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Development of L-H Transition Simulation Framework with BOUT++ code

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in collaboration with

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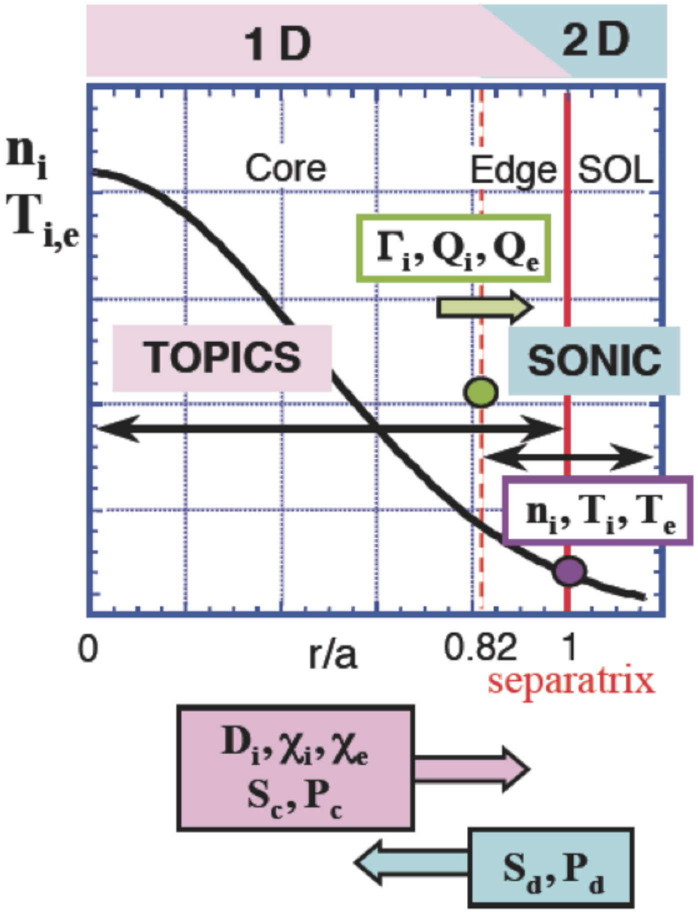
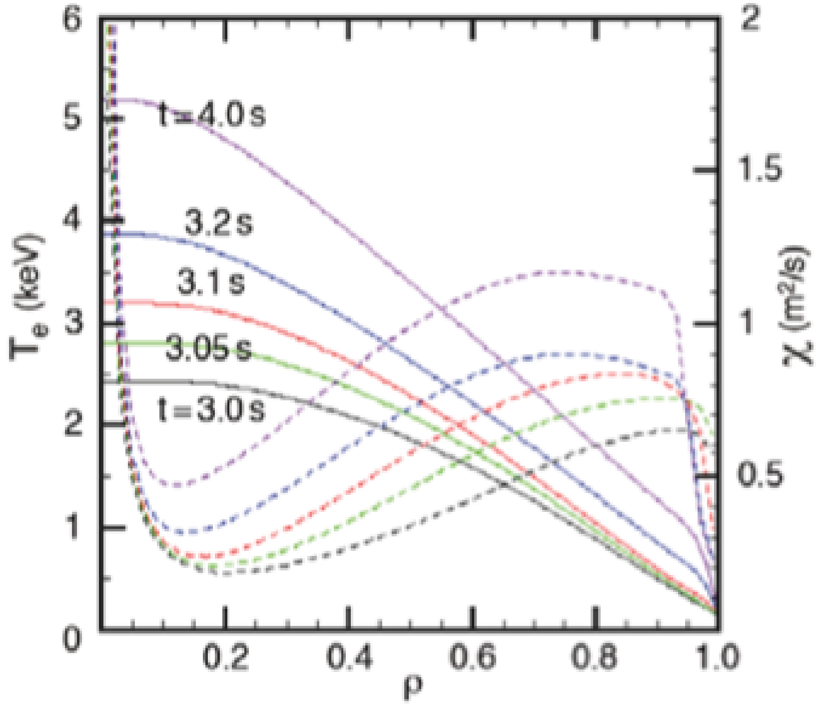
Motivation for L-H transition simulation by RMHD model

- **Understanding of L-H transition mechanism is one of key issues for**
  - **designing a high performance core plasma**
  - **controlling the heat load on plasma facing materials**
- **Integrated transport simulation framework for L-H transition in JAEA**[1]
  - 1.5D core transport code **TOPICS** + 2D SOL/divetor transport code **SONIC**
    - **CDBM turbulent transport model**
  - **Qualitative evaluation** was accomplished in this framework
    - **Spontaneous turbulence quench at edge region by mean flow shear**
  - **Quantitative evaluations** of L-H transition were not accomplished in this framework
    - **Pressure at pedestal top**
    - **Power threshold for L-H transition**





