



Benchmark of local and non-local neoclassical transport calculations in helical configurations

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Background of the work

- **Evaluation of neoclassical transport in stellarator / heliotron devices is important compared to that in tokamaks because of**
 - Relatively large amplitude
 - Strong dependence on radial electric field, especially at low-collisionality
 - Radial electric field estimated from ambipolar condition of NC flux
- **Complexity in solving drift-kinetic equation in helical configurations → mono-energy and local approximation are commonly used (DKES, GSRAKE, etc.)**
 - Reduction of the dimension of DKE to be solved from 5 to 3
 - In some W7AS and LHD ion-root discharges, fairly good agreement has been reported in the particle and energy flux b/w local NC calculations and transport analyses (from deposition profiles) at $r/a < 0.6$. (Dinklage *et al.*, IAEA 2012, Nuclear Fusion 2013)

Purpose of the work

- We further intend to improve the prediction of neoclassical contribution to the total radial fluxes and the E_r profile from ambipolar condition.
- In this work, it is investigated how much does **the non-local, 5D drift-kinetic simulation** (FORTEC-3D code) improve the evaluation of NC transport in the discharges in LHD, W7-AS, and TJ-II, which were precisely studied with local neoclassical transport codes.
- We would like to see in what condition local model is valid, and where the non-local effect becomes important for transport analysis.

Differences in local and non-local NC transport simulations

Drift-kinetic equation for $\delta f(r, \theta, \zeta, v, \xi) = f - f_M$:

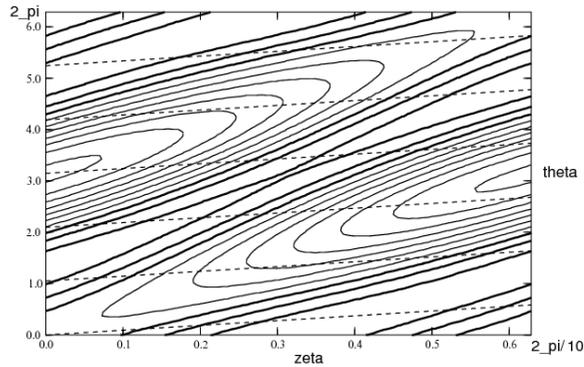
$$\frac{\partial \delta f}{\partial t} + \left[(\mathbf{v}_{\parallel} + \mathbf{v}_{E \times B} + \mathbf{v}_B) \cdot \nabla + \dot{v} \frac{\partial}{\partial v} + \dot{\xi} \frac{\partial}{\partial \xi} \right] \delta f = - \left(\mathbf{v}_B \cdot \frac{\partial}{\partial r} + \dot{v} \frac{\partial}{\partial v} \right) f_M(r, v) + C(\delta f)$$

Here, $C(\delta f)$ is the collision term, $\xi = v_{\parallel}/v$, and

$\mathbf{v}_B = \left(\frac{\mu}{e} \right) \frac{\mathbf{B} \times \nabla B}{B^2} + \left(\frac{mv^2 \xi^2}{eB} \right) \mathbf{b} \times \mathbf{b} \cdot \nabla \mathbf{b}$ represents the magnetic drift velocity.

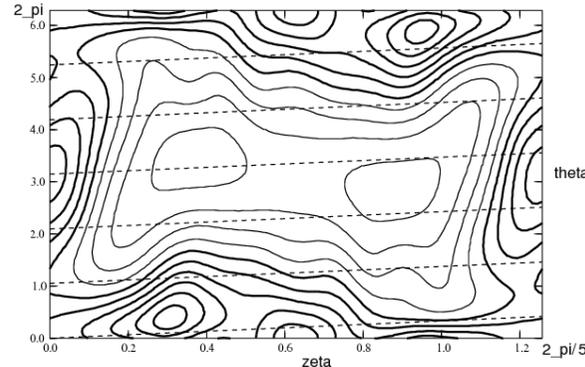
- **Local and mono-energy methods** [Reduced to 3-D (θ, ζ, ξ)]
 - **Simplified collision operator** : Adopt pitch-angle scattering operator
 - **Small-magnetic-drift & mono-energy approximations** :
 - Neglect the $[\mathbf{v}_B \cdot \nabla + \dot{v}(\partial/\partial v)]\delta f$ term ($\dot{v} = e\mathbf{v}_B \cdot \mathbf{E}_r/mv$)
 - **DKES**: Solves the DKE by using the variational principle.
 - **GSRAKE**: Ripple-averaged DKE (both passing and trapped particles). Simplification in the magnetic field spectrum.
- **Non-local, full-5D method**
 - **FORTEC-3D**: Solve the full 5-D DKE as it is, using the δf -PIC method.
 - Exact guiding-center trajectory including the $[\mathbf{v}_B \cdot \nabla + \dot{v}(\partial/\partial v)]\delta f$ term (what we call “non-local effect” here).
 - Pitch-angle & energy scattering collisions with conservation property.

Configuration of target plasmas



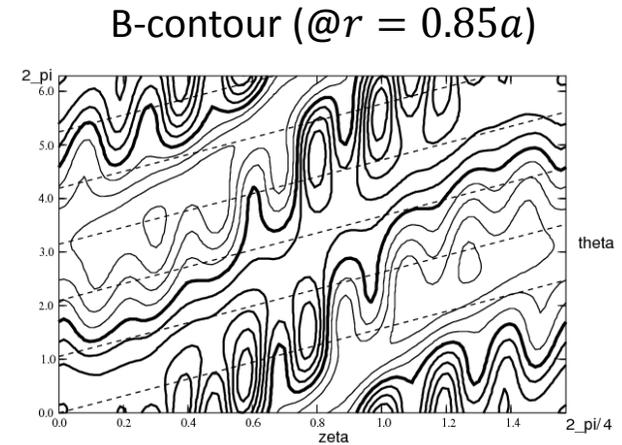
LHD (l=2, N=10 heliotron)
 (#109696 t=4.44s)

$1/\nu - \sqrt{\nu}$ regime (near the plateau boundary)



