Particle flux representations with FLR effects in the gyrokinetic model
ジャイロ運動論モデルにおける有限ラーマー半径効果を含む粒子フラックスの表現

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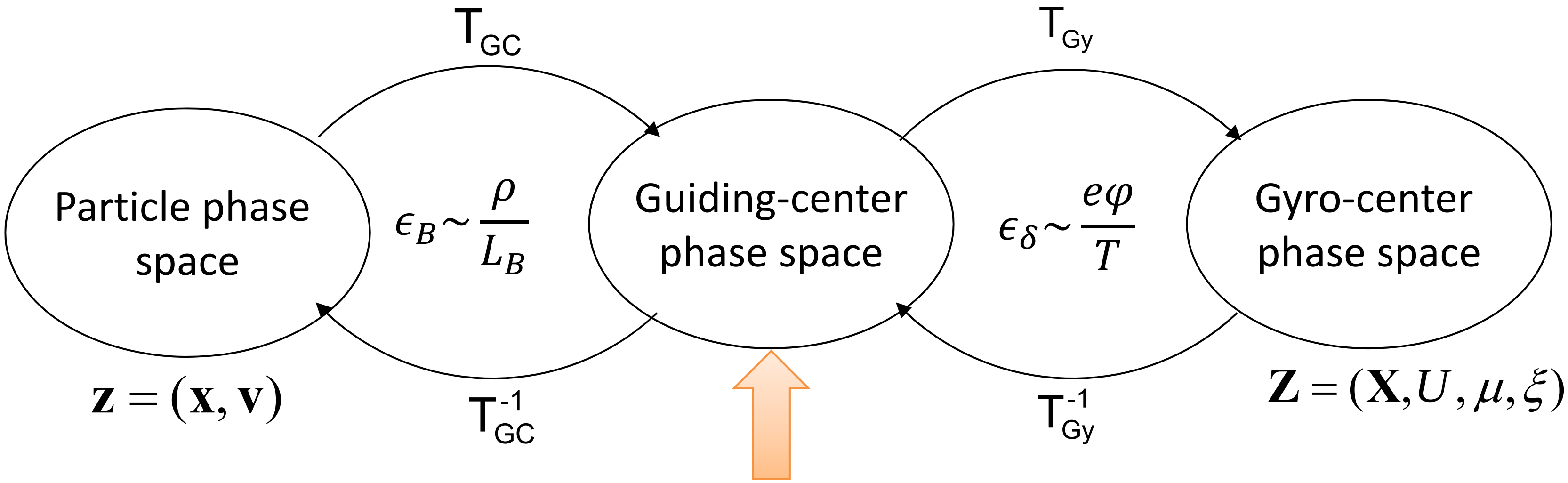
Introduction

- In general, simple fluid moments of the gyro-center distribution function (N, P, …) are different from physical fluid moments (n, p,…).
- In particular, gyro-center particle flux in standard gyrokinetic model does not include diamagnetic and polarization-drift terms.
- We have to consider push-forward representation of particle flux to recover the diamagnetic and polarization-drift terms.

Standard gyrokinetic formulation

[Hahm PoF 1988, Brizard JPP 1989]

Two-step phase space transformation to remove fast gyro-motion of a charged particle in a magnetic field.



Small potential perturbation φ is introduced after $\mathrm{T_{GC}}$.

$\mathrm{T_{GC}}$: guiding-center transformation (small parameter ϵ_B)

$\mathrm{T_{Gy}}$: gyro-center transformation (small parameter ϵ_δ)

Particle flux representation

Two abstract push-forward representations of particle flux $\boldsymbol{\Gamma} \equiv n[\mathbf{v}]$

conventional
$$\boldsymbol{\Gamma}(\mathbf{r}) = \int d^6\mathbf{Z}\,\mathcal{J}\left[\mathrm{T}_{\mathrm{GC}}^{-1}\mathbf{v}\right](\mathbf{Z})\left[\mathrm{T}_{\mathrm{Gy}}^{*}F\right](\mathbf{Z})\delta^3\left(\left[\mathrm{T}_{\mathrm{GC}}^{-1}\mathbf{x}\right](\mathbf{Z}) - \mathbf{r}\right)$$

pure push-forward
$$\boldsymbol{\Gamma}(\mathbf{r}) = \int d^6\mathbf{Z}\,\mathcal{J}\mathrm{T}_{\mathrm{Gy}}^{-1}\mathrm{T}_{\mathrm{GC}}^{-1}\mathbf{v}F(\mathbf{Z})\delta^3\left(\mathrm{T}_{\mathrm{Gy}}^{-1}\mathrm{T}_{\mathrm{GC}}^{-1}\mathbf{x} - \mathbf{r}\right)$$

- Conventionally the first representation is used to derive explicit push-forward representations of fluid moments from the standard gyrokinetic model.
- They give the same result, but more information on the gyro-center transformation is needed to derive the polarization flux from the conventional representation as shown below.

