モーメント･クロージャーを用いたジャイロ運動論的シミュレーション手法
Gyrokinetic simulation method with a moment closure

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- Drift kinetic equation and moment closure
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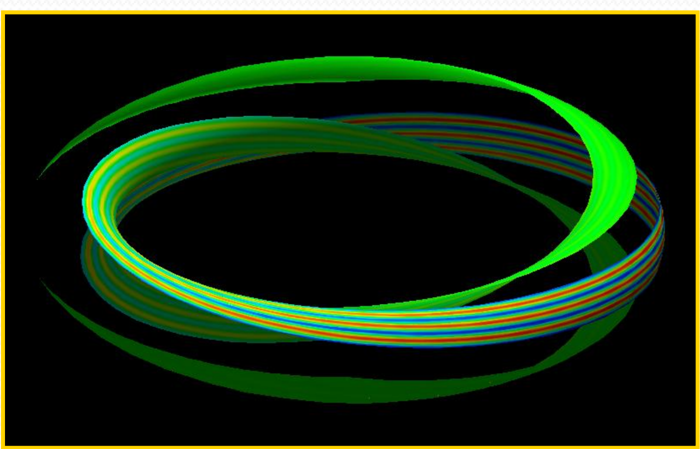
Gyrokinetic Simulation Code: GKV

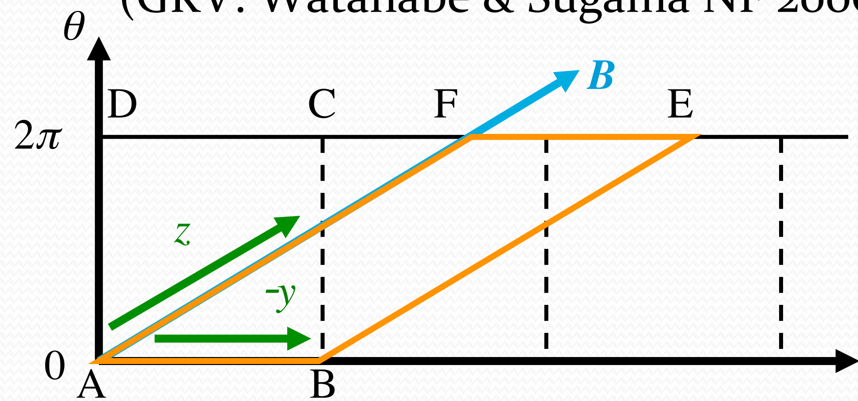
- Nonlinear gyrokinetic equation for perturbed gyrocenter distribution δf is numerically solved on the five-dimensional phase space, $(x, y, z, v_{||}, \mu)$

$$\left[\frac{\partial}{\partial t} + v_{\|}\hat{\mathbf{b}}\cdot\nabla + \mathbf{v}_d\cdot\nabla - \mu\left(\hat{\mathbf{b}}\cdot\nabla\Omega\right)\frac{\partial}{\partial v_{\|}}\right]\delta f + \frac{c}{B_0}\{\psi, \delta f\} = \left(\mathbf{v}_* - \mathbf{v}_d - v_{\|}\hat{\mathbf{b}}\right)\cdot\frac{e\nabla\psi}{T_i}F_M + C(\delta f)$$

  - Strong anisotropy of fluctuations is accurately resolved by using curvilinear coordinates along field lines.
  - High resolution of 5-D phase space.



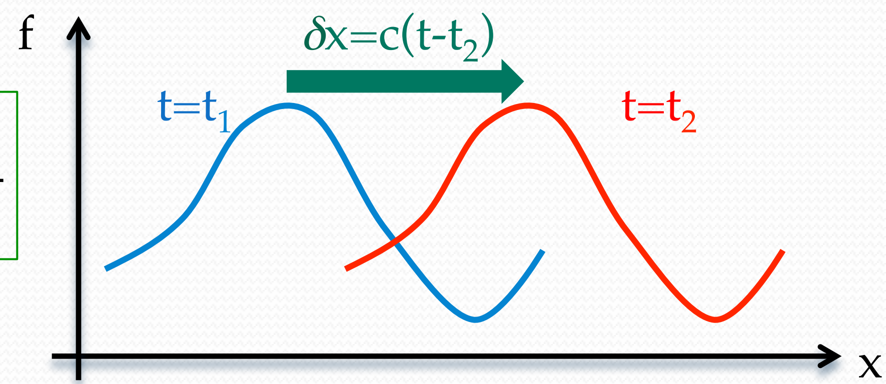
(GKV: Watanabe & Sugama NF 2006)