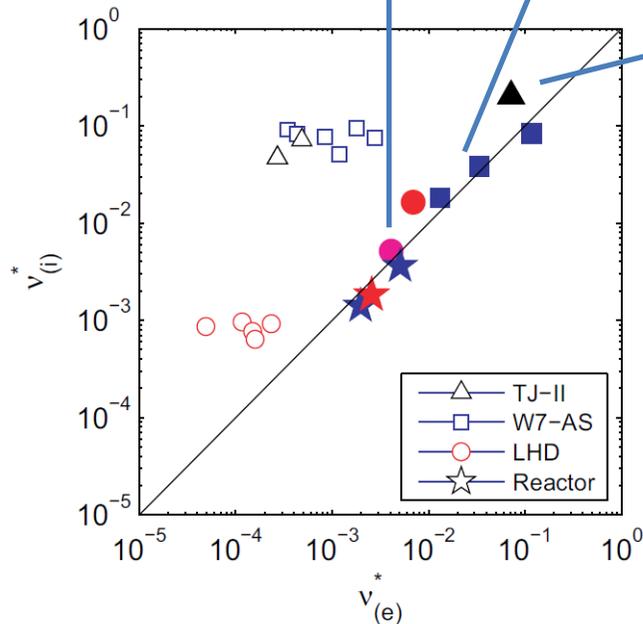
W7-AS (N=5 modular-coil helias)
 (#34609, t=0.3s)

Plateau regime (near the $1/\nu$ boundary)



TJ-II (N=4 heliac)
 (#19065, t=1.17s)

Plateau regime (near the $1/\nu$ boundary)

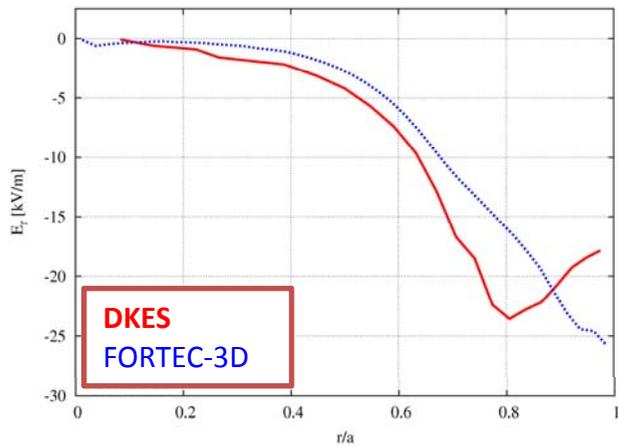


- The target discharges are characterized by $\nu_{*i} \sim \nu_{*e}$, since this situation is relevant to prospect the neoclassical transport in future reactors in stationary operation, $T_i \sim T_e$.
- The ambipolar condition in this condition is expected to be ion-root (negative E_r).

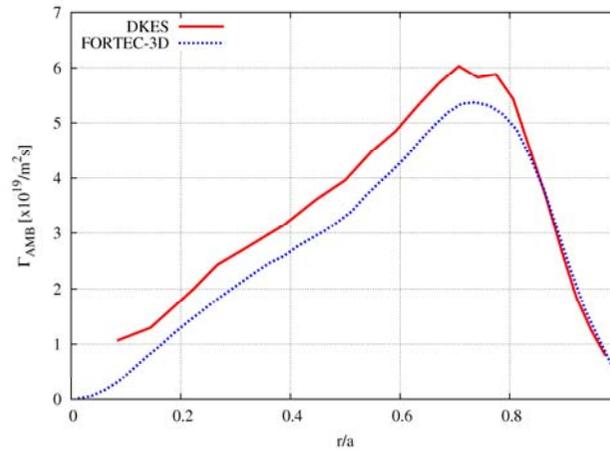
← Ion and electron collisionality of the discharges in the three devices (Dinklage et al., IAEA 2012, NF 2013)

Simulation result : (1) W7-AS (Comparison with DKES)

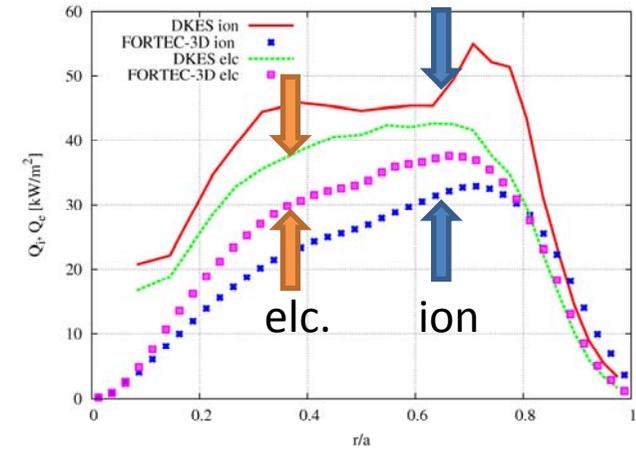
Ambipolar- $E_r : E_{amb}$



Ambipolar particle flux : Γ_{amb}

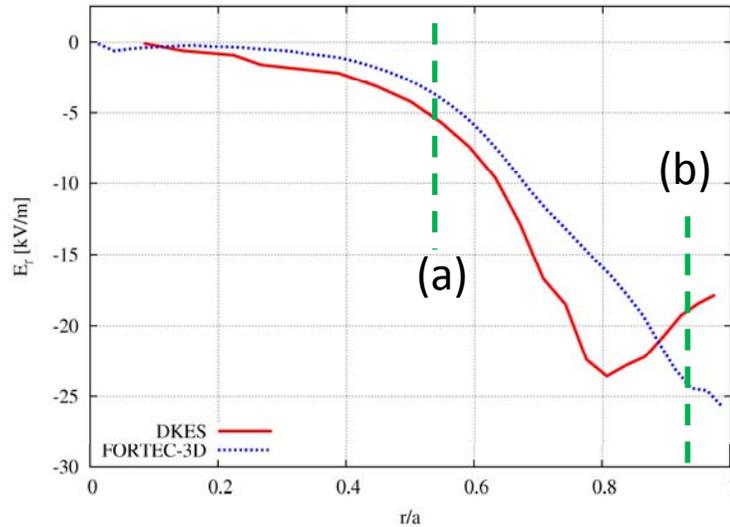


Energy flux : $Q_{i,e}(@E_{amb})$



- Ambipolar condition is determined by scanning the E_r profile which satisfies $\Gamma_i(r, E_r) = \Gamma_e(r, E_r)$.
- Difference in the E_{amb} b/w local and non-local codes is small in the core region but becomes larger towards the plasma boundary.
- On the contrary, difference in Γ_{amb} is small in the entire region.
- Most significant difference appears in Q_i .