Pull-back transformation of *F*

Generally

$$\mathrm{T}_{\mathrm{Gy}}^{*}F = F + \epsilon_\delta\{S_1, F\} + \frac{\epsilon_\delta^2}{2}\{S_1, \{S_1, F\}\} + \epsilon_\delta^2\{S_2, F\} + O\left(\epsilon_\delta^3\right)$$

S_1, S_2, ... : Scalar function generating the gyro-center transformation.

{,} : Guiding-center Poisson brackets [Littlejohn PoF 1981].

Equation determining S_1 [Brizard JPP 1989, Qin PoP 1998]

$$\frac{\partial S_1}{\partial t} + \{S_1, H_0\} = e\tilde{\varphi}$$

where

Guiding-center Hamiltonian $\quad H_0 = \frac{m}{2}U^2 + \mu B$

Oscillatory part of potential $\quad \tilde{\varphi} = \varphi(\mathbf{X} + \boldsymbol{\rho}_0) - \underset{\substack{\uparrow \\ \text{Gyro-average}}}{\langle \varphi(\mathbf{X} + \boldsymbol{\rho}_0) \rangle}$

Generating function S_1

Usually only the lowest order solution for S_1 is considered:

$$S_1^{(1)} = \frac{e}{\Omega}\int \tilde{\varphi} d\xi$$

Then $\mathrm{T}_{\mathrm{Gy}}^{*}F$ is approximated as $\mathrm{T}_{\mathrm{Gy}}^{*}F \simeq F + \frac{e\tilde{\varphi}}{B}\frac{\partial F}{\partial \mu}$

But we cannot obtain the polarization flux from the conventional representation with the lowest order S_1, because time derivative of potential does not appear at this order.

To obtain the polarization flux, we need the higher order solution for S_1 [Qin PoP 1999, Belova 2001]

$$S_1^{(2)} = -\frac{1}{\Omega}\left(\frac{\partial}{\partial t} + U\mathbf{b}\cdot\boldsymbol{\nabla}\right)\int d\xi\, S_1^{(1)}$$

where a constant magnetic field is assumed for simplicity.

Gyro-center displacement vector

When we use the pure push-forward representation, the higher order solution is not necessary.

In the pure push-forward representation

$$\mathrm{T}_{\mathrm{Gy}}^{-1}\mathrm{T}_{\mathrm{GC}}^{-1}\mathbf{x} = \mathbf{X} + \boldsymbol{\rho}_0 + \boldsymbol{\rho}_1 + \cdots$$

$$\mathrm{T}_{\mathrm{Gy}}^{-1}\mathrm{T}_{\mathrm{GC}}^{-1}\mathbf{v} = \dot{\mathbf{X}} + \dot{\boldsymbol{\rho}}_0 + \dot{\boldsymbol{\rho}}_1 + \cdots$$

Gyro-center displacement vector [Brizard PoP 2008]

$$\boldsymbol{\rho}_1 \equiv -\{S_1, \mathbf{X} + \boldsymbol{\rho}_0\}$$

$$= \frac{e}{m}\left(\frac{\partial S_1}{\partial \mu}\frac{\partial \boldsymbol{\rho}_0}{\partial \xi} - \frac{\partial S_1}{\partial \xi}\frac{\partial \boldsymbol{\rho}_0}{\partial \mu}\right) + \frac{\mathbf{B}^*}{mB_\parallel^*}\frac{\partial S_1}{\partial U} + \frac{1}{eB_\parallel^*}\mathbf{b} \times \boldsymbol{\nabla} S_1$$

In long wavelength limit, gyro-average of $\boldsymbol{\rho}_1$ with the lowest order S_1 is given by

$$\langle \boldsymbol{\rho}_1 \rangle = -\frac{\boldsymbol{\nabla}_\perp \varphi}{B\Omega}$$

The polarization flux emerges from $\dot{\boldsymbol{\rho}}_1$ directly.

Γ with FLR terms

Belova derived an explicit representation of Γ with FLR terms from the conventional representation with $S_1^{(2)}$ and S_2 as well as $S_1^{(1)}$ [Belova PoP 2001].

