

# Recent progress of toroidal full- $f$ gyrokinetic simulation based on GKNET

## Contents

1. Introduction
  - Toroidal full- $f$  gyrokinetic code GKNET
  - Recent progress based on GKNET
2. Flux-driven turbulent transport and ITB formation
  - Flux-driven turbulent transport coupled with mean flow
  - ITB formation in flux-driven ITG turbulence
3. Summary & Future Plans

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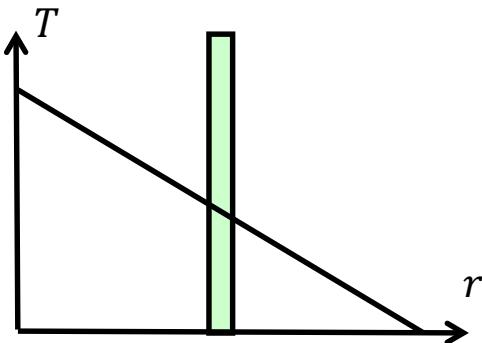
# Local/Global Gyrokinetics

## Local approach

$$\partial_t f_{eq} - [H, f_{eq}] = C(f_{eq}) + S$$

$$\partial_t \delta f - [H, \delta f] - [\delta H, f_{eq}] - [\delta H, \delta f] = C(\delta f)$$

*linear*      *driving*      *nonlinear*



Fixed Gradient

$$R/L_T \neq 0$$

$$T = \text{const}$$

- 😊 Very powerful tool to estimate turbulent transport process
- 😊 Computationally efficient  
-> multi-species, EM turbulence

GKV(JPN), GS2(US), GENE(GER), ...

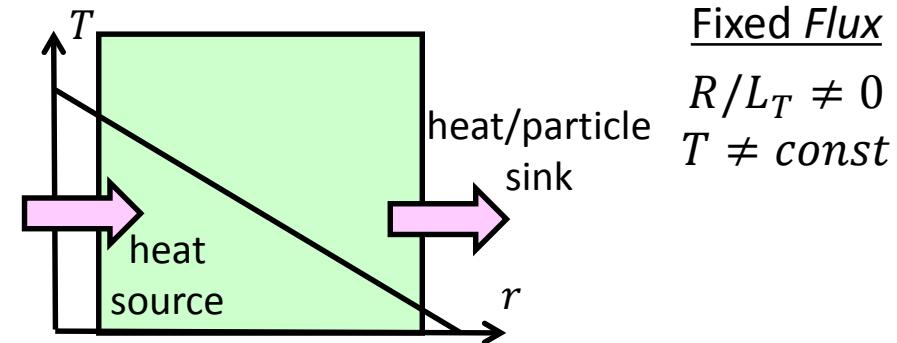
## Global approach

$$\partial_t f_{eq} - [H, f_{eq}] = C(f_{eq}) + S$$

$$\partial_t \delta f - [H, \delta f] - [\delta H, f_{eq}] - [\delta H, \delta f] = C(\delta f)$$

*linear*      *driving*      *nonlinear*

*self-consistently determined Mean  $E_r$*



Fixed Flux

$$R/L_T \neq 0$$

$$T \neq \text{const}$$

- 😊 Global profile shear effect can be taken into account (e.g.  $\omega_r$  shear)
- 😊 Mean  $E_r$  is self-consistently determined  
-> ITBs, L-H transition...

GT5D, GKNET(JPN), XGC(US), GYSELA(FRA), ...

# Toroidal Full-*f* Gyrokinetic Code GKNET

## GK Vlasov equation for ion

$$\frac{\partial f}{\partial t} + \frac{d\mathbf{R}}{dt} \cdot \frac{\partial f}{\partial \mathbf{R}} + \frac{d\mathbf{v}_{\parallel}}{dt} \frac{\partial f}{\partial \mathbf{v}_{\parallel}} = C_{coll}$$

$$\frac{d\mathbf{R}}{dt} \equiv \{\mathbf{R}, H\} = \mathbf{v}_{\parallel} \mathbf{b} + \frac{c}{eB_{\parallel}^*} \mathbf{b} \times (e\nabla \langle \phi \rangle_{\alpha} + m_i v_{\parallel}^* \mathbf{b} \cdot \nabla \mathbf{b} + \mu \nabla B)$$

$$\frac{d\mathbf{v}_{\parallel}}{dt} \equiv \{\mathbf{v}_{\parallel}, H\} = -\frac{\mathbf{B}_{\parallel}^*}{m_i B_{\parallel}^*} \cdot (e\nabla \langle \phi \rangle_{\alpha} + \mu \nabla B)$$

### Vlasov solver

- ✓ 4th-order Morinishi scheme + 4th-order RK-Gill scheme

[K. Imadera, et.al. 25th Fusion Energy Conference, TH/P5-8, Oct. 16, 2014.]

[K. Obrejan, K. Imadera, J. Q. Li and Y. Kishimoto Plasma Fusion Res. **10**, 3403042 (2015).]

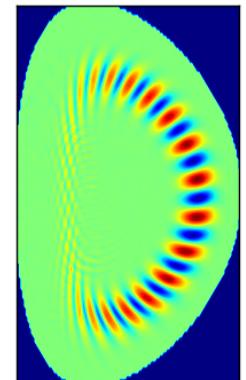
- ✓ Full-*f* (Global)
- ✓ Electrostatic
- ✓ Conservative

## GK quasi-neutrality condition

$$\phi - \langle \phi \rangle_{\alpha} + \frac{1}{T_{e0}(r)} (\phi - \langle \phi \rangle_{\alpha}) = \frac{1}{n_{i0}(r)} \iint \langle \delta f \rangle_{\alpha} B_{\parallel}^* d\mathbf{v}_{\parallel} d\mu$$

### Real space field solver

- ✓ Full-order FLR effect (without Tayler/Pade approximation)
- ✓ Field equation is solved in real space (not k-space)



# Recent Progress Based on GKNET

## Study of flux-driven ITG turbulence

### (A-1) Flux-driven turbulent transport couple with mean flow

[Y. Kishimoto, *et al.*, submitted to IAEA-2016]

[W. Wang, *et al.*, this workshop]

- ✓ Global profile shear effect of  $\omega_r$  and  $\omega_f$  on ballooning structure
- ✓ Intermittent turbulent transport coupled with radially extended ballooning structure

### (A-2) ITB formation in flux-driven turbulence

[K. Imadera, *et al.*, submitted to ICPP-2016 & IAEA-2016]

[S. Maeda, *et al.*, this workshop]

- ✓ ITB formation by toroidal momentum injection
- ✓ Momentum pinch originated from global profile shear effect of  $\omega_r$  and  $\omega_f$

## Development of GKNET

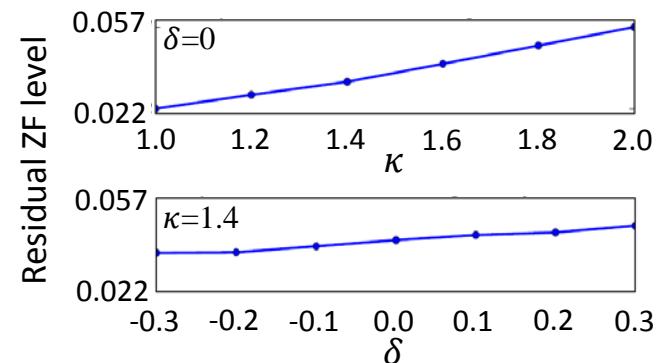
### (B-1) Development of real space field solver

[K. Obrejan, *et al.*, this workshop]