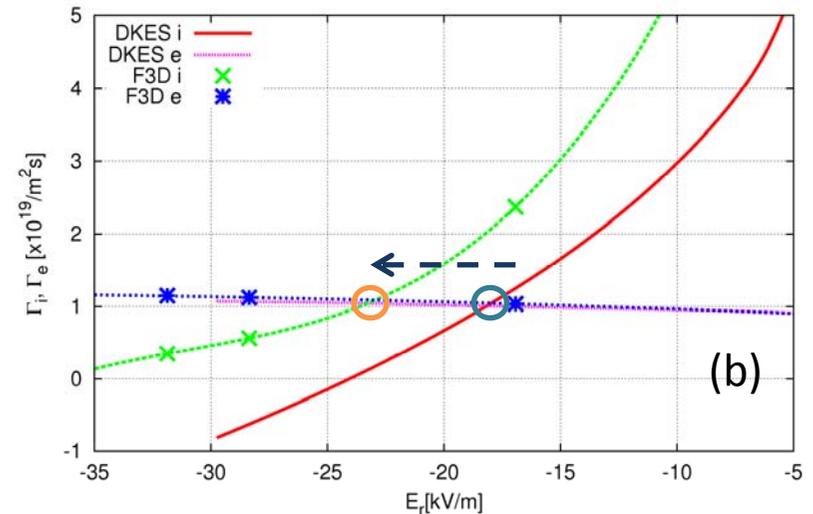
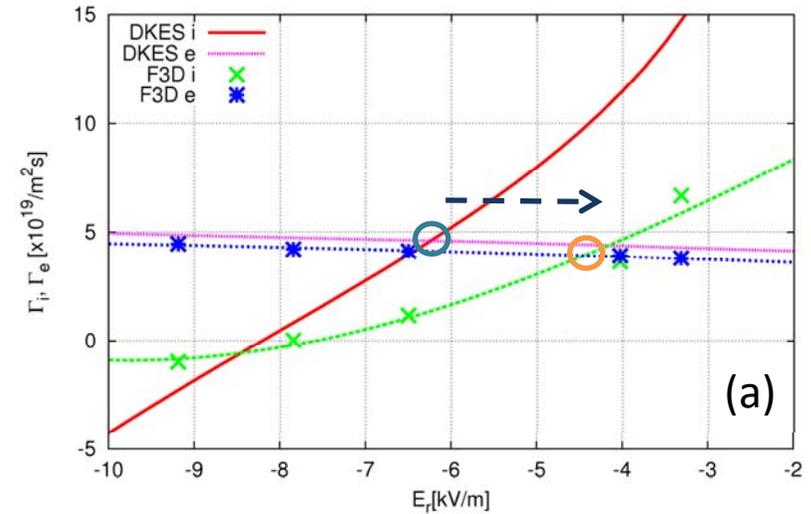
Finite magnetic drift causes the difference in ambipolar- E_r



A) $|E_r| \sim 0 \rightarrow$ Assumption $V_{E \times B} \gg V_B$ is not good.
 $V_B \cdot \nabla \theta$ term ($\propto \frac{\partial B}{\partial r}$) causes poloidal precession of ripple-trapped particles and prevents $1/\nu$ -type large NC flux even without $E \times B$ rotation.

B) $|E_r| \gg 0 \rightarrow$ Assumption $V_{E \times B} \gg V_B$ is valid.
 $V_B \cdot \nabla r$ term ($\propto \frac{\partial B}{\partial \theta}$) becomes more effective near the boundary, since magnetic ripple is larger there.

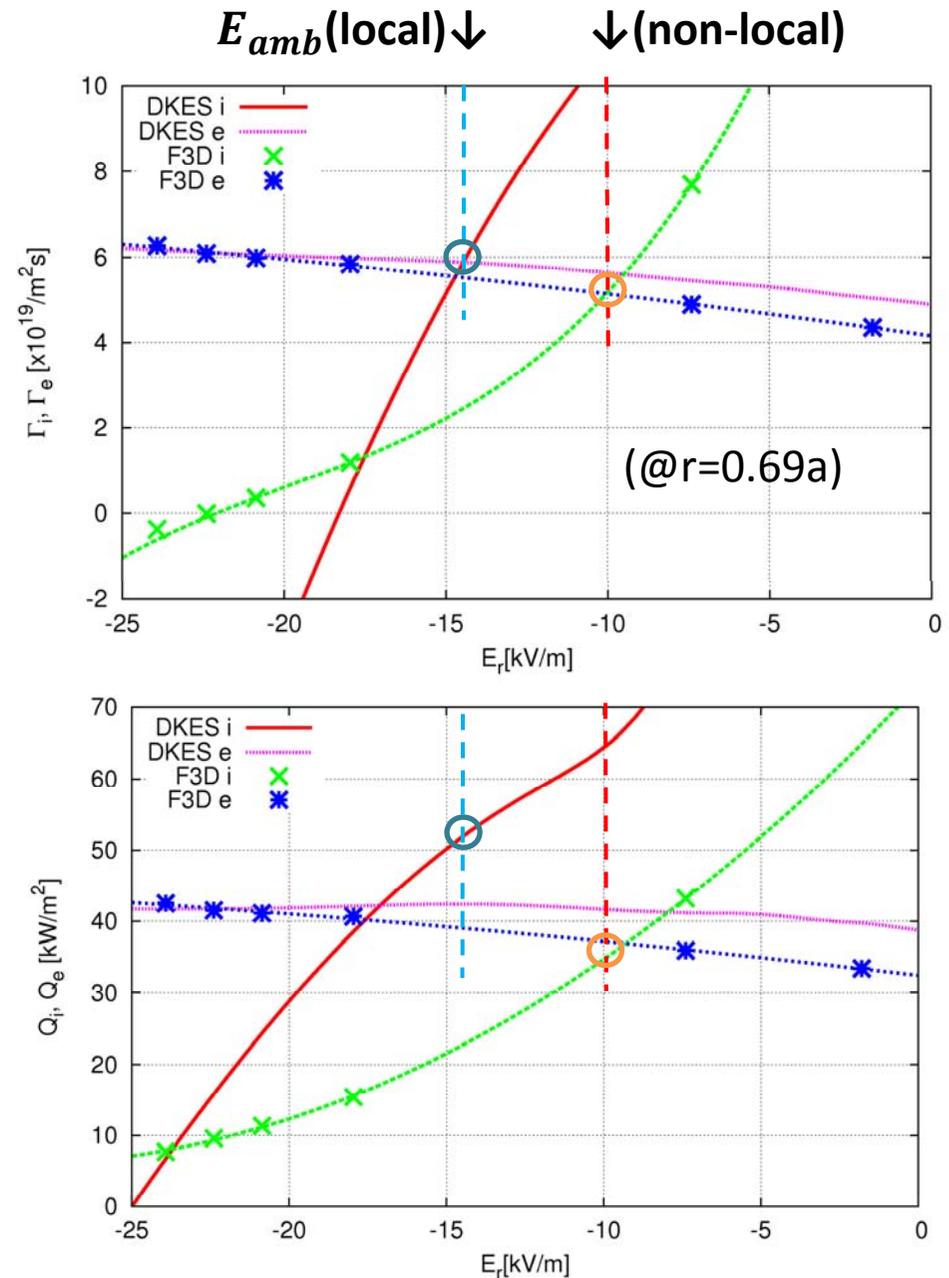
**Also, at $\rho > 0.9$, poloidal Mach number is $\cong 1$.
 \rightarrow Incompressible- $E \times B$ approximation used in DKES is not valid.**