(left): concept of integrated transport simulation framework by TOPICS + SONIC
(right): time evolutions of $T_{\rm e}$ and χ

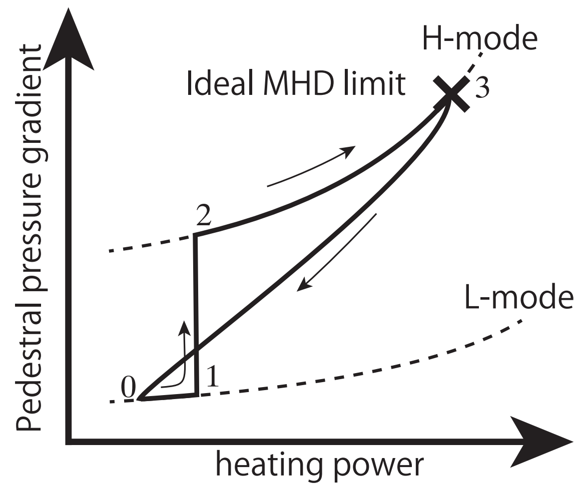
Improvement of turbulent transport model or development of first principal simulation framework are required for quantitative analysis of L-H transition.

- **Improvement of LH transition simulation frameworks through comparison with first principal and integrated transport simulations**
    - **From first principal simulation to integrated transport simulation**

- Improvement of CDBM turbulent transport model by introducing
  - **turbulence suppression by zonal flow shear**
  - **contribution from SOL/divertor transport**

- **From integrated transport simulation to first principal simulation**
  - Improvement of SOL/Divetor physics model in BOUT++ by
    - **Implementation of neutral particle/impurity transport model**
    - **Improvement of sink model**

- **Framework for whole-time edge plasma simulation**
  - **LH transition (1-2)**: **BOUT++**
  - **Pedestal formation (2-3)**: **TOPICS/SONIC**
  - **ELM collapse (3-0)**: **BOUT++**



In this study we have been developing L-H transition simulation framework with CDBM turbulence based on 3-D nonlinear edge MHD/turbulence code BOUT++

[1] M. Yagi *et al.*, Contrib. Plasma Phys. **52**, 372 (2012)

Minimal set for L-H transition simulation by RMHD model

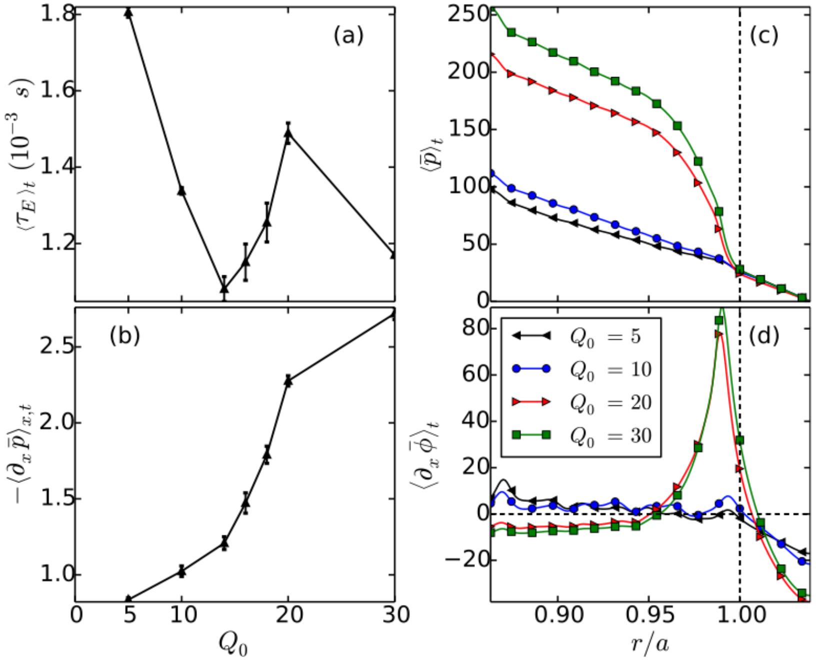
- **3D flux-driven simulations of L-H transition by 2-field RMHD model[2,3]**
  - **Resistive ballooning mode turbulence**
  - **Sheared mean flow by poloidal damping**
  - **Self-consistent pressure evolution by dynamic heat balance**

$$\frac{\partial U_{\mathrm{E}}}{\partial t} = -[\phi, U_{\mathrm{E}}] - B^2\nabla_{\parallel}J_{\parallel} + \boldsymbol{b}_0 \times \boldsymbol{\kappa}\cdot\nabla P + \mu_{\perp}\nabla_{\perp}^2 U_{\mathrm{E}} - \mu_{\mathrm{nc}}\left[\bar{U}_{\mathrm{E}} - \delta(k_{\mathrm{nc}}-1)\bar{U}_{\mathrm{D}}\right]$$

$$\frac{\partial P}{\partial t} = -[\phi, P] + \chi_{\parallel}\nabla_{\parallel}^2 P + \chi_{\perp}\nabla_{\perp}^2 P + S_p - L_p$$