Elongation  $\nearrow$  or triangularity  $\nearrow$



Residual zonal flow level  $\nearrow$



### (B-2) Introduction of kinetic electron

[R. Yoshida, *et al.*, this workshop]

Kinetic electron  $\rightarrow$  slab ITG instability  $\searrow$   
phase shift  
between  $\phi$  and  $\delta n_e$

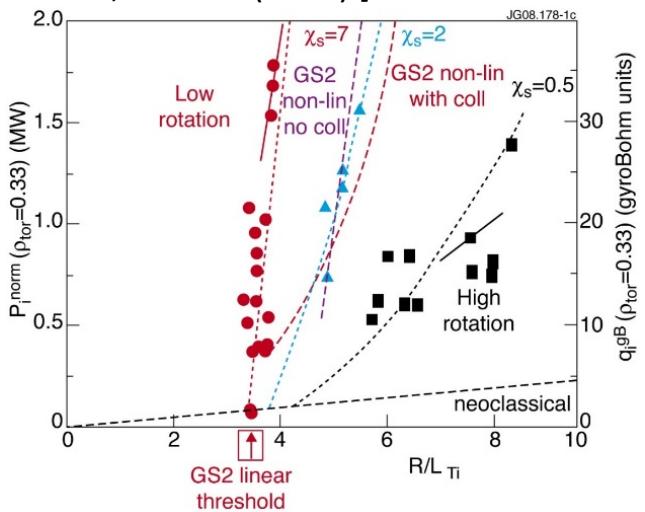
Kinetic electron  $\rightarrow$  GAM damping rate  $\nearrow$   
residual ZF level  $\rightarrow$

# Background - Profile Stiffness in Flux-driven System -

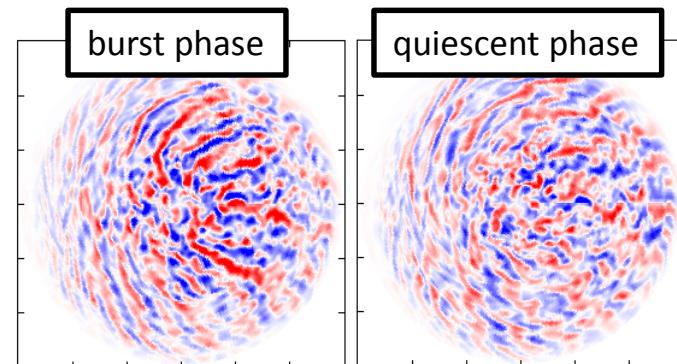
- ✓ Profile stiffness is a long standing problem, which may limit the overall performance of H-mode plasmas.
- ✓ In the JET experiment, while strong temperature profile stiffness is observed, it can be greatly reduced by co-current toroidal rotation in weak magnetic shear plasma.
- ✓ In our flux-driven ITG simulation, we also observe a stiff temperature profile in the absence of momentum source, where not only heat avalanches but also the explosive global transport coupled with the instantaneous formation of radially extended ballooning structure become dominant.



[P. Mantica, *et.al.*, Phys. Rev. Lett., 102, 175002 (2009).]



[K. Imadera, *et.al.*, 25th Fusion Energy Conference, TH/P5-8, Oct. 16 (2014).]



- A) Why radially extended structure is formed even in the presence of MF and ZF?
- B) What is the stabilization mechanism by co-current toroidal rotation?

# Purpose of This Work

## *Purpose of this work*

- A) Understand the origin of radially extended ballooning structure in flux-driven ITG turbulence with MF and ZF -> **profile stiffness**
- B) Control such structures by momentum injection -> **barrier formation**

## *Approaches*

### 1. Non-local first-order ballooning theory

- ✓ Notation of  $\theta_b$ ,  $\Delta r$  and  $\gamma$
- ✓ Impact of MF and toroidal rotation on toroidal ITG mode

### 2. Global GK ITG simulation w/o mom. source

- ✓ Impact of MF on profile stiffness

### 3. Global GK ITG simulation with mom. source

- ✓ Impact of momentum injection on profile stiffness

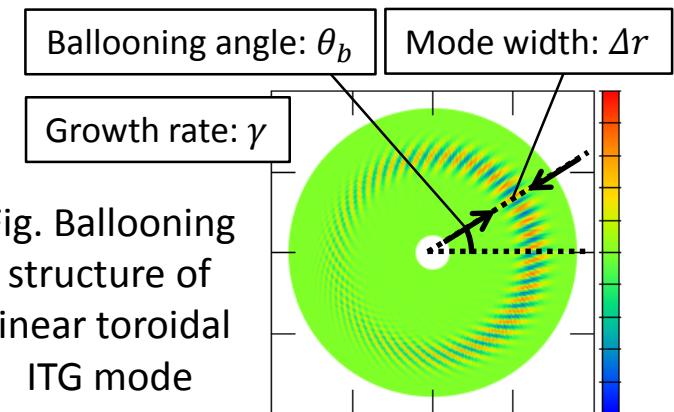


Fig. Ballooning structure of linear toroidal ITG mode

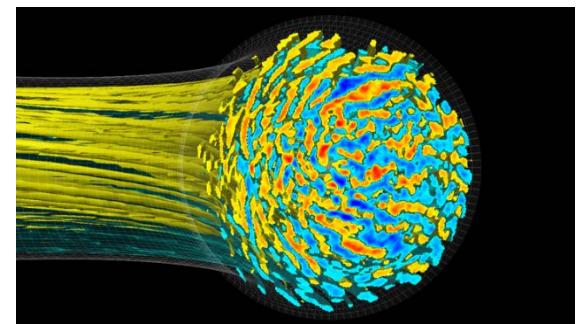


Fig. Typical structure of flux-driven toroidal ITG turbulence calculated by *GKNET*

# Non-Local Ballooning Theory

[Y. Kishimoto, et.al., Plasmas Phys. Controlled Fusion, **40**, A663 (1998).]

$$\theta_b = \mp \left| \frac{\partial_r(\omega_r + \omega_f)}{2k_\theta \gamma_0 \hat{s}} \right|^{1/3}$$

$$\Delta r = \left| \frac{\sin \theta_b}{k_\theta^2 \hat{s}^2 \theta_b^3} \right|^{1/2}$$

$$\gamma = \gamma_0 \cos \theta_b$$

Radial force balance

$$E_r - v_\theta B_\varphi + v_\varphi B_\theta - \frac{1}{n_i e} \frac{\partial p_i}{\partial r} = 0 \longrightarrow E_r = \frac{rB}{qR} U_{\parallel} - \frac{T_i}{e} \left( \frac{1}{L_n} + \frac{1-k}{L_{T_i}} \right)$$

$$n_i = n_{i0} \exp\left(-\frac{r}{L_n}\right), T_i = T_{i0} \exp\left(-\frac{r}{L_{T_i}}\right)$$