From the above observations, we may obtain Belova's result from the pure push-forward representation using only $S_1^{(1)}$.

We calculate Γ using the pure push-forward representation with $S_1^{(1)}$.

Generally, Γ is divided into 3 parts [Pfirsch ZNA 1984, Brizard PoP 2008] :

$$\mathbf{\Gamma} = \mathbf{\Gamma}_{\rm gy} + \mathbf{\Gamma}_{\rm pol} + \mathbf{\Gamma}_{\rm mag}$$

$\mathbf{\Gamma}_{\rm gy} = \int dU d\mu d\xi \dot{\mathbf{X}} F \mathcal{J}$: gyro-center flux

$\mathbf{\Gamma}_{\rm pol} = \frac{\partial}{\partial t}\mathbf{P}$: polarization flux (**P** is polarization vector)

$\mathbf{\Gamma}_{\rm mag} = \boldsymbol{\nabla} \times \mathbf{M}$: magnetization flux (**M** is magnetization vector)

Gyro-center flux

We consider a slab plasma, then Hamilton equations are

$$\dot{\mathbf{X}} = U\mathbf{b} + \frac{1}{B}\mathbf{b}\times\nabla\Phi, \quad \dot{U} = -e\mathbf{b}\cdot\nabla\Phi, \quad \dot{\mu} = 0, \quad \dot{\xi} = \Omega + \frac{e\Omega}{B}\frac{\partial\Phi}{\partial\mu}$$

Effective potential is given by

$$\Phi = \langle\varphi(\mathbf{X}+\boldsymbol{\rho}_0)\rangle - \frac{1}{2}\langle\{S_1,\tilde{\varphi}\}\rangle$$

Using the lowest order solution for S_1 and taking the long wavelength limit, we have

$$\Phi(\mathbf{X},\mu,t) \simeq \varphi(\mathbf{X},t) + \frac{\mu}{2e\Omega}\nabla_\perp^2\varphi(\mathbf{X},t) - \frac{1}{2B\Omega}|\nabla_\perp\varphi(\mathbf{X},t)|^2$$

(This is the same one in a gyro-kinetic model for flowing plasmas [Miyato JPSJ 2009])

Gyro-center flux is obtained immediately

$$\boldsymbol{\Gamma}_{\rm gy} = NV_{||}\mathbf{b} + N\mathbf{v}_E - \frac{N}{2B\Omega}\frac{\mathbf{b}\times\nabla|\nabla_\perp\varphi|^2}{B} + \frac{P_\perp}{2m\Omega^2}\frac{\mathbf{b}\times\nabla\nabla_\perp^2\varphi}{B}$$

Polarization flux・Magnetization flux

Polarization flux

$$
\begin{aligned}
\mathbf{\Gamma}_{\rm pol} &= \frac{\partial}{\partial t}\int d^3v\left[\langle\boldsymbol{\rho}_1\rangle F\mathcal{J} - \frac{1}{2}\nabla\cdot\left(\langle\boldsymbol{\rho}_0\boldsymbol{\rho}_0\rangle F\mathcal{J}\right)\right] \\
&= -\frac{\partial}{\partial t}\left[\frac{N}{B\Omega}\nabla_\perp\varphi + \frac{1}{2m\Omega^2}\nabla_\perp P_\perp\right]
\end{aligned}
$$

Magnetization flux

$$
\begin{aligned}
\mathbf{\Gamma}_{\rm mag} &= \nabla\times\left[-\frac{P_\perp}{eB}\mathbf{b} - \frac{P_\perp\nabla_\perp^2\varphi}{m\Omega^2 B}\mathbf{b} - \frac{3}{2}\frac{\nabla_\perp P_\perp\cdot\nabla_\perp\varphi}{m\Omega^2 B}\mathbf{b} - \nabla_\perp^2\frac{M_{20}}{4e^2\Omega}\mathbf{b}\right. \\
&\qquad \left. -N\frac{|\nabla_\perp\varphi|^2}{B^2\Omega}\mathbf{b} + \underbrace{N\frac{V_\parallel}{\Omega}\mathbf{v}_E - \nabla\times\frac{M_{11}B}{2m\Omega^2}\mathbf{b}}\right]
\end{aligned}
$$

$$
\approx\left[\nabla\cdot\left(\frac{NV_\parallel}{B\Omega}\nabla_\perp\varphi\right) + \nabla_\perp^2\frac{M_{11}}{2e\Omega}\right]\mathbf{b}
$$