$U_{\mathrm{E}} = \nabla_{\perp}^2\phi$, $U_{\mathrm{D}} = \nabla_{\perp}^2 P$, $J = S\nabla_{\parallel}\phi$, $\delta = d_{\mathrm{i}}/4B_0$, d_a: skin depth, $P_{\mathrm{i}} = P_{\mathrm{e}} = P/2$, $[f,g] = \boldsymbol{b}\times\nabla_{\perp}f\cdot\nabla_{\perp}g$, $\bar{f}$: (0,0)-component of f, S_{p}: heat source, L_{p}: heat sink, μ_{nc}, k_{nc}: quantities related to neoclassical flow/friction coefficients, $n_{\mathrm{i}} = \mathrm{const}$.

Red term describes poloidal damping



L-H transition simulation by L. Chônè [2]

[2] L. Chônè *et al.* Phys. Plasmas **21**, 070702 (2014)

[3] G.Y. Park et *al.*, IAEA-FEC 2014 TH-8-1

Derivation of poloidal damping term

length	$L_{\rm N} = R_{\rm mag}$	vorticity	$U_{\rm N} = 1/t_{\rm N}$
magnetic field	$B_{\rm N} = B_{\rm mag}$	vector potential(ish)	$\psi_{\rm N} = L_{\rm N}$
velocity	$V_{\rm N} = V_{A\rm N} = B_{\rm N}/\sqrt{\mu_0 m_{\rm i} n_{\rm i}}$	electrostatic potential	$\phi_{\rm N} = V_{\rm N} L_{\rm N} B_0$
time	$t_{\rm N} = L_{\rm N}/V_{\rm N}$	resistivity	$\eta_{\rm N} = \mu_0 V_{\rm N} L_{\rm N}$
viscosity	$\mu_{\rm N} = L_{\rm N}^2/t_{\rm N}$	hyper-resistivity	$\lambda_{\rm N} = \mu_0 \bar{V}_A L_{\rm N}^3$
pressure	$P_{\rm N} = B_{\rm N}^2/2\mu_0$	diffusivity	$\chi_{\rm N} = L_{\rm N}^2/t_{\rm N}$
current	$J_{\rm N} = -B_0/\mu_0 L_{\rm N}$	viscosity	$\mu_{\rm N} = L_{\rm N}^2/t_{\rm N}$
frequency	$\nu_{\rm N} = t_{\rm N}^{-1}$	heat source & sink	$W_{\rm N} = P_{\rm N}/t_{\rm N}$

- **Poloidal damping term is derived from**
  - Gianakon's heuristic closure for parallel viscous force[3]
  - Radial force balance with poloidal rotation of ion
  - Parallel force balance of ion in neoclassical transport theory[4]
- **Gianakon's heuristic closure for parallel viscous force in SI unit**

$$\nabla \cdot \overleftrightarrow{\Pi}_{\rm i\|} = m_{\rm i} n_{\rm i} (\langle B^2 \rangle / B_\theta^2) \left(\mu_{\rm i1} \bar{V}_{\rm i\theta} + \mu_{\rm i2} \bar{W}_{\rm i\theta} \right) \boldsymbol{e}_\theta, \quad \boldsymbol{W}_{\rm i} = 2\boldsymbol{q}_{\rm i}/5P_{\rm i} \tag{1}$$

$$\boldsymbol{B} = \nabla\zeta \times \nabla\psi + B_\zeta \nabla\zeta, \quad \boldsymbol{e}_\theta = \sqrt{g}\nabla\zeta \times \nabla\psi, \quad g^{\psi\zeta} = g^{\theta\zeta} = 0, \quad g^{\zeta\zeta} = R^{-2}$$

- (ψ, θ, ζ) is the orthogonal toroidal coordinates
- Red colored term is additional term expressing contribution from heat flux
- Contribution from heat flux results in offset poloidal flow term