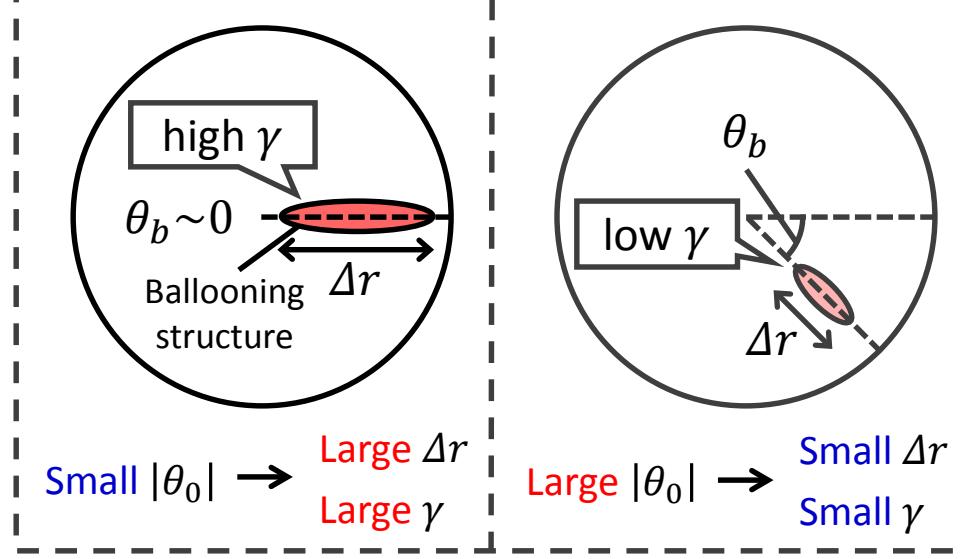
Eigenfrequency + Doppler shift frequency

$$\omega_r + \omega_f \sim \frac{k_\theta}{eB} \left[ \left( \frac{2}{R_0} - \frac{1}{L_n} - \frac{1-k}{L_{T_i}} \right) T_i - \frac{erB}{qR} U_{\parallel} \right]$$

Diamagnetic drift

Mean flow

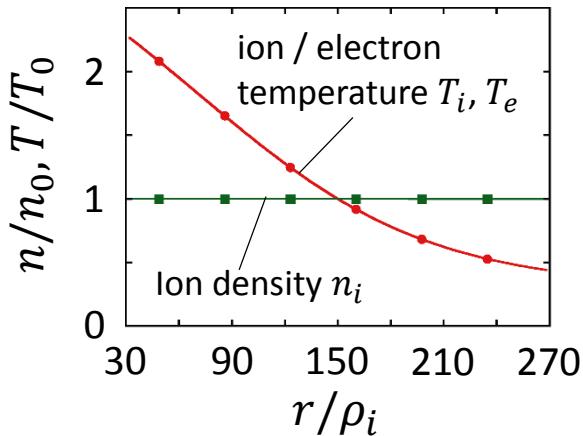
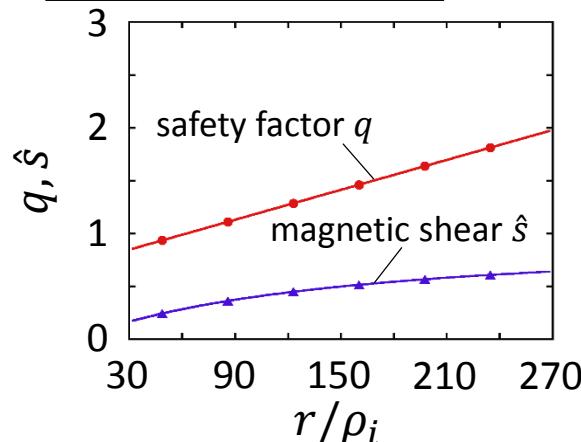
Toroidal rotation



- ✓ Cancellation by mean flow
- ✓ Impact of toroidal rotation

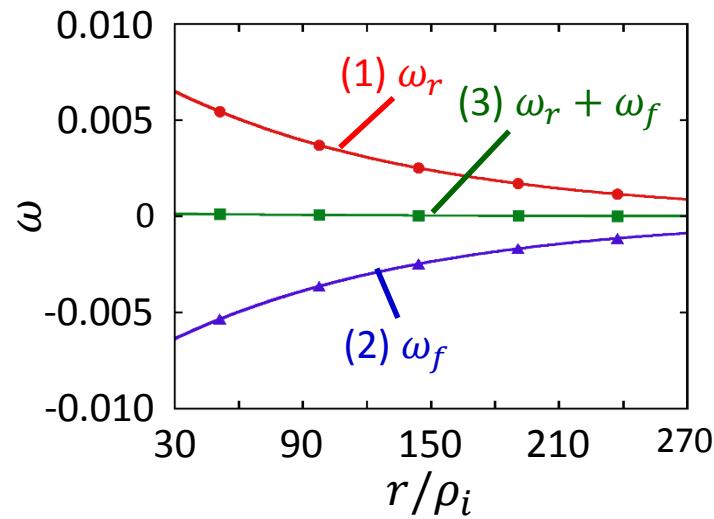
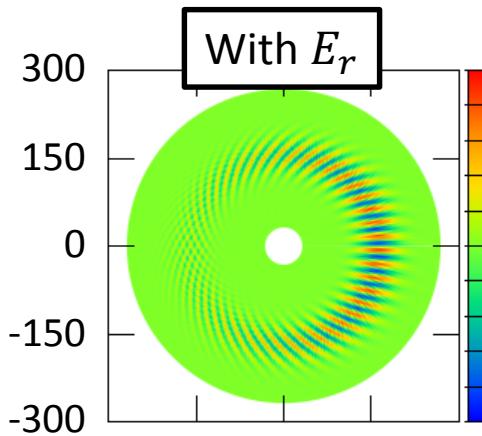
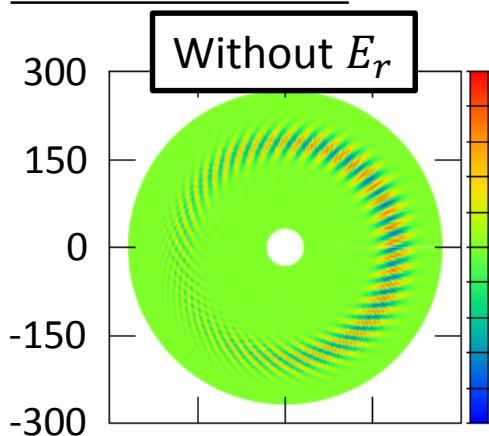
# Linear Global GK ITG Simulation

## Simulation condition



Parameter	Value
$a_0/\rho_i$	300
$a_0/R_0$	0.36
$(R_0/L_n)_{r=a_0/2}$	0
$(R_0/L_{T_i})_{r=a_0/2}$	6.92

## Numerical results



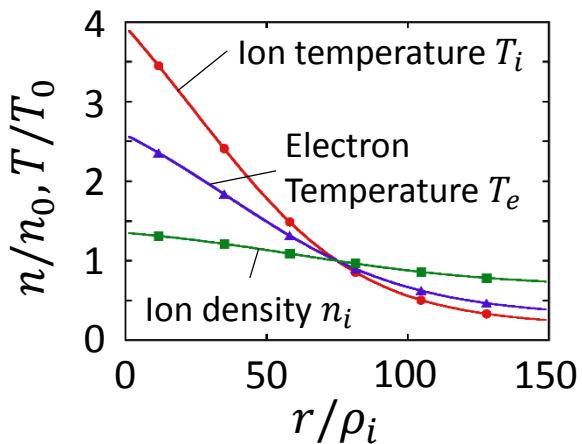
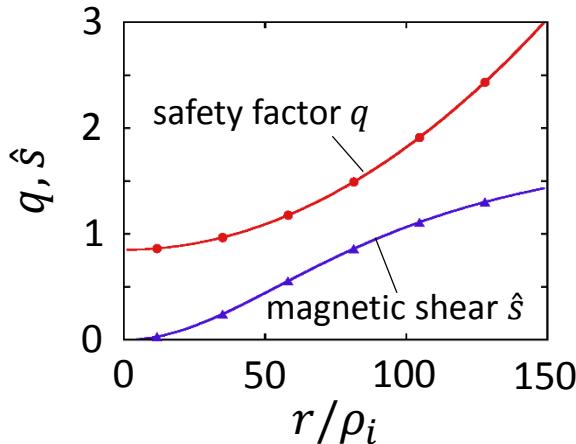
$\gamma$	$0.07 \sim 0.12$
$\theta_b$	$0.5 \sim 0.6$
$\Delta r$	$28 \sim 42$