Numerical difficulties in multi-species gyrokinetic simulation

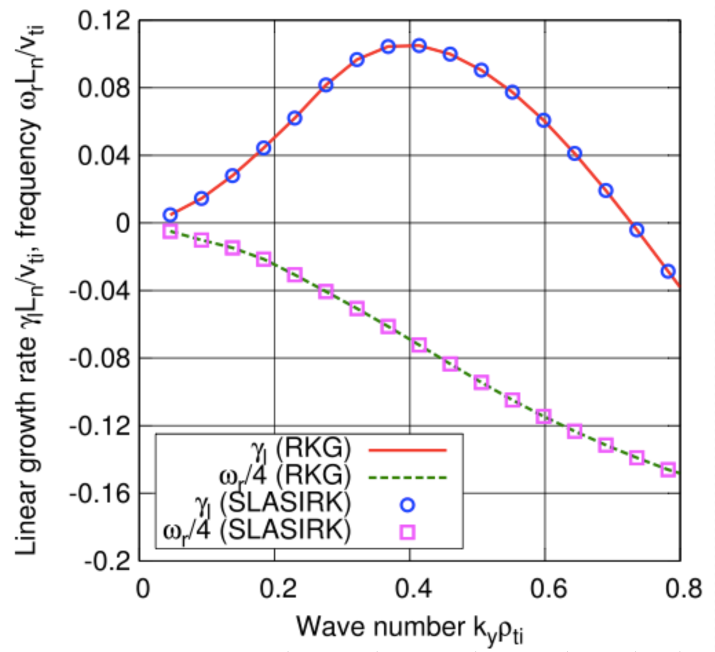
- Multi-species gyrokinetic (GK) simulation suffers from separation of typical time scales of ions and electrons
  - Fast electron motion along field lines restricts the time-step size of explicit schemes. (say, Δt<1.e-4 R_0/C_s for LHD)
  - It also leads to slow convergence of recursive solvers in implicit time integrations.
  - Semi-Lagrangian (SL) method can trace drift motion of particles with time steps beyond the CFL condition.

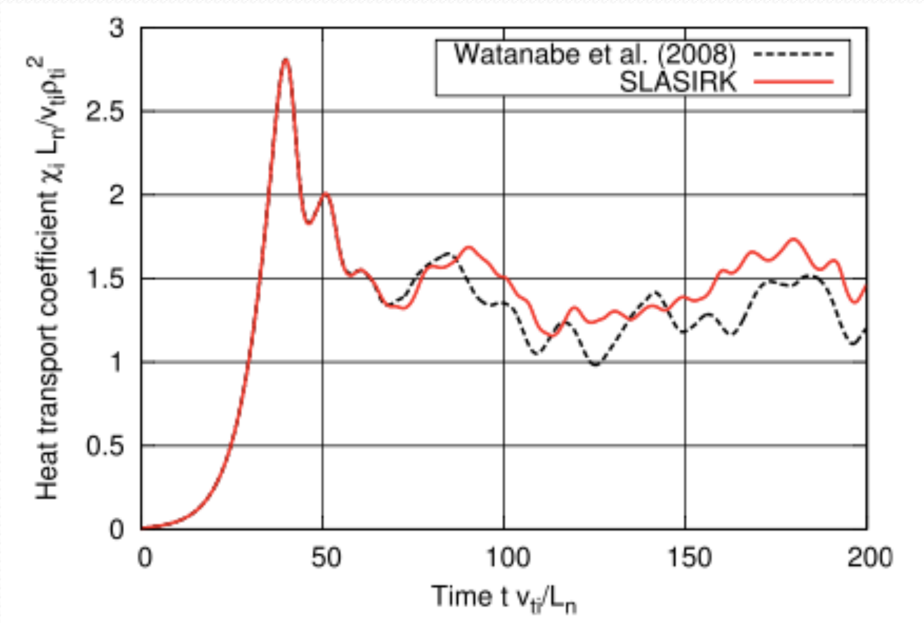
$$\frac{\partial f}{\partial t} = -c\frac{\partial f}{\partial x}$$



Semi-Lagrangian scheme applied to GKV – Early work (Maeyama et al CPC 2012)

- Semi-Lagrangian scheme applied to GKV code could successfully reduce the computational costs.





- Linear ITG benchmark with adiabatic electrons; Δt=0.005 for RKG but Δt=0.1 for Semi-Lagrangian + iRK
- Nonlinear benchmark for the ITG turbulence was also successful.

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  - It also leads to slow convergence of recursive solvers in implicit time integrations.
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  - But, SL for GK equation is often numerically unstable to electromagnetic fluctuations, because the wave propagation direction may be different from those of particle motions.

=> We need new numerical techniques for multi-species GK.