- **Ion poloidal flow by radial force balance w/ poloidal rotation**

$$\nabla \bar{P}_{\mathrm{i}} \cdot \nabla \psi = e_{\mathrm{i}} n_{\mathrm{i}} (-\nabla \bar{\phi} + \bar{\boldsymbol{V}}_{\mathrm{i}} \times \boldsymbol{B}) \cdot \nabla \psi$$

$$\bar{V}_{\mathrm{i}\theta} = \frac{\sqrt{g} g^{\psi\psi}}{B_\zeta} \frac{\mathrm{d}}{\mathrm{d}\psi} \left(\bar{\phi} + \frac{\bar{P}_{\mathrm{i}}}{e_{\mathrm{i}} n_{\mathrm{i}}} \right), \quad \frac{\langle B^2 \rangle}{B_\theta^2} \bar{V}_{\mathrm{i}\theta} \boldsymbol{e}_\theta = \frac{B_0}{B_p^2} \boldsymbol{b}_0 \times \nabla \left(\bar{\phi} + \frac{\bar{P}_{\mathrm{i}}}{e_{\mathrm{i}} n_{\mathrm{i}}} \right) \tag{2}$$

- **Ion poloidal heat flow determined by parallel force balance in $\varepsilon \ll 1$ limit**

$$\hat{\mu}_{\mathrm{i}1} \hat{V}_{\mathrm{i}}^{\theta} + \hat{\mu}_{\mathrm{i}2} \hat{W}_{\mathrm{i}}^{\theta} = 0, \qquad \varepsilon = \frac{r}{R}, \quad \hat{f}_{\mathrm{i}}^{\theta}(\psi) = \frac{f^\theta}{B^\theta}, \quad \hat{\mu}_{\mathrm{i}j} = \frac{3 \langle (\nabla_\parallel B)^2 \rangle}{\langle B^2 \rangle} \mu_{\mathrm{i}j}$$

$$\hat{\mu}_{\mathrm{i}2} \hat{V}_{\mathrm{i}\theta} + \hat{\mu}_{\mathrm{i}3} \hat{W}_{\mathrm{i}\theta} = l_{22}^{\mathrm{ii}} \left(\frac{\bar{V}_{2\mathrm{i}} B}{\langle B^2 \rangle} + \hat{W}_{\mathrm{i}} \right), \quad l_{22}^{\mathrm{ii}} \gg \hat{\mu}_{\mathrm{i}j}, \quad \bar{V}_{2\mathrm{i}} = -\frac{B_\zeta}{e_{\mathrm{i}} B} \frac{\mathrm{d}\bar{T}_{\mathrm{i}}}{\mathrm{d}\psi}, \quad \bar{V}_{\mathrm{i}\theta} = -\frac{\mu_{\mathrm{i}2}}{\mu_{\mathrm{i}1}} \bar{W}_{\mathrm{i}\theta}$$

$$\bar{W}_{\mathrm{i}\theta} = \frac{\sqrt{g} g^{\psi\psi}}{B_\zeta} \frac{\mathrm{d}}{\mathrm{d}\psi} \left(\frac{P_{\mathrm{i}}}{e_{\mathrm{i}} n_{\mathrm{i}}} \right), \qquad \frac{\langle B^2 \rangle}{B_\theta^2} \bar{W}_{\mathrm{i}\theta} \boldsymbol{e}_\theta = \frac{B_0}{B_p^2} \boldsymbol{b}_0 \times \nabla \left(\frac{\bar{P}_{\mathrm{i}}}{e_{\mathrm{i}} n_{\mathrm{i}}} \right) \tag{3}$$

- **From (1)-(3), Gianakon's heuristic closure becomes**

$$\nabla \cdot \overleftrightarrow{\Pi}_{\mathrm{i}\parallel} = m_{\mathrm{i}} n_{\mathrm{i}} \mu_{\mathrm{i}1} \frac{B_0}{B_p^2} \boldsymbol{b}_0 \times \nabla \psi \left[\frac{\mathrm{d}}{\mathrm{d}\psi} \left(\bar{\phi} + \frac{\bar{P}_{\mathrm{i}}}{e_{\mathrm{i}} n_{\mathrm{i}}} \right) - k_{\mathrm{nc}} \frac{\mathrm{d}}{\mathrm{d}\psi} \left(\frac{\bar{P}_{\mathrm{i}}}{e_{\mathrm{i}} n_{\mathrm{i}}} \right) \right]$$