$\gamma$	$0.15$
$\theta_b$	$0$
$\Delta r$	$49$

$$\theta_b = \mp \sqrt[3]{\frac{\partial_r(\omega_r + \omega_f)}{2k_\theta \gamma_0 \hat{s}}}$$

# Nonlinear Flux-Driven GK ITG Simulation

## Simulation condition



Parameter	Value
$a_0/\rho_i$	150
$a_0/R_0$	0.36
$(R_0/L_n)_{r=a_0/2}$	2.22
$(R_0/L_{T_i})_{r=a_0/2}$	10.0
$(R_0/L_{T_e})_{r=a_0/2}$	6.92
$\nu_*$	0.28
$P_{\text{in}}$	4, 8, 16, 24 [MW]

## Source operator

$$S_{src} = A_{src}(r)\tau_{src}^{-1}[f_M(2\bar{T}) - f_M(\bar{T})]$$

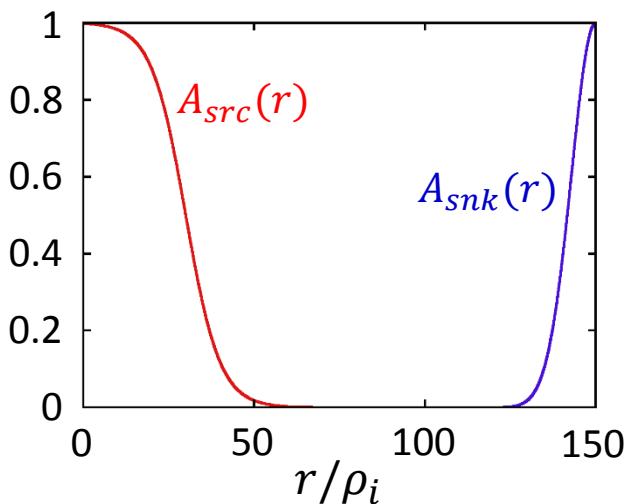
- ✓ Constant power input near magnetic axis

## Sink operator

$$S_{snk} = A_{snk}(r)\tau_{snk}^{-1}[f(t) - f(t=0)]$$

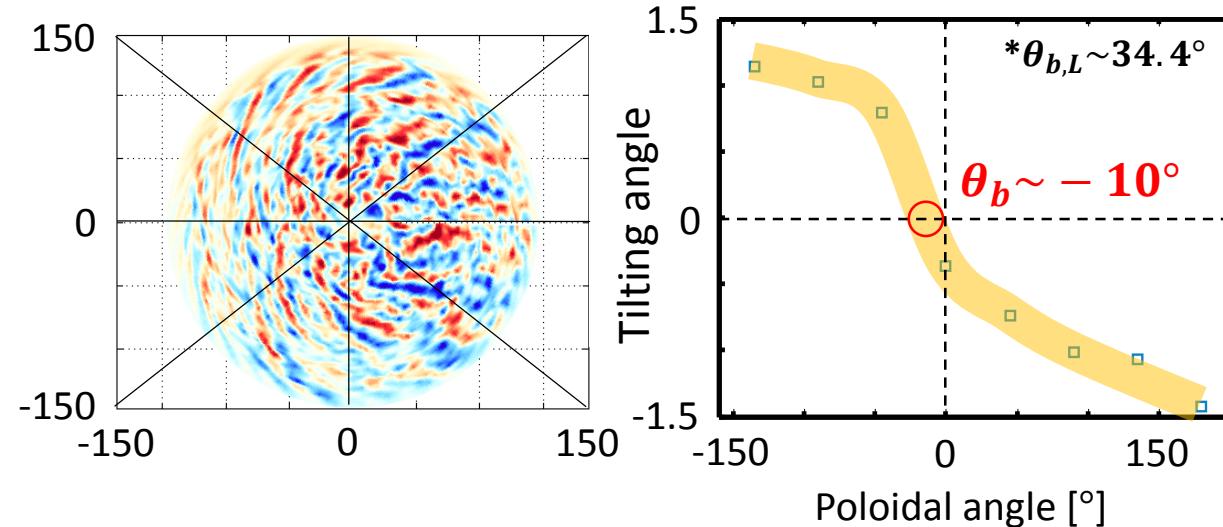
- ✓ Krook-type operator to  $f$  in boundary region

[Y. Idomura, et. al., Nucl. Fusion, **49**, 065029 (2009).]

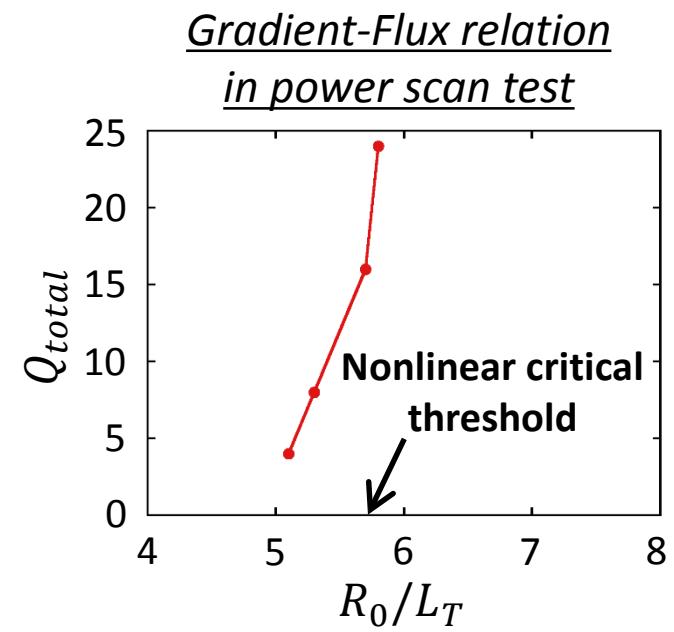
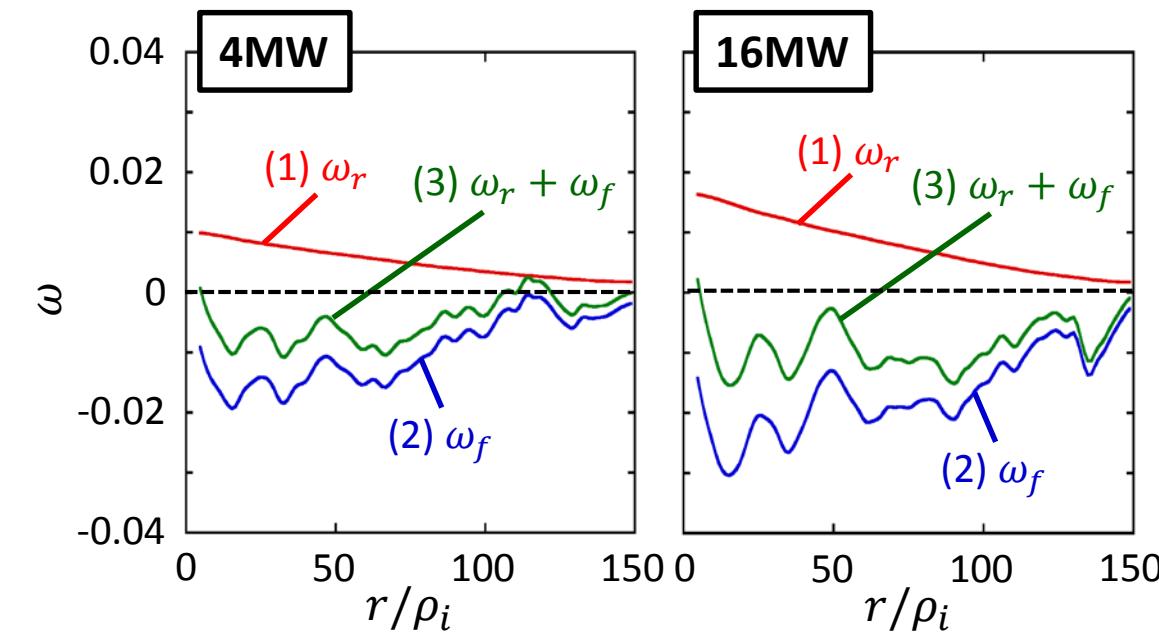


