Neoclassical transport and flow analysis in Heliotron J plasmas

Graduated School of Energy Science
Kyoto university

Kenji Nishioka

1:Background and Objective

Realization of Fusion energy

Analysis of plasma transport is required

Radial transport : related to the loss of particle and heat

Parallel transport : affects to the magnetic geometry

Experimental researches about the plasma transport is carried out in many devices

Experimental research about the parallel transport (flow) in Heliotron J

C^{6+} flow measurement by CXRS
-> controllability of the parallel flow by the magnetic configuration

Physical mechanism?
(neoclassical or anomalous?)

We apply the consistent NC transport theory to analyze the plasma flow and viscosity in Heliotron J

2:Analysis method

NC transport : caused by the distortion of the distribution function from Maxwellian

$$f_a = f_{aM} + f_{a1}$$

Moment method (approximate the f_{a1} by polynomial expansion) is applied in this research

Equation for the f_{a1} -> **The drift kinetic equation (DKE)**

$$\boxed{(V_{\parallel} + V_E) f_{a1} - C_a^L(f_{a1}) = -\mathbf{v}_{da} \cdot \nabla f_{aM} + \frac{e_a}{T_a} v_{\parallel} B \frac{\langle B E_{\parallel} \rangle}{\langle B^2 \rangle} f_{aM}}$$

Integration of DKE -> parallel momentum balance

$$\left[-\begin{bmatrix}\mathbf{M}_a & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \mathbf{M}_N\end{bmatrix}+\left\langle B^2\right\rangle\begin{bmatrix}\mathbf{\Lambda}_{aa} & \cdots & \mathbf{\Lambda}_{aN}\\ \vdots & \ddots & \vdots\\ \mathbf{\Lambda}_{Na} & \cdots & \mathbf{\Lambda}_{NN}\end{bmatrix}\right]\cdot\begin{bmatrix}\mathbf{U}_a\\ \vdots\\ \mathbf{U}_N\end{bmatrix}=\begin{bmatrix}\mathbf{N}_a & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \mathbf{N}_N\end{bmatrix}\cdot\begin{bmatrix}\mathbf{X}_a\\ \vdots\\ \mathbf{X}_N\end{bmatrix}-\begin{bmatrix}\mathbf{Z}_a\\ \vdots\\ \mathbf{Z}_N\end{bmatrix}\left\langle BE_\parallel\right\rangle$$

Viscosity damping **Friction damping** **Thermodynamic force driven** **Inductive Electric field**

$\mathbf{M}_a$ Parallel viscosity matrix

$\mathbf{N}_a$ Viscosity matrix due to thermodynamic force

$\mathbf{\Lambda}_{ab}$ Parallel friction matrix

$\mathbf{U}_a$ Moments of parallel flow

$\mathbf{X}_a$ Thermodynamic force (radial gradient of pressure and electrostatic potential)

Radial electric field (Er) dependence of M_a and N_a ➡ **Consistent E_r** is required

Radial flux - parallel flow relation

$$\begin{bmatrix}\Gamma_a\\ -q_a/T_a\end{bmatrix}=\begin{bmatrix}\mathbf{N}_a{}^{\mathrm{Tr}} & \mathbf{L}_a\end{bmatrix}\cdot\begin{bmatrix}\mathbf{U}_a\\ \mathbf{X}_a\end{bmatrix}$$

$\mathbf{L}_a$ Neoclassical diffusion matrix

$\Gamma_a, q_a/T_a$ Neoclassical particle and heat flux

E_r which satisfies $\sum e_a\Gamma_a(E_r)=0$ ➡ Consistent analysis with ambipolar E_r

Matrices L_a, M_a, and N_a

-> obtained from the numerical solution of the DKE

However…

Accuracy of the solutions degrades in collisional, collisionless, and strong E_r limits

Therefore…

combination of numerical and analytical solutions

-> solution in arbitrary collision frequency and E_r

We apply this moment method to analyze

1: parallel ion flow in NB heated plasmas

2: effect of E_r and friction on the bootstrap current

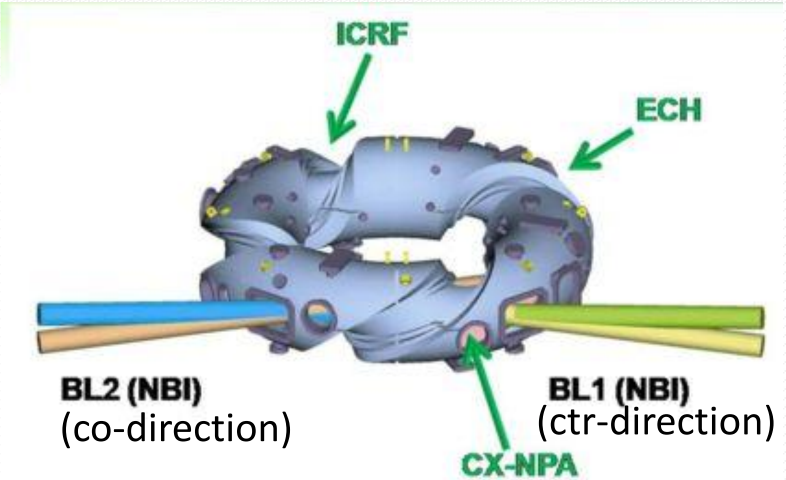
3:Ion flow analysis in NB-heated plasmas