Finite magnetic drift can cause large difference in Q_i

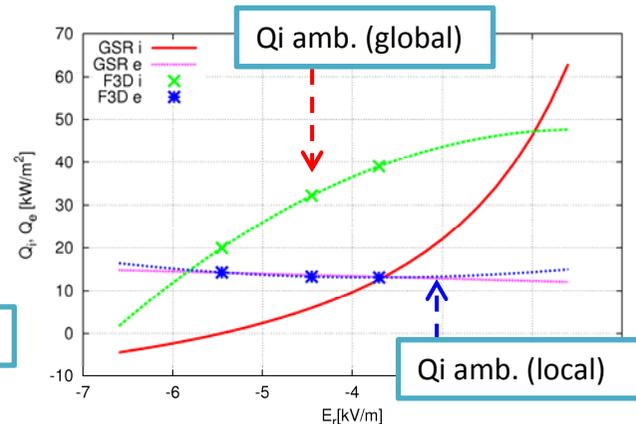
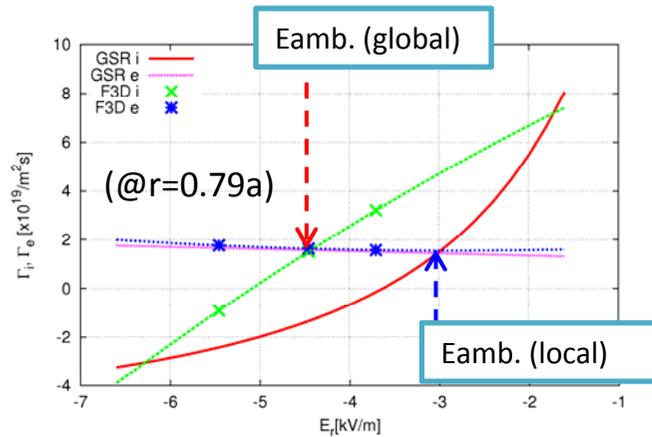
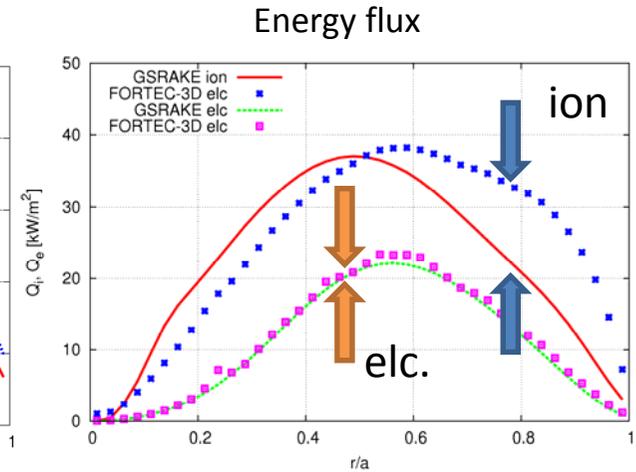
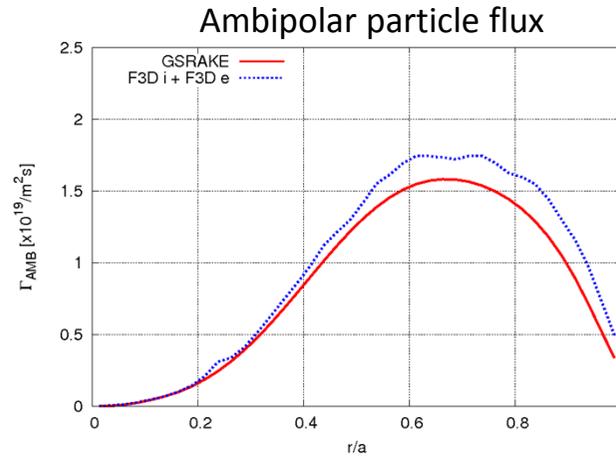
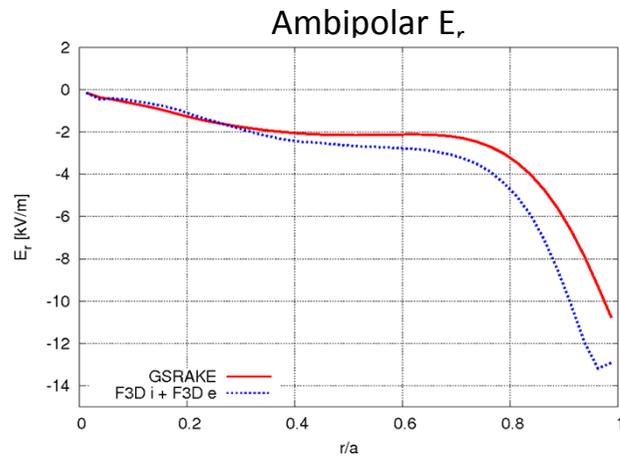
- Even if E_{amb} changes, difference in Γ_{amb} is small.
→ because of weak dependence of Γ_e on E_r .