# Poloidal Symmetry and Profile Stiffness

2D spatial correlation analysis for potential structure (16MW)



✓ Ballooning angle is smaller than that estimated from linear analysis without  $E_r$ .



# Discussion - How we can break profile stiffness ? -

$$\text{Radial force balance: } E_r + \frac{k}{e} \frac{\partial T_i}{\partial r} - \frac{rB}{qR} U_{\parallel} - \frac{1}{n_i e} \frac{\partial p_i}{\partial r} = 0$$

- ✓ Mean flow shear recovers the symmetry or weakly reverses the ballooning angle so that **its stabilization effect is small**.



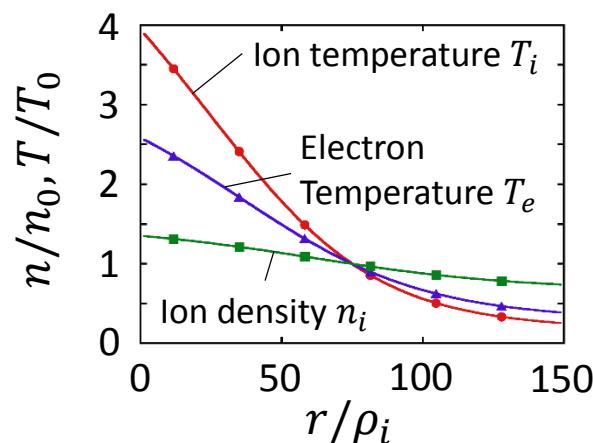
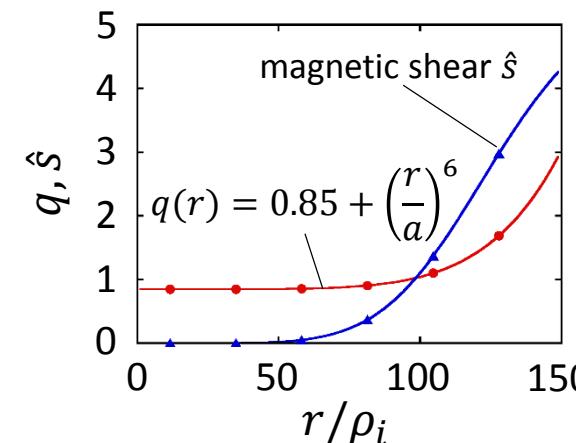
- ✓ **Toroidal rotation** can change the mean flow shear through radial force balance, by which **we may enhance its stabilization effect**.



- ✓ Especially, **toroidal rotation in outer region with small safety factor (weak/reversed magnetic shear)** can be effective.

# Flux-Driven ITG Simulation with Momentum Source

## Simulation condition



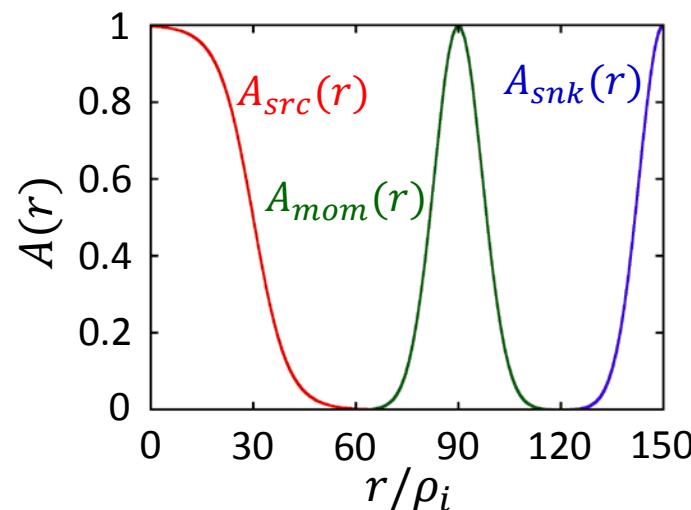
Parameter	Value
$a_0/\rho_i$	150
$a_0/R_0$	0.36
$(R_0/L_n)_{r=a_0/2}$	2.22
$(R_0/L_{T_i})_{r=a_0/2}$	10.0
$(R_0/L_{T_e})_{r=a_0/2}$	6.92
$\nu_*$	0.28
$P_{\text{in}}$	4 [MW]
$T_{\text{in}}$	11.2 [N·m]

## Momentum source operator

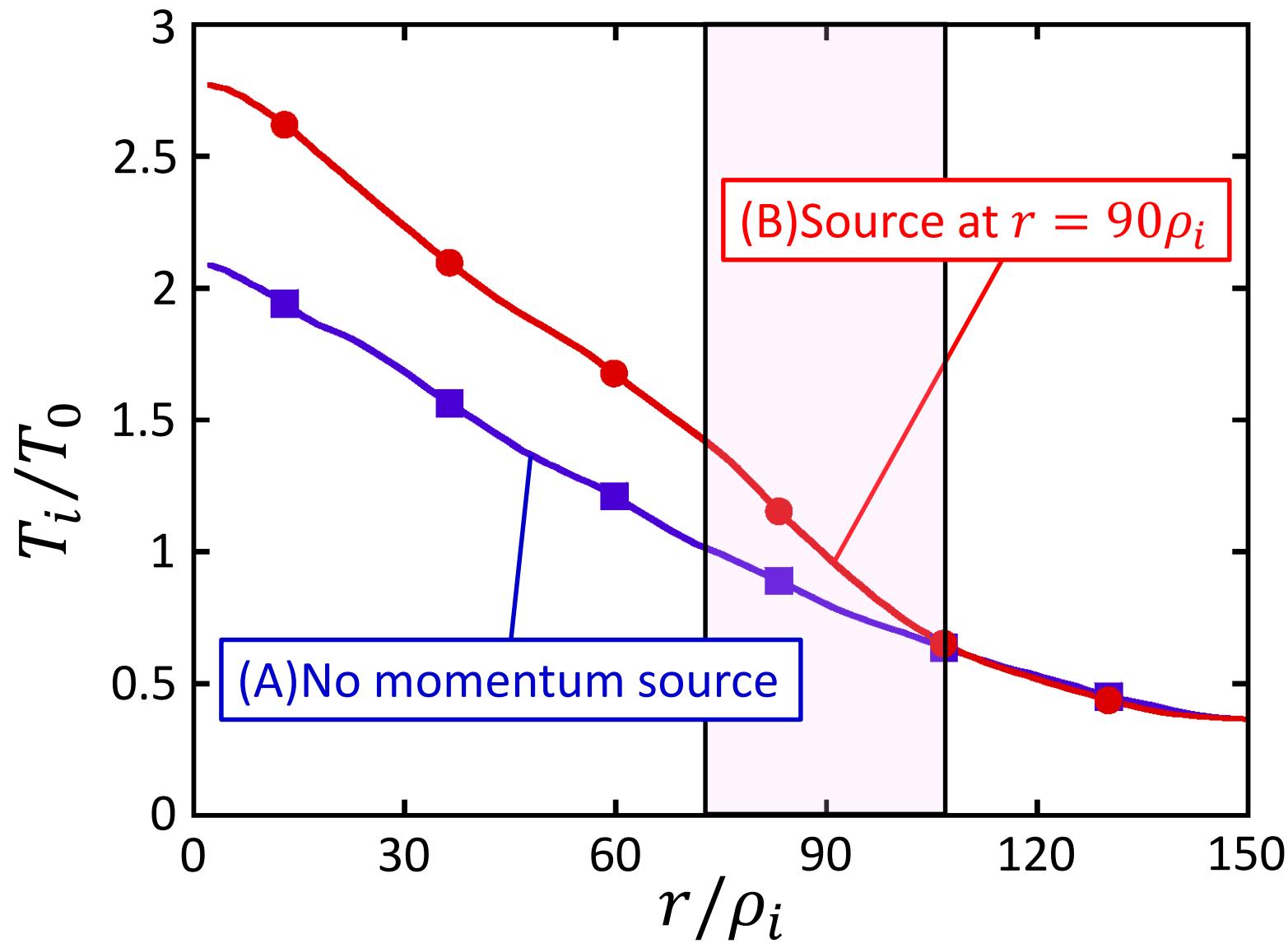
$$S_M = \tau_M^{-1} A(r) [f_{LM}(n_0, 0.5\nu_{ti}, T_0) - f_{LM}(n_0, 0, T_0)]$$