Red terms describes radial force balance with neoclassical poloidal flow

- Interpolation formula for μ_{i1} and k_{nc}

$$\mu_{i1} = \frac{0.66\varepsilon^{1/2}\nu_i}{(1+1.03\nu_{i*}^{1/2}+0.31\nu_{i*})(1+0.66\varepsilon^{3/2}\nu_{i*})}$$

$$k_{nc} = \frac{1}{1+\nu_{i*}^2\varepsilon^3}\left(\frac{1.17-0.35\nu_{i*}^{1/2}}{1+0.7\nu_{i*}^{1/2}} - 2.1\nu_{i*}^2\varepsilon^3\right)$$

- **Gianakon's heuristic closure results in poloidal damping term**

$$\boldsymbol{b}\cdot\nabla\times\nabla\cdot\overleftrightarrow{\Pi}_{i\|} = m_i n_i \mu_{nc}\left[\nabla_\perp^2\frac{\bar{\phi}}{B_0} - (k_{nc}-1)\nabla_\perp^2\frac{\bar{P}_i}{e_i n_i B_0}\right], \quad \mu_{nc} = \mu_{i1}\frac{B_0^2}{B_p^2}$$

$$\xrightarrow{\text{normalize}} \mu_{nc}\left[\bar{U}_E - \delta(k_{nc}-1)\bar{U}_D\right],$$

  - with self-consistently evolving coefficients $\mu_{nc}(\nu_{i*}) \to \mu_{nc}(P), \quad k_{nc}(\nu_{i*}) \to k_{nc}(P)$

[4] T.A. Gianakon et al., Phys. Plasmas 9, 536 (2002)
[5] S.P Hirshman and D.J. Sigmar, Nucl. Fusion **21**, 1079 (1981)

Preliminary simulations by simplified 2-field model by BOUT++

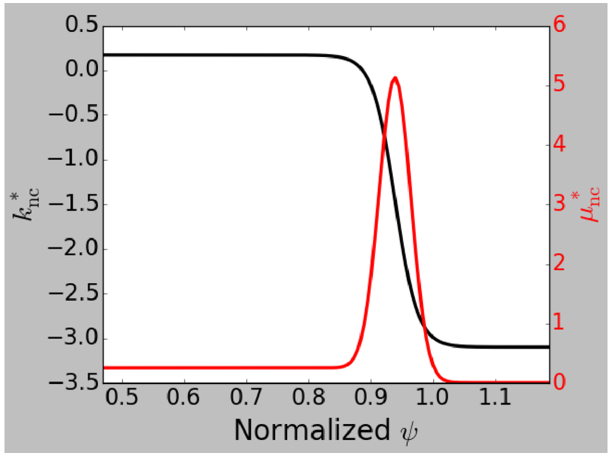
- **Simplified 2-field RMHD model for LH transition**

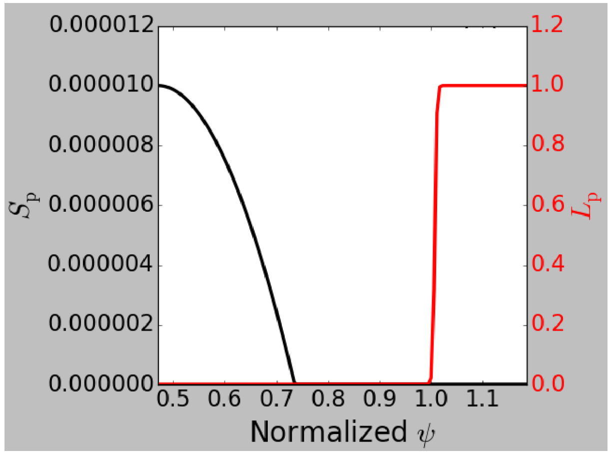
$$\frac{\partial U_{\rm E}}{\partial t} = -\left[\phi, U_{\rm E}\right] - B_0^2 \nabla_\parallel J_\parallel + \boldsymbol{b}_0 \times \boldsymbol{\kappa} \cdot \nabla P + \mu_\perp \nabla_\perp^2 U - \mu_{\rm nc}^* \left[\bar{U}_{\rm E} - \delta k_{\rm nc}^* \bar{U}_{\rm D}\right]$$

$$\frac{\partial P}{\partial t} = -\left[\phi, P\right] + \chi_\perp \nabla_\perp^2 P + \chi_\parallel \nabla_\parallel^2 P + S_p - P L_p$$

$$\eta = \mu_\perp = \chi_\perp = 10^{-6}, \quad \chi_\parallel = 10^{-2}$$

  - **Simplified poloidal damping coefficients**
    - Profiles of $\mu_{\rm nc}^*$ and $k_{\rm nc}^*$ are given by hyperbolic functions by reference to Ref. [2,3]
    - Interaction between E_r shear and diamag. flow through $\mu_{\rm nc}$ and $k_{\rm nc}$ are neglected
  - **Parabolic heat source and step heat sink**