- Not only because of difference in E_{amb} but also the difference in the dependence on E_r causes the large difference in Q_i .



Simulation result : (2) LHD

(Comparison with GSRAKE)



➤ **Local & mono-energy solutions of Γ_i and Q_i tend to be peaky at $E_r \rightarrow 0$ since it neglects $v_{B,r}$ drift which is important at there.**

⇒ Results in difference in E_{amb}

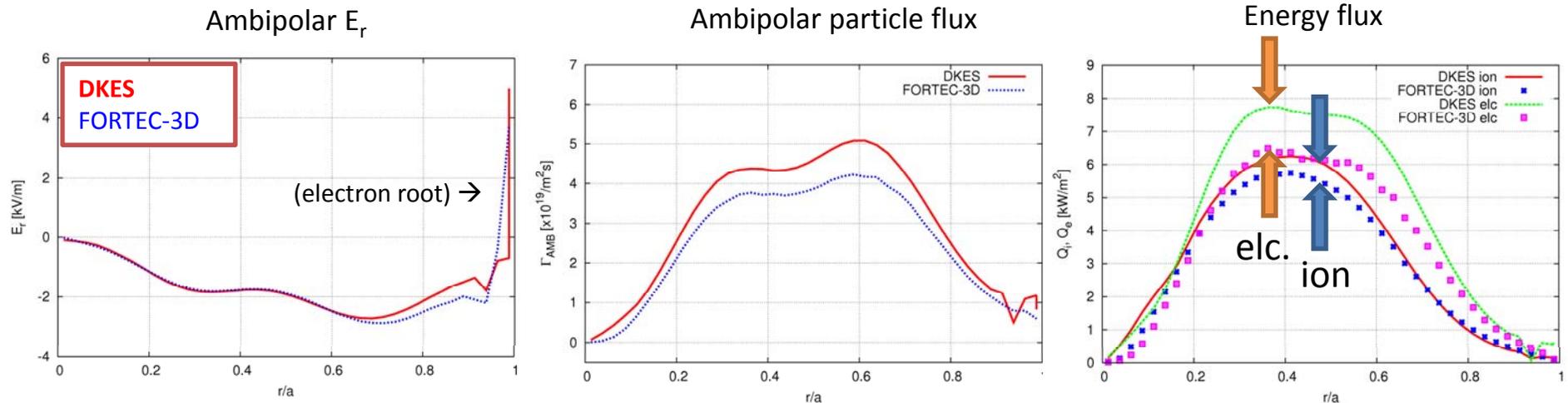
➤ Since Γ_e and Q_e depend on E_r weakly and zero-orbit-width approximation is valid for electrons, **ambipolar- Γ and Q_e differs only slightly b/w two calculation methods.**

➤ **Difference in amb- Q_i between local and global calculations are much more significant than that in Q_e .**

➡ Same tendency as in the W7-AS case

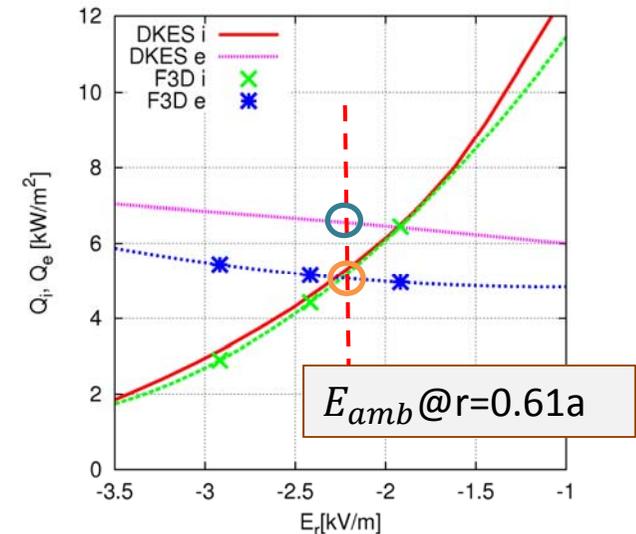
Simulation result : (3) TJ-II

(Comparison with DKES)



