)\nabla_\perp^2\phi = k_\parallel\nabla_\perp^2(k_\parallel\phi) - \frac{m^2}{r^2}\Omega' p_0'\phi$$

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- Without dissipation, $\eta = \nu = \chi_\perp = 0$, the eigen-mode equation reduces to :

$$-\gamma_I^2 \nabla_\perp^2 \phi = k_\parallel \nabla_\perp^2 (k_\parallel \phi) - \frac{m^2}{r^2} \Omega' p_0' \phi$$

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- Same eigenmode problem, hence $\boxed{\omega(\omega - \omega_i^\star) = -\gamma_I^2}$ [Kulsrud 1963, Stringer 1975, Connor 1984, Sugama 1988]

$$\Longrightarrow \begin{cases} \underline{\text{Growth rate :}} \quad \Re(\omega) = \gamma_I \sqrt{1 - \left(\dfrac{\omega_i^\star}{2\gamma_I}\right)^2} \\ \\ \underline{\text{Frequency :}} \quad \Im(\omega) = \dfrac{\omega_i^\star}{2} \end{cases}$$

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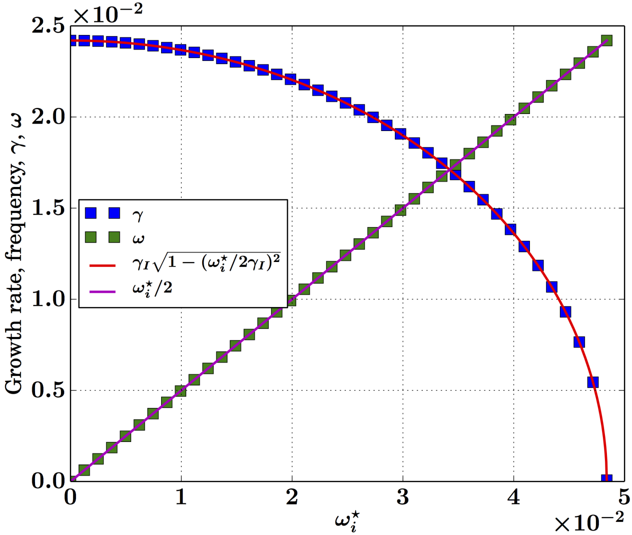
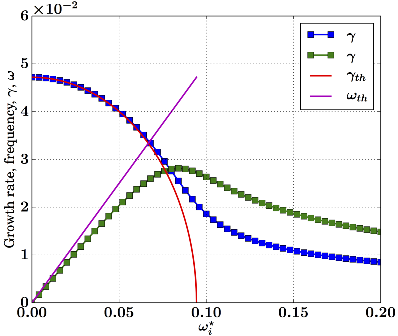


With dissipation, picture is changed

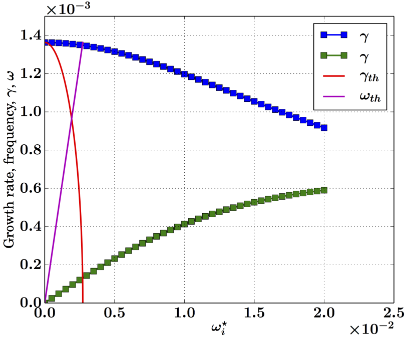
- When η, ν, $\chi_\perp \neq 0$, the picture is changed because it cannot be reduced to the ideal case
- If dissipation is weak, $\gamma_D \simeq \gamma_I$ and we can expect similar relation :

$$\boxed{\omega(\omega - \omega_i^\star) = -\gamma_D^2}$$

$\gamma_D \simeq \gamma_I$: works well for small $\omega_i^\star$



$\gamma_D \sim 0.1\gamma_I$: Does not work



Reduced MHD : conclusion

- The dissipation modifies the ideal growth rate $\gamma_I \longrightarrow \gamma_D$
- As a result the stabilization influence of ion diamagnetic effect is modified
- $\omega(\omega - \omega_i^\star) = -\gamma_D^2$ approximately if $\gamma_D \simeq \gamma_I$
- The ion diamagnetic effect **seems always stabilizing**

Outline

MHD model of MIPS code : Hazeltine-Meiss [Hazeltine 1992]

$$\begin{cases} \mathbf{v} = \mathbf{v}_E + \mathbf{v}_{\parallel,i} \\ \mathbf{v}_i = \mathbf{v} + \mathbf{v}_i^\star \end{cases}$$