Experiments: **C^{6+} flow** in the same direction as injection direction of the neutral beams is observed

➡ **Apply the moment method estimation for the clarification of contribution of NC component**

Application to NB heated plasmas

Effect of external momentum should be taken into account



NBI systems on Heliotron J

Modified DKE

$$\boxed{v_{\|}\mathbf{b}\cdot\nabla f_{a1}-\left(\sum_{b\neq \mathrm{f}}C_{ab}\left(\langle f_{a\mathrm{M}}\rangle, f_{b1}\right)+\sum_{b\neq \mathrm{f}}C_{ab}\left(f_{a1},\langle f_{b\mathrm{M}}\rangle\right)+C_{ab}\left(\langle f_{a\mathrm{M}}\rangle, f_{\mathrm{f}}\right)\right) = -\mathbf{v}_{\mathrm{d}a}\cdot\nabla f_{a\mathrm{M}}+\frac{e_a}{T_a}v_{\|}B\frac{\langle BE_{\|}\rangle}{\langle B^2\rangle}f_{a\mathrm{M}}}$$

taking velocity moments

Modified parallel momentum balance equation

$$\left[-\begin{bmatrix}\mathbf{M}_a & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \mathbf{M}_N\end{bmatrix}+\langle B^2\rangle\begin{bmatrix}\mathbf{\Lambda}_{aa} & \cdots & \mathbf{\Lambda}_{aN}\\ \vdots & \ddots & \vdots\\ \mathbf{\Lambda}_{Na} & \cdots & \mathbf{\Lambda}_{NN}\end{bmatrix}\right]\cdot\begin{bmatrix}\mathbf{U}_a\\ \vdots\\ \mathbf{U}_N\end{bmatrix}=\begin{bmatrix}\mathbf{N}_a & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \mathbf{N}_N\end{bmatrix}\cdot\begin{bmatrix}\mathbf{X}_a\\ \vdots\\ \mathbf{X}_N\end{bmatrix}-\begin{bmatrix}\mathbf{Z}_a\\ \vdots\\ \mathbf{Z}_N\end{bmatrix}\langle BE_{\|}\rangle-\begin{bmatrix}\mathbf{C}_a\\ \vdots\\ \mathbf{C}_N\end{bmatrix}\langle BF_{\|1}\rangle$$

External source term

($\mathbf{C}_a$: momentum transfer ratio, $\langle BF_{||f1}\rangle$: total momentum input)

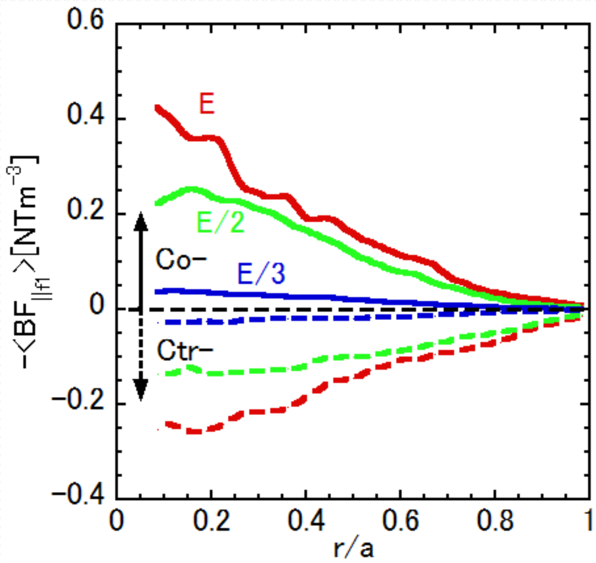
External source also affects to the flow-driven radial flux and resulting ambipolar E_r

Plasma parameters

Mix ratio of the plasma	$n_{\mathrm{e}}:n_{\mathrm{D}}:n_{\mathrm{C}}=1:0.82:0.03$
Density profile	$n_{\mathrm{e}}(r)=1.5\times10^{19}(1-(r/a)^2)\mathrm{m}^{-3}$
T_e profile	$T_e=300(1-(r/a)^2)\mathrm{eV}$
T_i profile	$T_I=175(1-(r/a)^{1.57})^{1.11}\mathrm{eV}$
Major and minor radius	$R=1.2\mathrm{m},\ r=0.16\mathrm{m}$