This modifies $NV_\parallel\mathbf{b}$ in $\mathbf{\Gamma}_{\rm gy}$

Total particle flux

$$
\begin{aligned}
\boldsymbol{\Gamma} \;=\; & \left[NV_{\|} + \nabla_\perp^2 \frac{M_{11}}{2e\Omega} + \nabla \cdot \left(\frac{NV_{\|}}{B\Omega} \nabla_\perp \varphi \right) \right] \mathbf{b} \\
& + N\mathbf{v}_E - \frac{N}{2B\Omega} \frac{\mathbf{b} \times \nabla |\nabla_\perp \varphi|^2}{B} + \frac{P_\perp}{2m\Omega^2} \frac{\mathbf{b} \times \nabla \nabla_\perp^2 \varphi}{B} \\
& - \frac{\partial}{\partial t} \left[\frac{N}{B\Omega} \nabla_\perp \varphi + \frac{1}{2m\Omega^2} \nabla_\perp P_\perp \right] \\
& + \nabla \times \left[-\frac{P_\perp}{eB}\mathbf{b} - \frac{P_\perp \nabla_\perp^2 \varphi}{m\Omega^2 B}\mathbf{b} - \frac{3}{2} \frac{\nabla_\perp P_\perp \cdot \nabla_\perp \varphi}{m\Omega^2 B}\mathbf{b} - \nabla_\perp^2 \frac{M_{20}}{4e^2\Omega}\mathbf{b} - N \frac{|\nabla_\perp \varphi|^2}{B^2 \Omega}\mathbf{b} \right]
\end{aligned}
$$

Using a vector relation

$$
\begin{aligned}
& p\mathbf{b} \times \nabla \nabla_\perp^2 \varphi - (\nabla_\perp^2 p)\mathbf{b} \times \nabla \varphi \\
= \;& \mathbf{b} \times \nabla (p \nabla_\perp^2 \varphi) - \mathbf{b} \times \nabla \{ (\nabla_\perp \varphi) \cdot (\nabla_\perp p) \} \\
& -2\mathbf{b} \times \nabla p \cdot \nabla \nabla_\perp \varphi - \nabla_\perp (\mathbf{b} \times \nabla \varphi \cdot \nabla p)
\end{aligned}
$$

to delete the 3rd term in the 2nd line,

Recover Belova's result

$$
\begin{aligned}
\boldsymbol{\Gamma} \;=\;& \left[NV_\| + \nabla_\perp^2 \frac{M_{11}}{2e\Omega} + \nabla\cdot\left(\frac{NV_\|}{B\Omega}\nabla_\perp\varphi\right)\right]\mathbf{b} \\
&+N\mathbf{v}_E - \frac{N}{2B\Omega}\frac{\mathbf{b}\times\nabla|\nabla_\perp\varphi|^2}{B} + \frac{\nabla_\perp^2 P_\perp}{2m\Omega^2}\mathbf{v}_E \\
&-\frac{\partial}{\partial t}\frac{N}{B\Omega}\nabla_\perp\varphi - \frac{\mathbf{b}\times\nabla P_\perp\cdot\nabla}{m\Omega^2 B}\nabla_\perp\varphi \\
&-\frac{1}{2m\Omega^2}\nabla_\perp\left[\frac{\partial P_\perp}{\partial t} + \mathbf{v}_E\cdot\nabla P_\perp\right] \longleftarrow \text{This is small} \\
&+\nabla\times\left[-\frac{P_\perp}{eB}\mathbf{b} - \frac{P_\perp\nabla_\perp^2\varphi}{m\Omega^2 B}\mathbf{b} - \frac{3}{2}\frac{\nabla_\perp P_\perp\cdot\nabla_\perp\varphi}{m\Omega^2 B}\mathbf{b} - \nabla_\perp^2\frac{M_{20}}{4e^2\Omega}\mathbf{b} - N\frac{|\nabla_\perp\varphi|^2}{B^2\Omega}\mathbf{b}\right]
\end{aligned}
$$

Neglecting the 4th line recovers Belova's result obtained from the conventional representation.