$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho \mathbf{v} + \rho \mathbf{v}_i^\star) = S_\rho + \nabla \cdot (D \nabla \rho) \\ \rho \left(\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} + \mathbf{v}_i^\star \cdot \nabla \mathbf{v}_\perp \right) = \mathbf{J} \times \mathbf{B} - \nabla p + \nu \nabla^2 \mathbf{v} \\ \partial_t p_s + \mathbf{v} \cdot \nabla p + \Gamma p \nabla \cdot \mathbf{v} = S_p + \nabla \cdot (\chi_\perp \nabla p) \\ \partial_t \mathbf{B} = -\nabla \times \mathbf{E} \\ \mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J} \end{cases}$$

- d_i is the normalized ion skin depth

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$$\boxed{d_i = K / \sqrt{n_0}}$$

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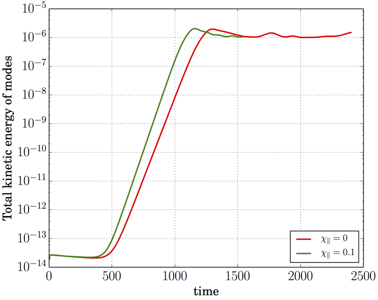
n_0 (m^{-3})	1×10^{18}	1×10^{19}	1×10^{20}
d_i	0.23	0.072	0.023

Simulation with the MIPS code

- The interchange mode dynamics is investigated using MIPS code [Todo 2010]
- The MHD model is Hazeltine-Meiss model [Hazeltine 1992]
- Numerical scheme is **explicit** 4^{th} order finite differences
- Equilibrium **HINT2 code** [Suzuki 2006], no assumption of flux surface
- Parallel heat conductivity is being developed but **not routinely used** yet

- Parallel diffusion important for
  - Correct growth rates, note that it can be destabilizing due to terms like $\mathbf{b}_0 \cdot \nabla \left(\tilde{\mathbf{b}} \cdot \nabla T_0 \right)$
  - **Correction of pressure flattening** in nonlinear phase



Diamagnetic frequency $\omega_i^\star$ compared to γ_I, γ_D

- Since $\omega_i^\star$ depends on the density, the **stabilization will depend on the density**, which varies in typically $10^{18} - 10^{20}$ m^{-3} in the experiment.

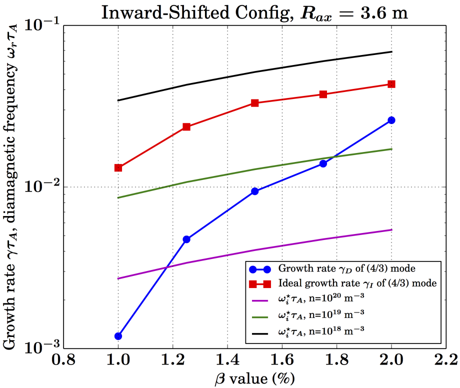
- $\eta = 10^{-7}$, $\nu = \chi_\perp = 10^{-6}$

- $\beta = 2\% \Rightarrow \gamma_D \simeq \gamma_I/2$

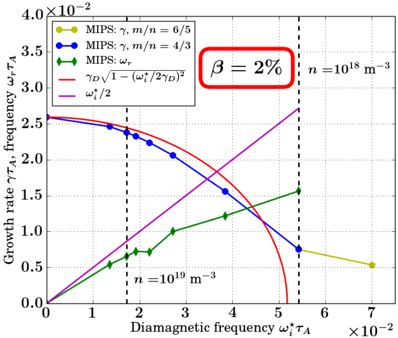
- $\beta = 1\% \Rightarrow \gamma_D \simeq \gamma_I/10$

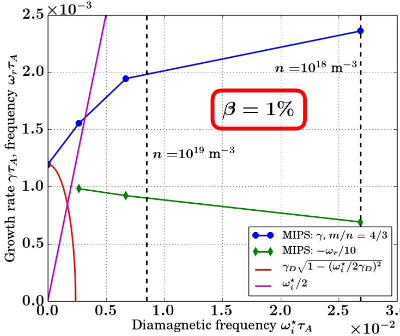
- $\omega_i^\star = 2\gamma_I$ for $n_0 = 10^{18}$ m^{-3}

- Study $(\beta, \omega_i^\star)$ space



Diamagnetic stabilization depends on γ_D/γ_I





- Linear **stabilization**
- Rotation in **ion direction**
- $4/3$ mode $\longrightarrow$ $6/5$ mode
- Suggests full stabilization of $4/3$ mode

- Linear **destabilization**
- Rotation in **electron direction**
- No change in mode structure
- Destabilization mechanism not understood