$$f_{LM}(n, U_{||}, T) = \frac{n}{\sqrt{2\pi T^3/m_i^3}} \exp \left[ -\frac{0.5(v_{||} - U_{||})^2 + \mu B}{T/m_i} \right]$$

- ✓ We compare two cases;
  - (A) without momentum source
  - (B) with momentum source at  $r = 90\rho_i$



# Impact of Momentum Source - 1



✓ Strong impact of momentum source at outer region on temperature build up.

# Impact of Momentum Source - 2

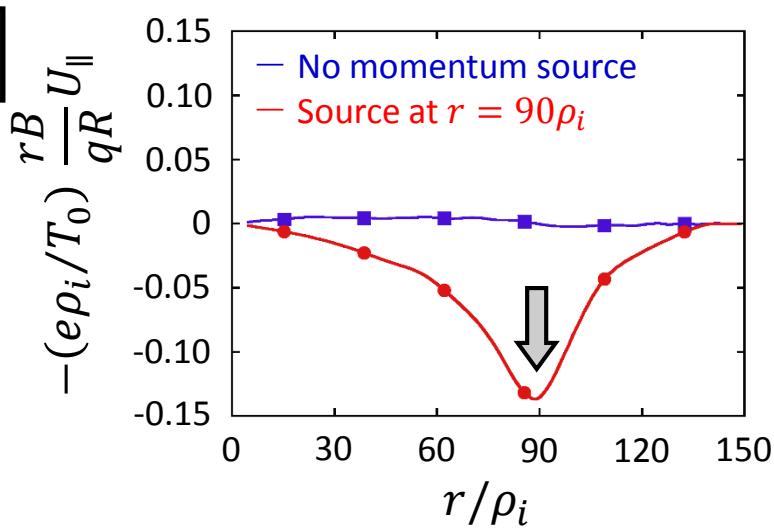
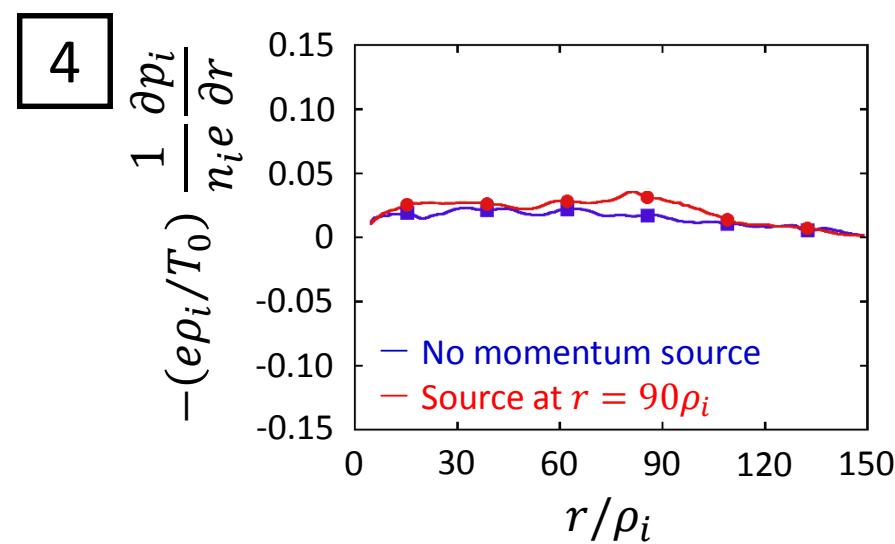
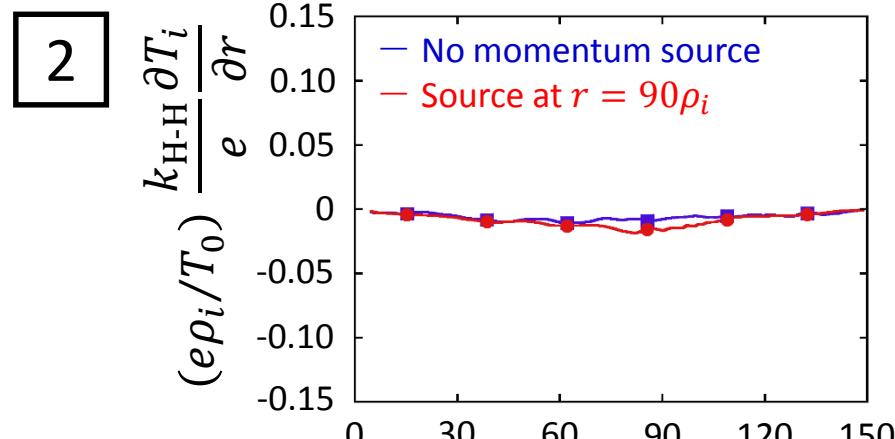
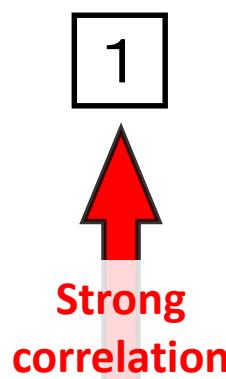
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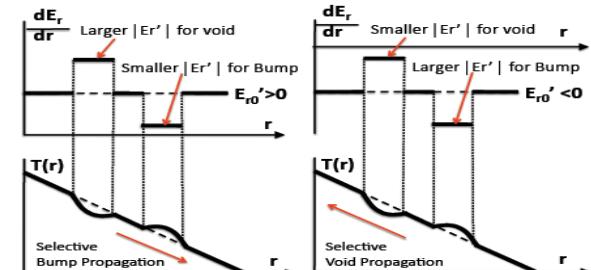
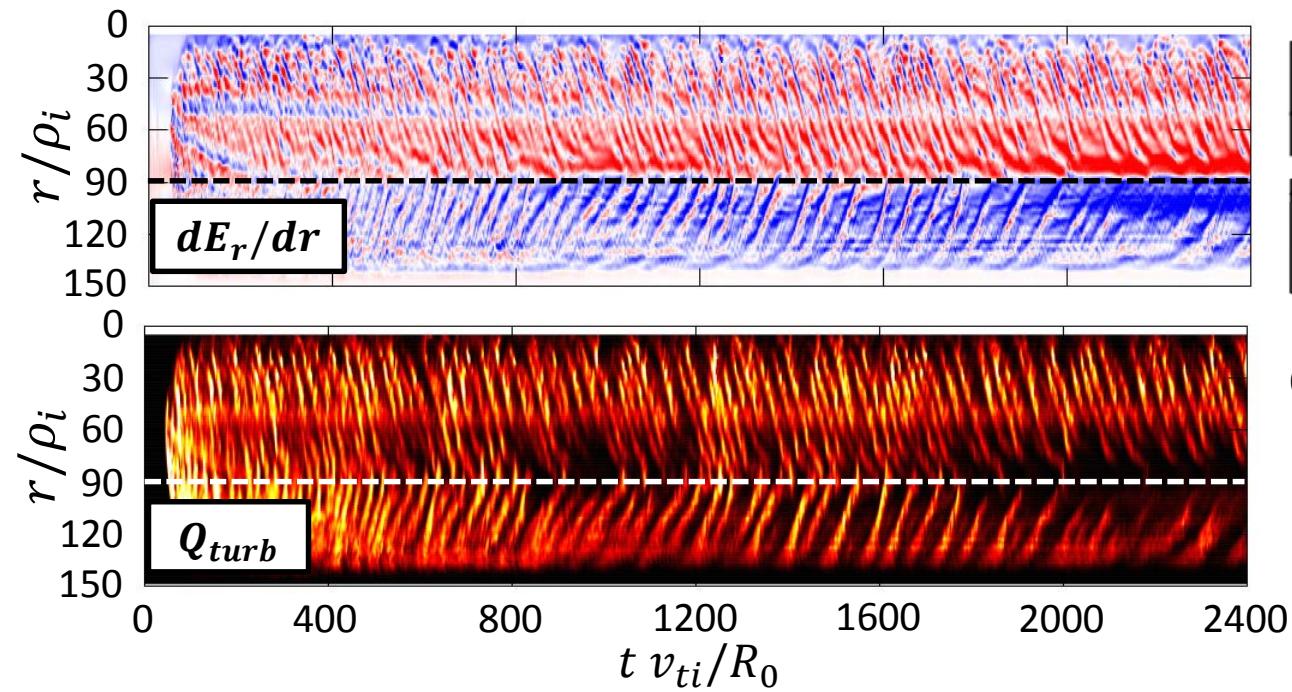
3