Alternate representation

Using another vector relation,

$$
\begin{aligned}
& p\mathbf{b} \times \nabla\nabla_\perp^2\varphi - (\nabla_\perp^2 p)\mathbf{b} \times \nabla\varphi \\
&\quad = \mathbf{b} \times \nabla(p\nabla_\perp^2\varphi) + \nabla_\perp(\mathbf{b} \times \nabla\varphi \cdot \nabla p) \\
&\quad\quad -2\mathbf{b} \times \nabla\varphi \cdot \nabla\nabla_\perp p - \mathbf{b} \times \nabla\{(\nabla_\perp\varphi) \cdot (\nabla_\perp p)\}
\end{aligned}
$$

and $\mathbf{v}_E \cdot \nabla P_\perp \simeq -\partial_t P_\perp$

$$
\begin{aligned}
\mathbf{\Gamma} \;=\; & \left[NV_\parallel + \nabla_\perp^2 \frac{M_{11}}{2e\Omega} + \nabla \cdot \left(\frac{NV_\parallel}{B\Omega}\nabla_\perp\varphi\right)\right]\mathbf{b} \\
& + N\mathbf{v}_E - \frac{N}{2B\Omega}\frac{\mathbf{b} \times \nabla|\nabla_\perp\varphi|^2}{B} + \frac{\nabla_\perp^2 P_\perp}{2m\Omega^2}\mathbf{v}_E \\
& - \frac{\partial}{\partial t}\frac{N}{B\Omega}\nabla_\perp\varphi - \frac{1}{m\Omega^2}\left(\frac{\partial}{\partial t} + \mathbf{v}_E \cdot \nabla\right)\nabla_\perp P_\perp \\
& + \nabla \times \left[-\frac{P_\perp}{eB}\mathbf{b} - \frac{3}{2}\frac{P_\perp\nabla_\perp^2\varphi}{m\Omega^2 B}\mathbf{b} - \frac{\nabla_\perp P_\perp \cdot \nabla_\perp\phi}{m\Omega^2 B}\mathbf{b} - \nabla_\perp^2\frac{M_{20}}{4e^2\Omega}\mathbf{b} - N\frac{|\nabla_\perp\varphi|^2}{B^2\Omega}\mathbf{b}\right]
\end{aligned}
$$

Transformation to particle fluid moments

Using push-forward representations of particle fluid moments

$$n = N + \frac{1}{2eB\Omega}\nabla_\perp^2 P_\perp + \nabla\cdot\left(\frac{N}{\Omega B}\nabla_\perp\varphi\right)$$

$$nu_\parallel = NV_\parallel + \nabla_\perp^2\frac{M_{11}}{2e\Omega} + \nabla\cdot\left(\frac{NV_\parallel}{B\Omega}\nabla_\perp\varphi\right)$$

$$p_\perp = P_\perp + \frac{B}{2e\Omega}\nabla_\perp^2 M_{20} + 2\nabla\cdot\left(\frac{P_\perp}{\Omega B}\nabla_\perp\varphi\right)$$

Gyrofluid moments are transformed as

$$\begin{aligned}\mathbf{\Gamma} \;=\;& nu_\parallel\mathbf{b} + n\mathbf{v}_E \\ & -\left(\frac{\partial}{\partial t} + \mathbf{v}_E\cdot\nabla\right)\frac{n}{B\Omega}\nabla_\perp\varphi - \frac{1}{m\Omega^2}\left(\frac{\partial}{\partial t} + \mathbf{v}_E\cdot\nabla\right)\nabla_\perp p_\perp \\ & +\nabla\times\left[-\frac{p_\perp}{eB}\mathbf{b} + \frac{1}{2}\frac{p_\perp\nabla_\perp^2\varphi}{m\Omega^2 B}\mathbf{b} + \frac{\nabla_\perp p_\perp\cdot\nabla_\perp\varphi}{m\Omega^2 B}\mathbf{b} + \nabla_\perp^2\frac{m_{20}}{4e^2\Omega}\mathbf{b}\right]\end{aligned}$$

A FLR corrected particle continuity equation is obtained immediately.

$$\partial_t n + \nabla\cdot\mathbf{\Gamma} = 0$$

Summary

- We have derived an explicit push-forward representations of particle flux with FLR terms including the polarization drift term from the pure push-forward abstract representation using only the lowest order solution of S_1, $S_1^{(1)}$, in the standard electrostatic gyrokinetic model.
- Belova's result, which was derived from the conventional representation with $S_1^{(2)}$, S_2 as well as $S_1^{(1)}$, has been recovered from our result through a vector relation and neglecting small terms.
- We have also derived the other form of representation and then the FLR corrected continuity equation retaining the time derivative term of $\nabla_\perp p_\perp$ by transforming the gyrofluid moments to the particle fluid moments.
- Electromagnetic generalization in the p_z formulation [Hahm-Lee-Brizard 1988] is easier for the pure push-forward representation.

詳細は Miyato et al., Physics of Plasmas **22**, 012103 (2015)