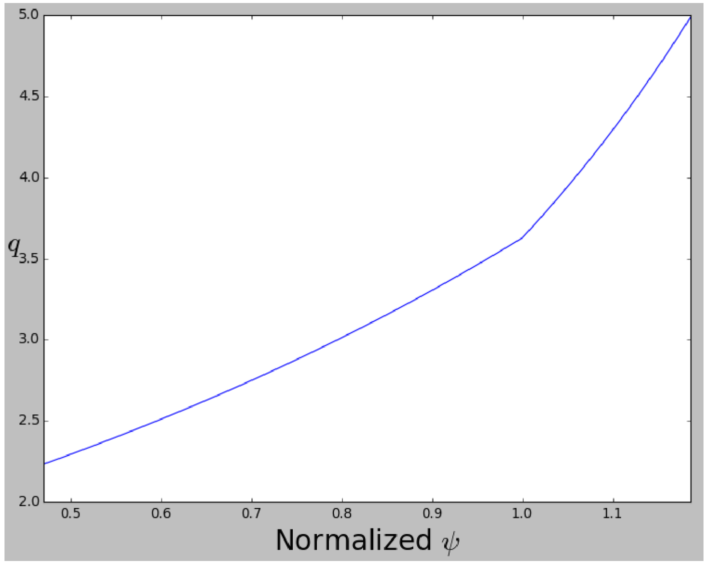
We have investigated impact of poloidal damping on RMB turbulence

- **Computational grid generated by circular equilibrium generator of BOUT++**

  - **Input parameters**
    $R_0 = 300[\mathrm{cm}],\ a_0 = 75[\mathrm{cm}],\ B_0 = 4[\mathrm{T}],$
    $\beta_p = 0.01,\ \Delta = 5[\mathrm{cm}],\ n_\mathrm{i} = 1.0 \times 10^{19}[\mathrm{m}^{-3}]$

    $$q(\rho) = 1.5 + 2\rho^2, \quad \rho = r/a_0$$

    $$p_1(\rho) = 0.5 \left(1 - \tanh\left[\frac{(\rho - 0.4)}{0.225}\right]\right)$$



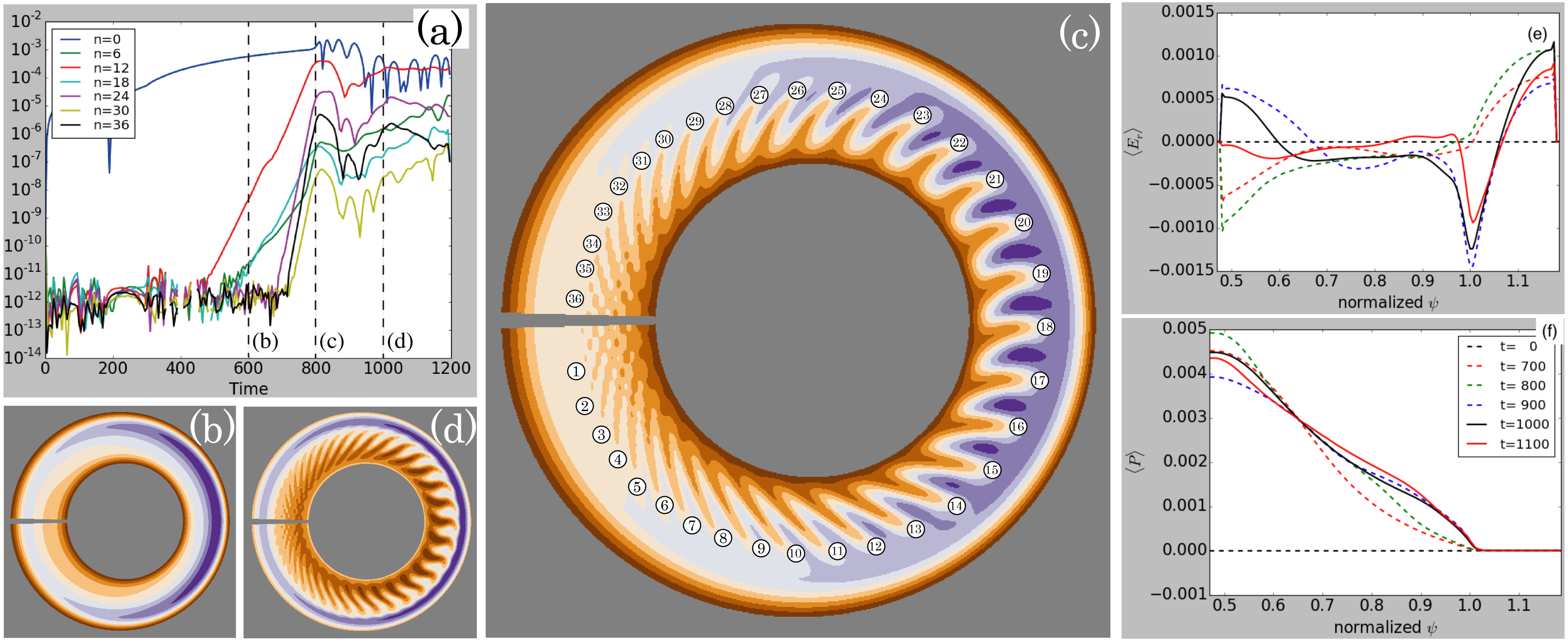
  - **Computational domain**:
    radial: $0.47 \le \psi \le 1.19$, poloidal: $0 \le \theta \le 2\pi$, toroidal: $0 \le \zeta \le 2\pi/6$