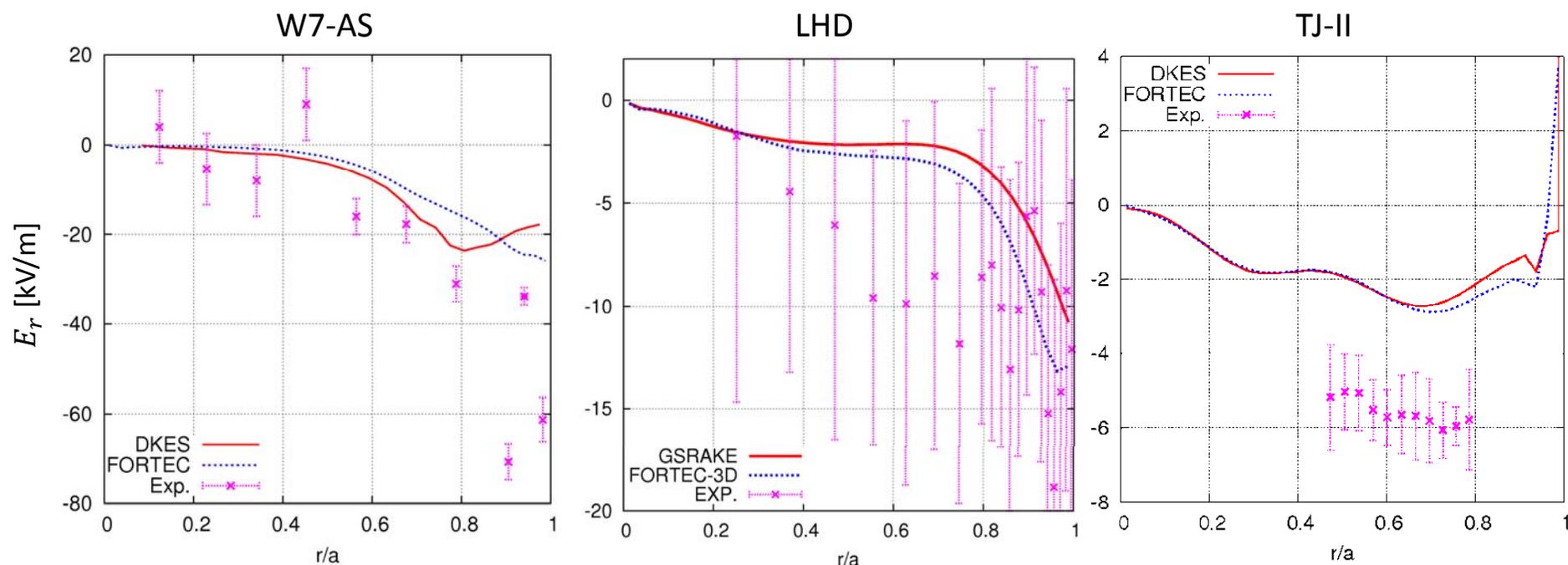
- As the plasma is **most collisional among the 3 cases**, non-local effect is least expected to appear in this case.
- **Ambipolar- E_r** coincides well b/w local and non-local simulations.
- However, **difference b/w two solution is more significant in Q_e** than in Q_i .

✓ Finite $V_B \cdot \nabla\theta$ term, which is not negligible compared to $V_{E \times B} \cdot \nabla\theta$ term for electrons at the ion-root E_r , seems to significantly affect the evaluation of Q_e in the TJ-II configuration.



Comparisons with experimental analysis

(1) Radial electric field



- Measured E_r profile (HIBP or CXRS) < NC ambipolar condition, but reasonable agreement is found in LHD and W7-AS cases.
- For TJ-II case, non-local NC simulation cannot resolve the discrepancy.
- Some unconsidered mechanism of ion particle loss other than the bulk ion NC flux is required to explain the difference.
 - ✓ Loss of fast ions from NBI heating? Impurity ion transport?

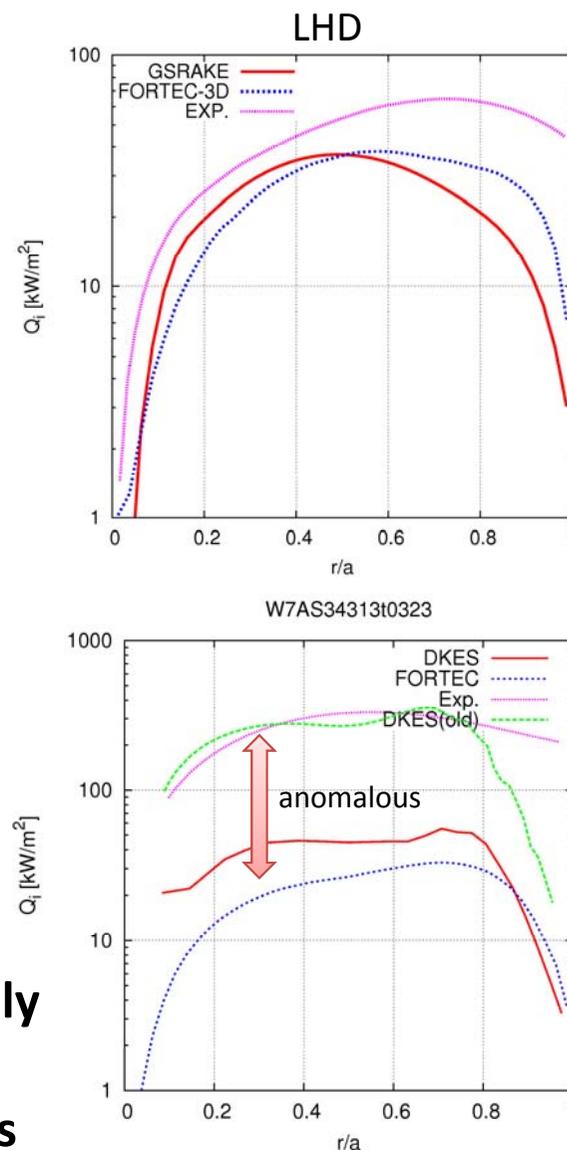
Comparisons with experimental analysis

(2) Radial energy flux

Energy fluxes were analyzed by **TASK3D** or **ASTRA** codes from the heat deposition profile.

- **In the LHD case**, good agreement of Q_i between local NC flux from GSRAKE and experiment analysis within factor 2 (in the core region) has been reported.
- Non-local NC calculation also changes the estimation of $Q_i(\text{NC})$ at ion-root by factor 2 from the local one.
- **In the W7-AS case**, it is found that previous DKES solution (lower accuracy in the MHD equilibrium and low resolution in solving DKES) differs much from the new DKES and FORTEC-3D solutions, though it agrees better with ASTRA in the core region.
- Contribution of anomalous transport to the energy flux is almost one order larger than that from neoclassical transport, according to the improved calculations.

Improvement of the evaluation of neoclassical Q_i is really important for the quantitative accuracy of transport analysis, especially if the neoclassical energy transport is dominant.