Drift kinetic equation and moment closure

Revisiting the drift kinetic equation

- Let us consider the linearized DK equation for electrons

$$\frac{\partial f_e}{\partial t} + v_\parallel \frac{\partial f_e}{\partial z} = -\frac{q_e}{T_{e0}} v_\parallel F_M \frac{\partial}{\partial z}\left(\frac{\partial \phi}{\partial z} + \frac{1}{c}\frac{\partial A_\parallel}{\partial t}\right)$$

- 0th , 1st, and 2nd order moments are given by

$$n_e = \int f_e d^3v \qquad n_0 U = \int v_\parallel f_e d^3v \qquad n_e T_0 + n_0 T_\parallel = m_e \int v_\parallel^2 f_e d^3v$$

- Quasi-neutrality and the Ampere's law

$$\frac{q_i^2 n_0}{T_i} k_\perp^2 \rho_i^2 \phi = q_e n_e \qquad k_\perp^2 A_\parallel = \frac{4\pi}{c} q_e n_0 U$$

- The above equations describe the parallel electron motion and the kinetic Alfven waves

Equations of low-order moments and moment-extracted kinetic eq.

- The 0th and 1st order moment equations are given by

$$\frac{\partial n_e}{\partial t} = -n_0 \frac{\partial U}{\partial z}$$

$$n_0 m_e \frac{\partial U}{\partial t} = -q_e n_0 \left(\frac{\partial \phi}{\partial z} + \frac{1}{c} \frac{\partial A_\parallel}{\partial t} \right) - \frac{\partial}{\partial z} \left(n_e T_{e0} + n_0 T_\parallel \right)$$

- Distribution function h_e where the low-order moments are extracted.

$$f_e = \frac{n_e}{n_0} F_M + \frac{m_e}{T_{e0}} U v_\parallel F_M + h_e \qquad n_0 T_\parallel = m_e \int v_\parallel^2 h_e d^3 v$$

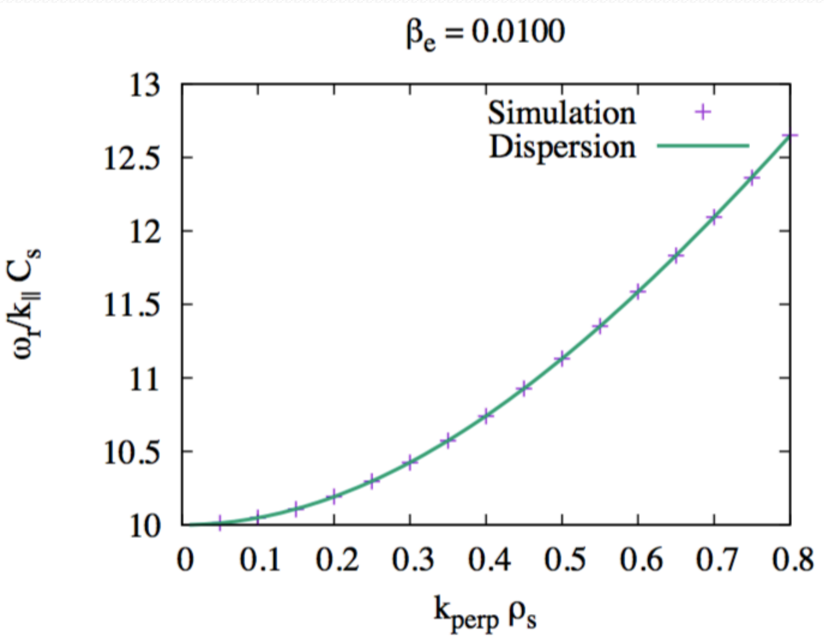
- The remnant drift kinetic equation for h_e

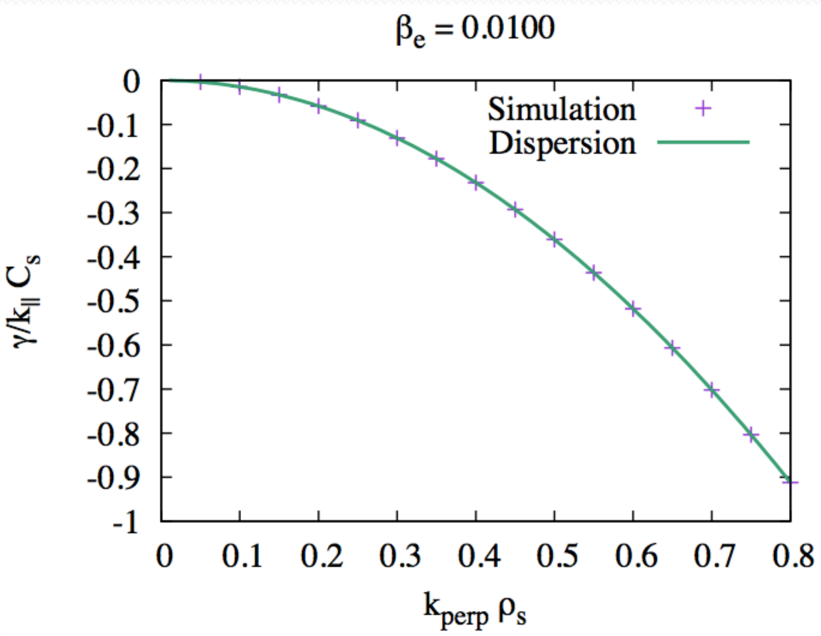
$$\frac{\partial h_e}{\partial t} + v_\parallel \frac{\partial h_e}{\partial z} = \left(1 - \frac{m_e v_\parallel^2}{T_{e0}} \right) F_M \frac{\partial U}{\partial z} + \frac{v_\parallel}{T_{e0}} F_M \frac{\partial T_\parallel}{\partial z}$$