4

Radial force balance:  $E_r + \frac{k}{e} \frac{\partial T_i}{\partial r} - \frac{rB}{qR} U_{\parallel} - \frac{1}{n_i e} \frac{\partial p_i}{\partial r} = 0$

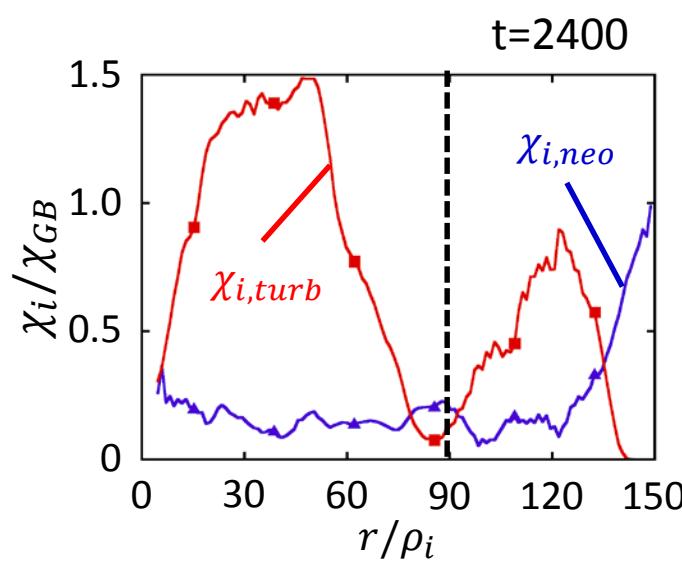
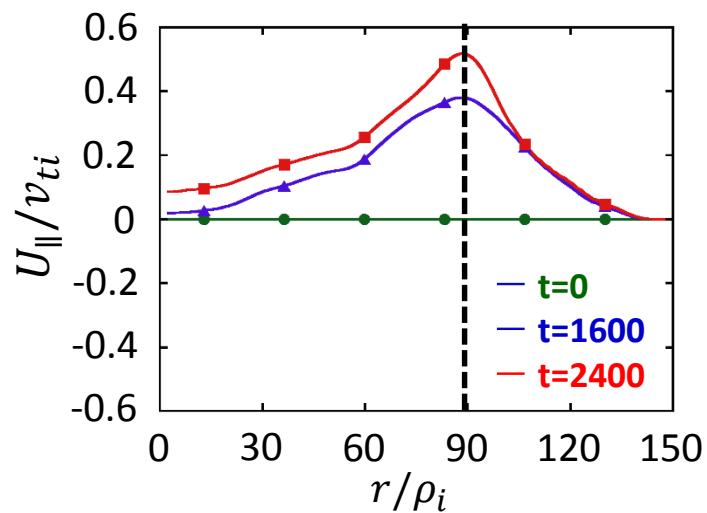