Summary

- FORTEC-3D non-local neoclassical transport code was applied to LHD, W7-AS and TJ-II to see the difference in the ambipolar NC flux and E_r from those evaluated by the local, mono-energy approximation codes.
- In ion-root plasmas, **the ambipolar E_r profile in these configurations estimated from local & mono-energy codes is similar to that is obtained by non-local simulation.**
- Though E_{amb} profiles are similar between local and non-local NC simulations, the **magnetic drift term**, which is neglected in local NC codes, is found to **alter the Q_i at the ion-root as large as by factor 2.**
- We plan to extend this analysis to more collisionless cases, in which non-local NC calculation is expected to be more important.

backup slides

Procedure to find the ambipolar condition by FORTEC-3D

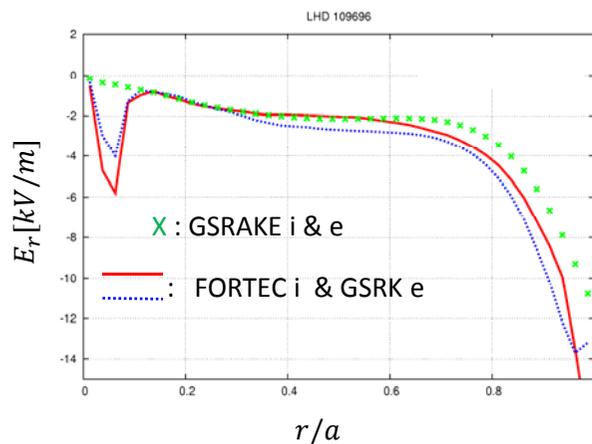
- FORTEC-3D can solve DKE and the time evolution E_r for only a single particle species at once. **In previous studies, only ion neoclassical transport was solved by FORTEC-3D**, while table of $\Gamma_e(r, E_r)$ was prepared from another local code.
- To determine the ambipolar condition from **both Γ_i and Γ_e by FORTEC-3D** code, the following three steps are used.

1: Run a simulation and solve time evolution of E_r until it reaches a steady state solution.

$$\epsilon_{\perp} \frac{\partial}{\partial t} E_r = -e (Z_i \Gamma_i - \Gamma_e)$$

FORTEC-3D
Local code

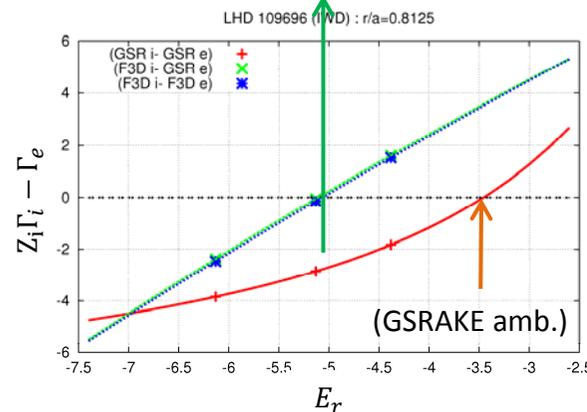
1st guess of amb. $E^{(1)}$



2: Using three E_r profiles ($E^{(1)}(r), E^{(1)}(r) \pm \Delta E$), neoclassical fluxes (Γ, Q) for ion and electron are evaluated by FORTEC-3D. (Note : E_r is time constant here.)

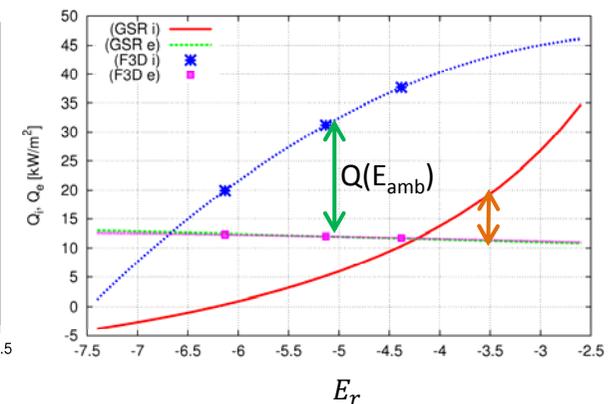
(Γ, Q) dependence on E_r around $E^{(1)}$.