  - **Resolution**:
    radial$\times$ poloidal $\times$ toroidal $= 132 \times 128 \times 65$

  - **Positions of radial boundary**:
    Core boundary: radial index $= 0$, $\rho = 0.6$, $\psi = 0.47$
    Separatrix: radial index $= 98$, $\rho = 1.0$, $\psi = 1.0$
    SOL boundary: radial index $= 132$, $\rho \simeq 1.2$, $\psi = 1.19$

  - **Boundary conditions**
    $\partial U_\mathrm{E}/\partial\psi = 0,\quad \partial\phi/\partial\psi = 0,\quad \partial P/\partial\psi = 0$ **at core boundary**
    $U_\mathrm{E} = 0,\quad \phi = 0,\quad P = 0$ **at SOL boundary**

- **Simulation results of w/ poloidal damping case**
  - toroidal mode structure at $\psi \simeq 0.76$ on outer mid-plane (a)
  - poloidal plot on $\zeta = 0$ plane at $t = 600\tau_A$ (b), $t = 800\tau_A$ (c), $t = 1000\tau_A$(d)
  - time evolution of flux surface averaged E_r (e) and and P (f)

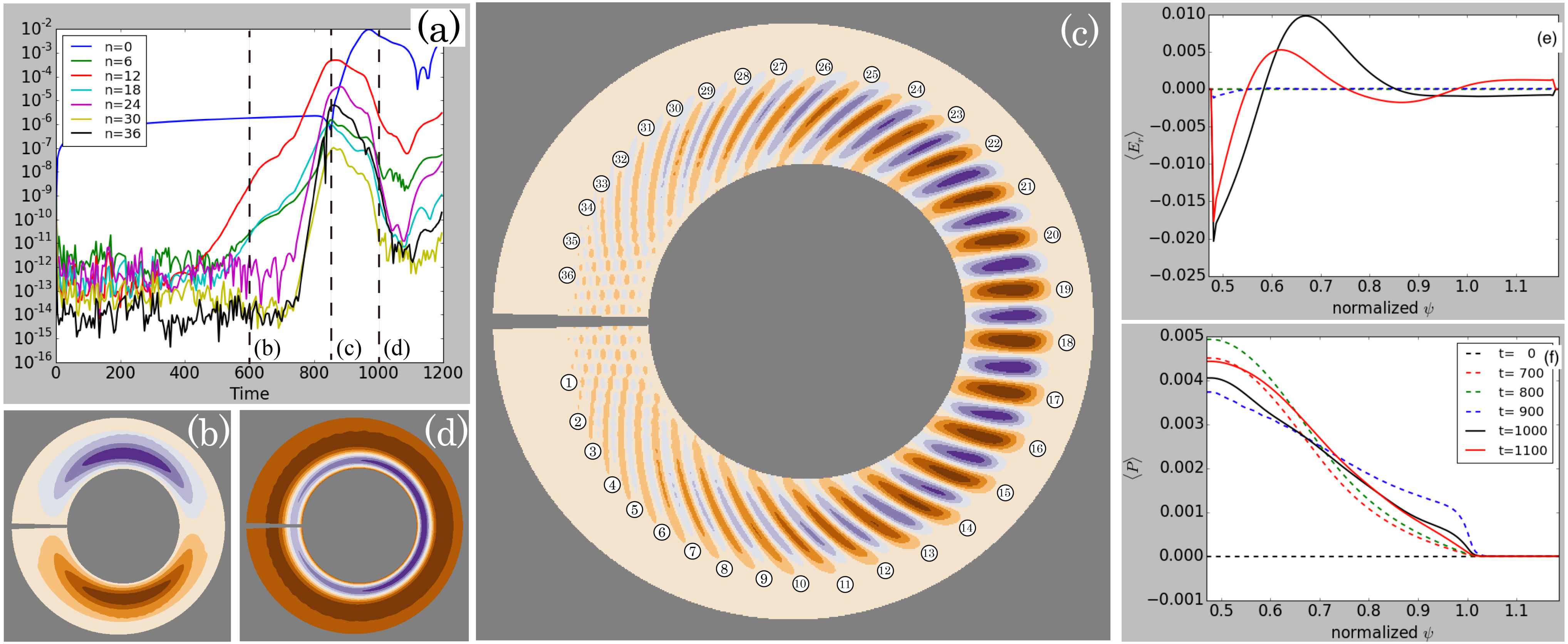


* **RBM structure $m = 36$ ($q = 3$, $n = 12$) was observed at $t = 800\tau_A$**

* **Strongly sheared E_r was observed at the vicinity of separatrix**

* **Pedestal-like P profile was obtained**

- **Simulation results of w/o poloidal damping case**
  - toroidal mode structure at $\psi \simeq 0.76$ on outer mid-plane (a)
  - poloidal plot on $\zeta = 0$ plane at $t = 600\tau_A$ (b), $t = 850\tau_A$ (c), $t = 1000\tau_A$(d)
  - time evolution of flux surface averaged E_r (e) and and P (f)



* **RBM structure $m = 36$ ($q = 3$, $n = 12$) was observed at $t = 850\tau_A$**
* **After $t \simeq 850\tau_A$, simulation was broken and unphisical results are obtained. Improvement of simulation settings is required**

Summary and conclusions

- **Summary**
  - **The detailed derivation of poloidal damping term was described**
    - NC poloidal damping term was derived from
      - Gianakon's closure for parallel viscous force
      - Radial force balance with poloidal flow of ion
      - Parallel force balance of ion in neoclassical transport theory