# Impact of Momentum Source - 3

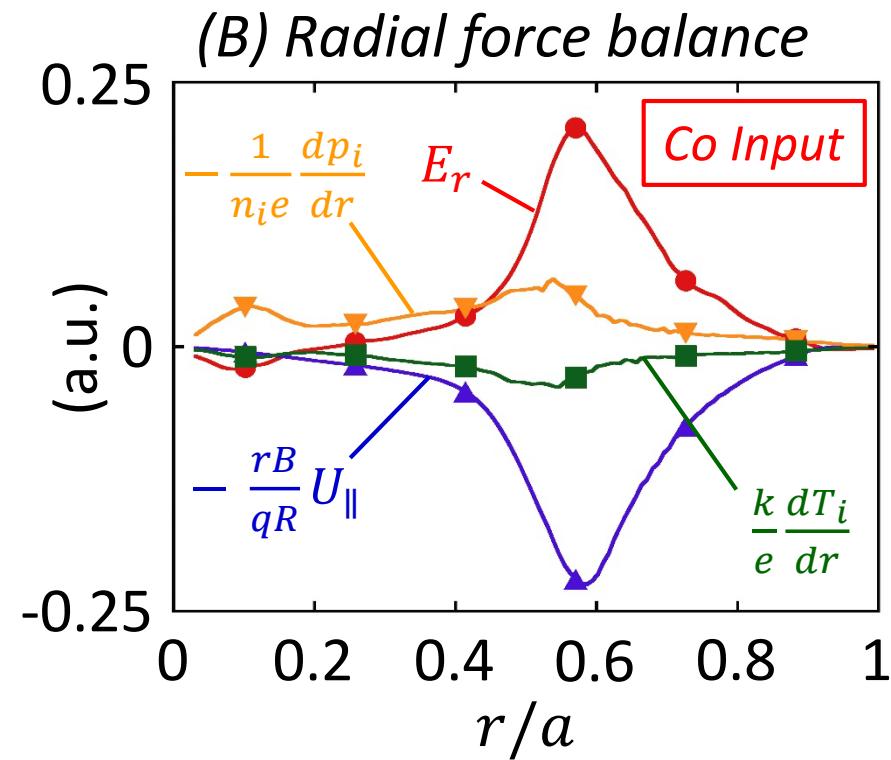
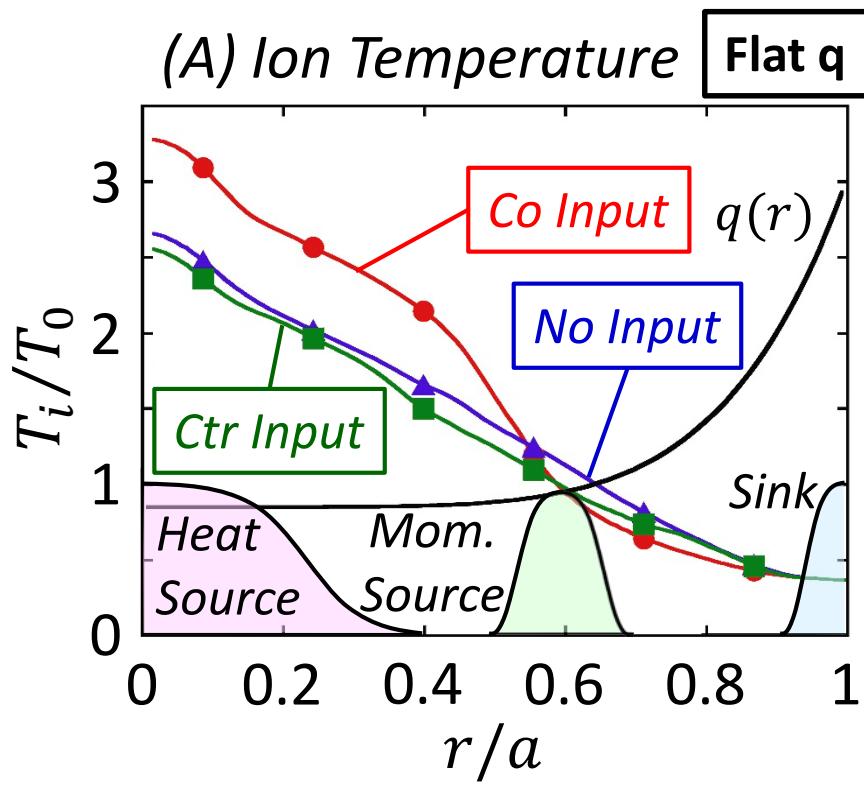


[Y. Idomura, et al. Nucl. Fusion, **49**, 065029 (2009).]  
 [M. Kikuchi and M. Azumi, Rev. Mod. Phys. **84**, 1807 (2012).]

✓ Strong  $E_r$  shear triggered by toroidal rotation in outer region suppresses the turbulence, leading to a **transport barrier formation**.



# Effect of Rotation Direction

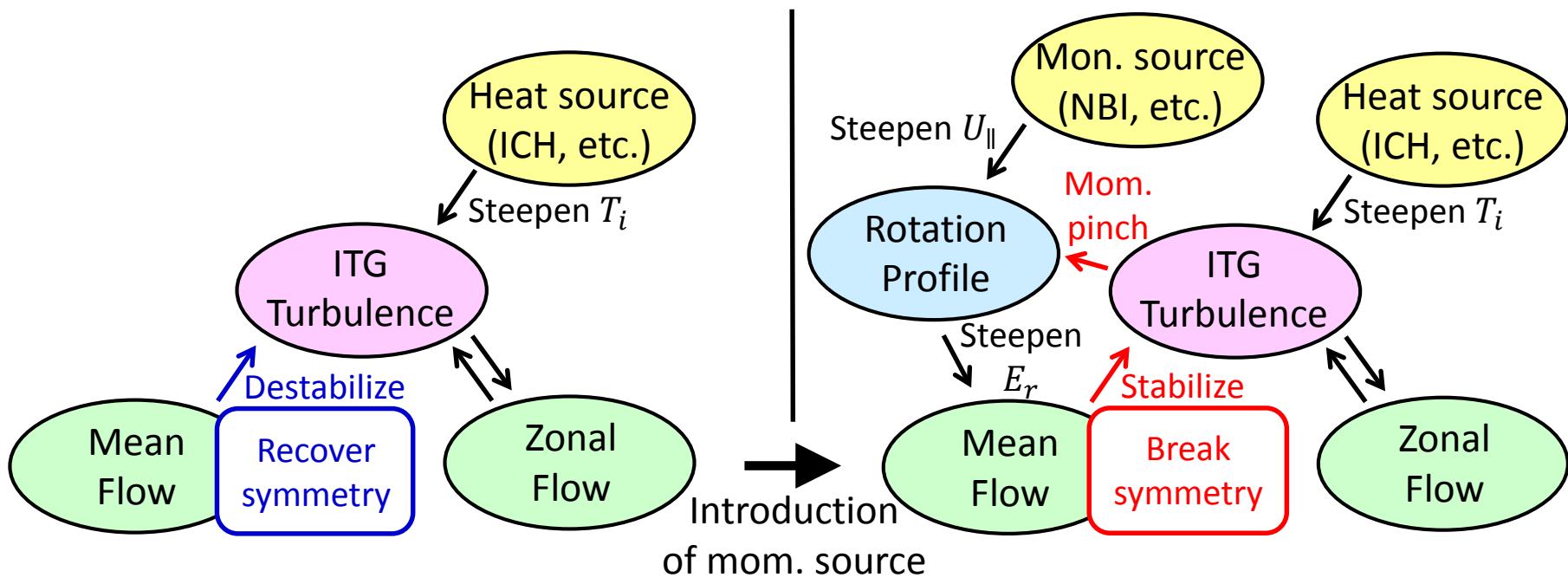


- ✓ Only co-current toroidal rotation can benefit the ITB formation in weak magnetic shear plasma.
- qualitative agreement with the observations in the JET experiment

# Summary

## Summary

- ✓ We have newly developed **5D toroidal full-*f* gyrokinetic code GKNET**.
- ✓ We found that a momentum source can change the mean  $E_r$  through the radial force balance, **leading to ITB formation**.
- ✓ The underlying mechanism is identified to originate from **a positive feedback loop between the enhanced mean  $E_r$  shear and resultant momentum pinch**, which can be observed **only in co-input case**.



# Future Plans

## Future Plans

Flux-driven turbulent transport  
couple with mean flow



ITB formation in  
flux-driven ITG turbulence



Introduction of  
kinetic electron

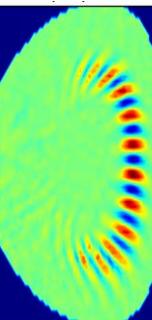


Fig. Poloidal structure  
of global toroidal ITG  
mode in positive (left)  
and negative (right) D  
shape.

(Yoshida)

(Kevin)

Magnetic shaping effect  
on ZF/GAM dynamics



ITB formation in  
flux-driven ITG/TEM turbulence

Magnetic shaping effect  
on ITG/TEM instability

- ✓ Opposite ballooning angle
- ✓ Density transport
- ✓ Momentum transport



- ✓ Impact of elongation and  
triangularity on ITG/TEM  
turbulence



Control of barrier formation by multi-sources and magnetic shape