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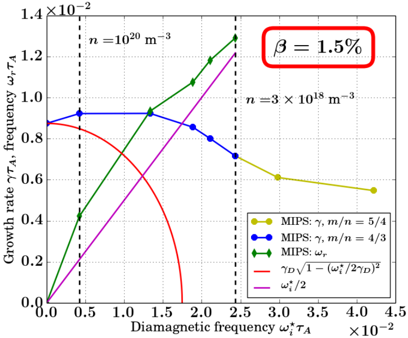
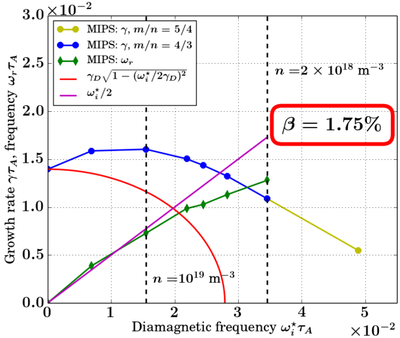
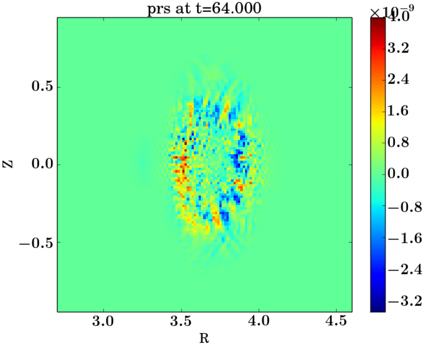




- For small $\omega_i^\star$, the mode is **destabilized**
- For larger $\omega_i^\star$, the mode is **stabilized**
- With increasing β, the influence of **dissipation becomes weaker** and the curves become closer
- When the mode is seen to change helicity, less than 50% stabilization

Change of rotation direction in non-linear phase

- $\beta = 2\%$: rotation in ion direction in linear phase
- In the non-linear phase, the mode rotation changes to electron



- The reversal of rotation direction is not due to a local inversion of pressure gradient
- The slightly more external 5/4 mode takes over
- This causes the change of rotation direction
- In this sense the linear phase is not really relevant

In experiment usually **electron direction** observed [Takemura 2013]

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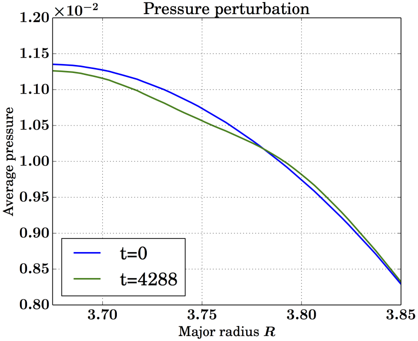

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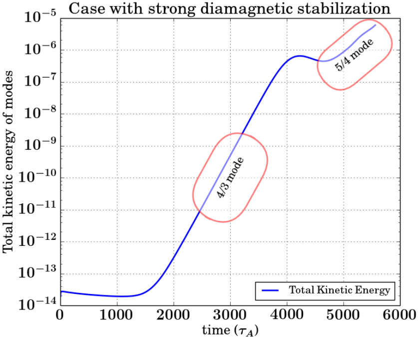

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Summary and future work

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- Limited diamagnetic **stabilization** when $\beta = 2\%$ (requires low density)
- Diamagnetic **destabilization** when $\beta = 1\%$
- In between, destabilization for low $\omega_i^\star$, weak stabilization for larger $\omega_i^\star$
- Other 5/4 mode **rotating in electron direction** becomes dominant when 4/3 mode is stabilized
- This mode can also be **seen in non-linear phase** when 4/3 mode is most linearly unstable mode
- Reduced MHD helpful to interpret results, but does not explain observed destabilization
- Overall two-fluid effects seem poor candidate to explain better stability

Summary and future work

Future works

- Ongoing analytic work on the dispersion relation of the resistive diamagnetic interchange mode, in the spirit of [Ara 1978] for the kink/tearing mode :

$$\left(\hat{\lambda}(\hat{\lambda}-i\hat{\lambda}_i)\right)^{1/2} = \hat{\lambda}_I \frac{\Lambda^{9/4}\Gamma\left((\Lambda^{3/2}-1)/4\right)}{8\Gamma\left((\Lambda^{3/2}+5)/4\right)}$$

$$\Lambda = \left(\hat{\lambda}(\hat{\lambda}-i\hat{\lambda}_i)(\hat{\lambda}-i\hat{\lambda}_e)\right)^{1/3}$$

- Study the non-linear phase
- Investigate the influence of more **realistic flows** on the stability by putting a source of momentum

Thank you for your attention