$E^{(2)}$ from FORTEC-3D i&e

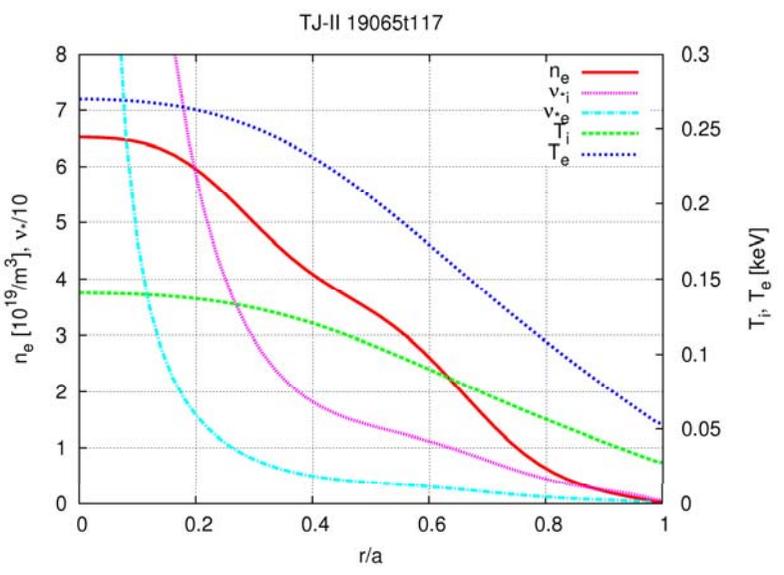
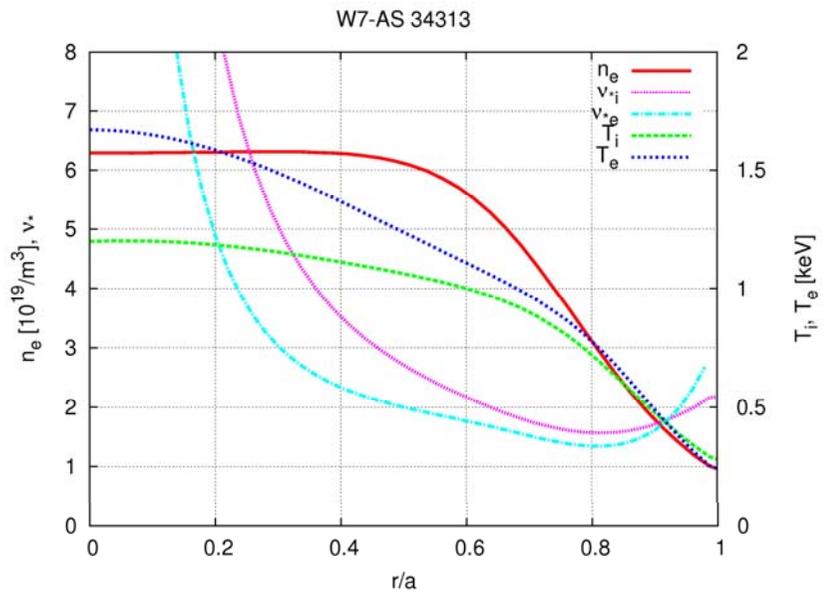
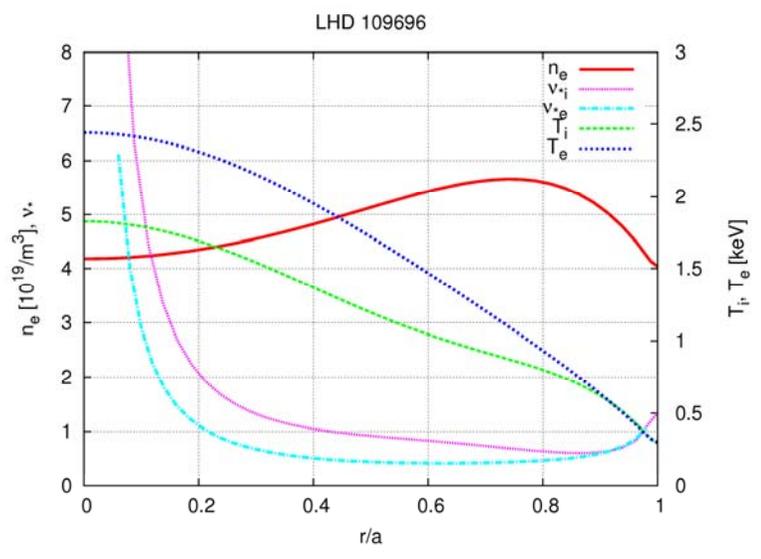


3: From fitting curve of $(Z_i \Gamma_i - \Gamma_e) [E_r]$, determine the ambipolar E_r ($E^{(2)}$). Also, using the fitting curves for Q , estimate energy flux at the ambipolar condition ($Q_i(E^{(2)}), Q_e(E^{(2)})$).

Amb. flux interpolation



density , temperature profiles

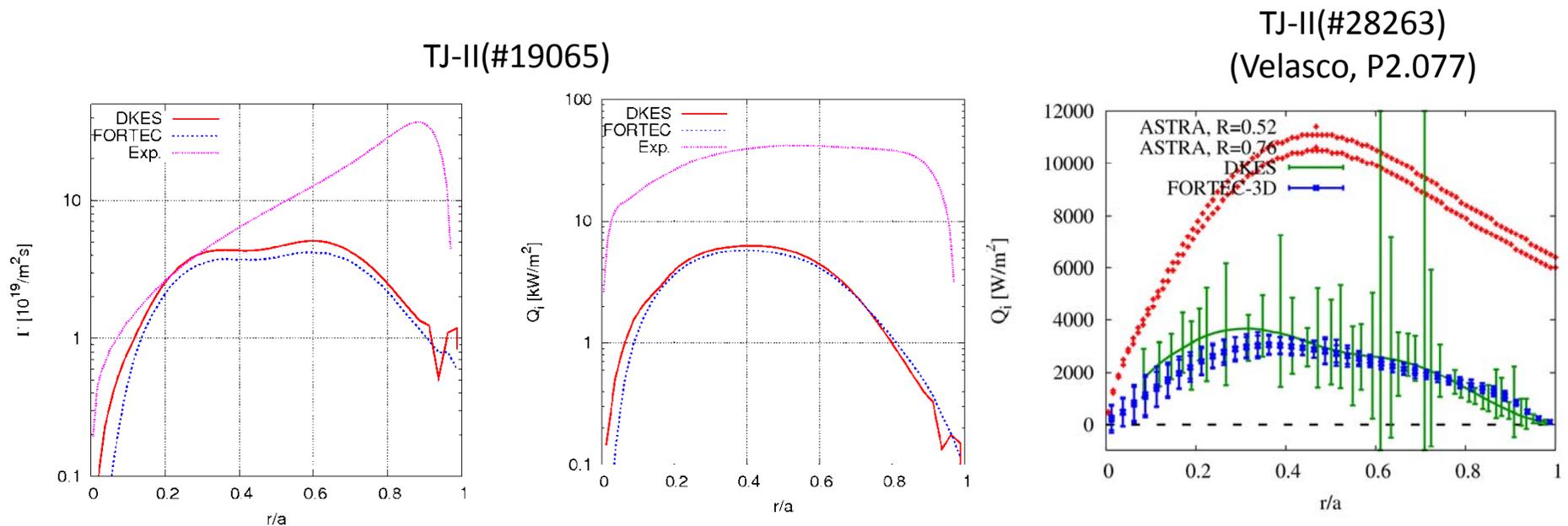


Comparisons with experimental analysis

Radial particle / energy flux in TJ-II

The TJ-II case analyzed here has $10 \times$ difference b/w $Q_i(\text{NC})$ and $Q_i(\text{Exp.})$.

However, we also found a case where $Q_i(\text{Exp.}) = 2 \sim 3 \times Q_i(\text{NC, local}) \cong Q_i(\text{NC, non-local})$



Comparisons with experimental analysis

Radial particle / energy flux in W7-AS

- Particle and energy fluxes in the W7-AS discharges was analyzed by ASTRA code considering the particle and heat deposition profiles.