  - **Preliminary simulations by 2-field RMHD model + simplified poloidal damping + dynamic heat balance were demonstrated by BOUT++**
    - RBM were observed in both w/ and w/o NC poloidal damping term
    - Poloidal damping generated strongly sheared radial electric field and pedestal-like pressure in the vicinity of the separatrix
- **Future work**
  - **Development of postscripts for turbulence analysis**
  - **Implementation of self-consistent μ_{nc} and k_{nc}**
  - **LH transition simulation by 3-field RMHD model**

(extra) L-H transition simulation framework by 3-field RMHD

- **3-field RMHD model for L-H transition with CDBM turbulence**

$$\frac{\partial \Psi}{\partial t} = -\left[\phi, \Psi\right] - \frac{1}{B_0}\nabla_{\parallel}^{0}\left(B_0\phi\right) + d_{\mathrm{i}}^{*}\nabla_{\parallel}P + \eta J_{\parallel} - \lambda\nabla_{\perp}^{2}J_{\parallel}$$

$$\frac{\partial U}{\partial t} = -\left[\varphi, U\right] + \frac{d_{\mathrm{i}}^{*}}{2}\left(\left[P, U\right] + \left[\varphi, U_{\mathrm{D}}\right] + \nabla_{\perp}^{2}\left[P, \varphi\right]\right)$$
$$- B_0^2\nabla_{\parallel}J_{\parallel} + \boldsymbol{b}_0 \times \boldsymbol{\kappa} \cdot \nabla P + \mu_{\perp}\nabla_{\perp}^{2}U - \mu_{\mathrm{nc}}\left[\bar{U}_{\mathrm{E}} - \delta(k_{\mathrm{nc}} - 1)\bar{U}_{\mathrm{D}}\right]$$

$$\frac{\partial P}{\partial t} = -\left[\phi, P\right] + \chi_{\perp}\nabla_{\perp}^{2}P + \chi_{\parallel}\nabla_{\parallel}^{2}P + S_p - L_p$$

$$\Psi = \psi - d_{\mathrm{e}}^2 J_{\parallel}, \quad \varphi = \phi + d_{\mathrm{i}}^{*}P, \quad U = \nabla_{\perp}^{2}\varphi, \quad J_{\parallel} = \nabla_{\perp}^{2}\psi, \quad d_{\mathrm{i}}^{*} = d_{\mathrm{i}}/4B_0$$

  - **CDBM turbulence + poloidal damping + dynamic heat balance**
  - **2-field RMHD model for L-H transition is reproduced if**
    - Two-fluid effects are neglected $(d_{\mathrm{e}}, d_{\mathrm{i}}^{*} \to 0)$
    - Electrostatic turbulence is assumed, $(J_{\parallel} = \eta^{-1}\nabla_{\parallel}\phi)$