with a constraint of $\int h_e d^3 v = \int v_\parallel h_e d^3 v = 0$

New formulation exactly describes the kinetic Alfven wave

- Numerical solution of the DK equation with the moment extracted formulation successfully describes the KAWs.

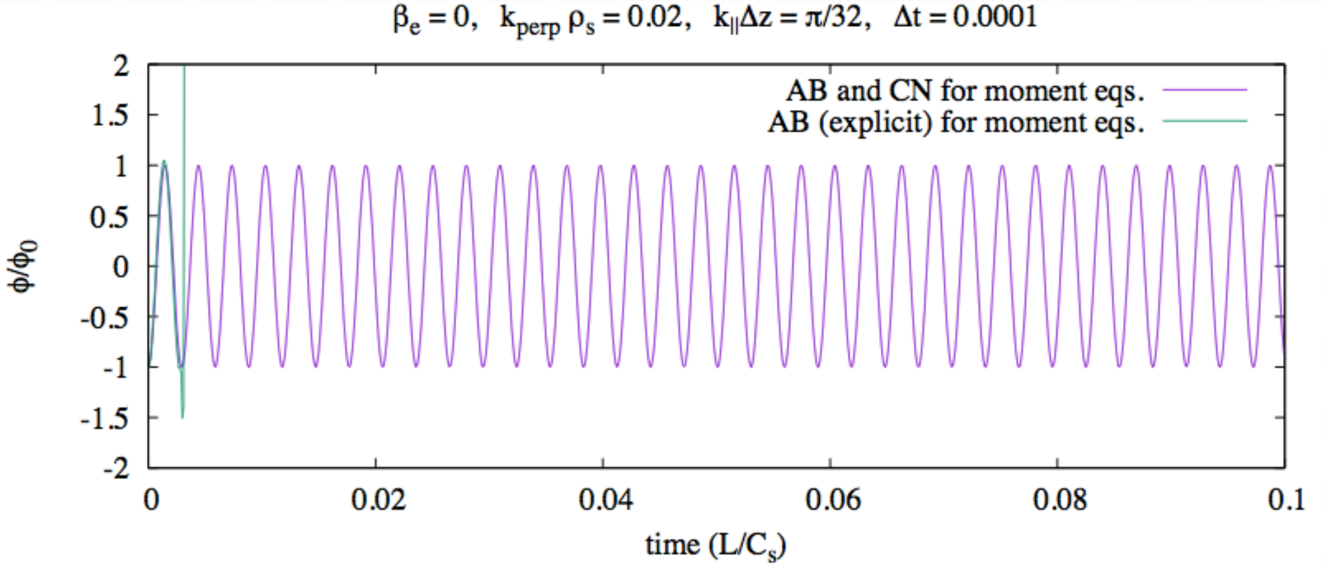




- Ion polarization is included in the form of a long wave-length limit.

Application of an implicit solver to the fluid equations

- Adams-Bashforth + Crank-Nicolson
- An explicit integrator is used for the temperature gradient
- Stable solutions can be obtained even with a larger time step size than that given by the Courant number for ω_H.



- Explicit integrator is unstable for $k_{perp}\rho_s < 0.12$, while the implicit can be successfully applied to $k_{perp}\rho_s < 0.01$.

Summary

- We developed a numerical scheme for handling the fast electron motion in gyrokinetic simulation.
  - Solve a kinetic equation, of which 0th and 1st order moments are extracted, and the electron fluid equations.
  - Wave propagation can be solved implicitly, while an explicit scheme is applied to the kinetic equation.
  - Modified Maxwellian distribution of which 0th and 2nd order moments exactly satisfy the identities.
  - Application of the semi-Lagrangian scheme has also been tested, demonstrating its numerical stability and efficiency, where kinetic and fluid equations can be solved explicitly with a time step size beyond the CFL limit.