External momentum sources
(Obtained by FIT3d code)

solid : co- dashed : ctr-



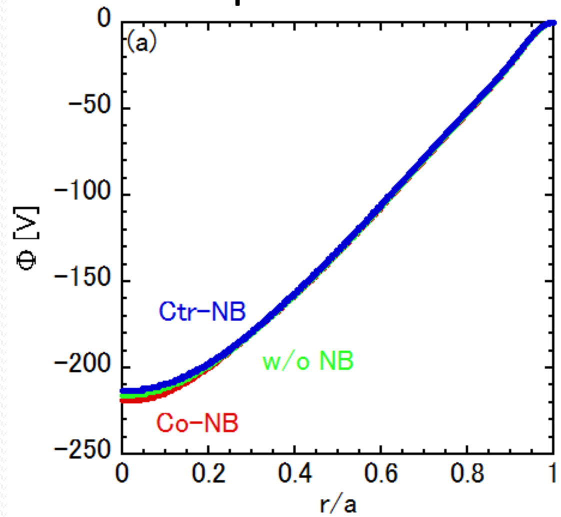
Parallel ion flow estimation

External momentum

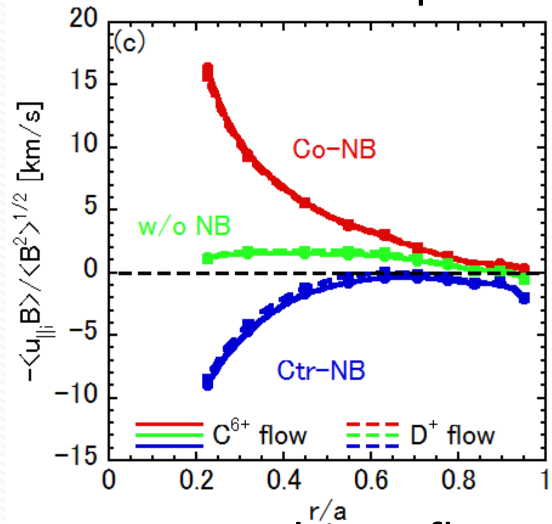
Flow: significantly influenced

Ambipolar E_r: slight change

Ambipolar condition



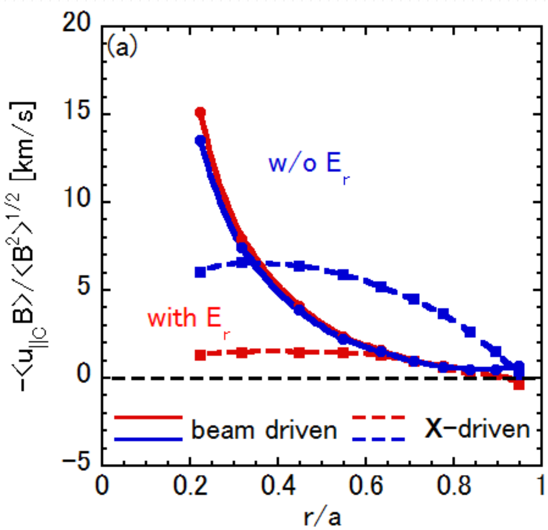
Ion flow in NB plasma



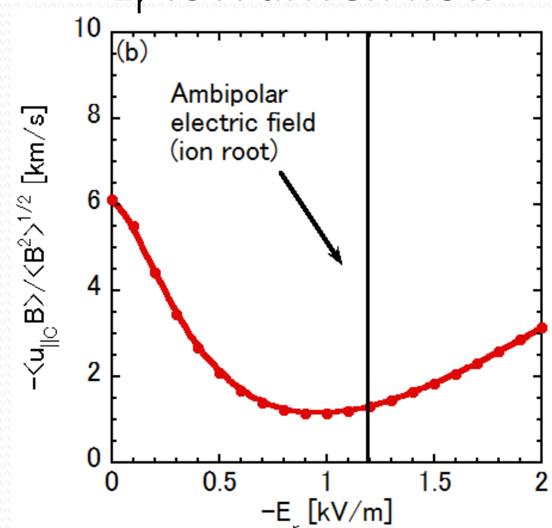
Thermodynamic force driven (**X driven) flow strongly depends on E_r**

->Necessity of considering ambipolar condition

beam and X driven flow



E_r vs X driven flow



Analysis of mechanism of parallel ion flow

Momentum balance for particle species *a*

$$\mathbf{M}_a \cdot \mathbf{U}_a - \left\langle B^2 \right\rangle \sum_b \left(\mathbf{\Lambda}_{ab} \cdot \mathbf{U}_b \right) + \mathbf{N}_a \cdot \mathbf{X}_a - \mathbf{C}_a \left\langle BF_{\|f1} \right\rangle = 0$$

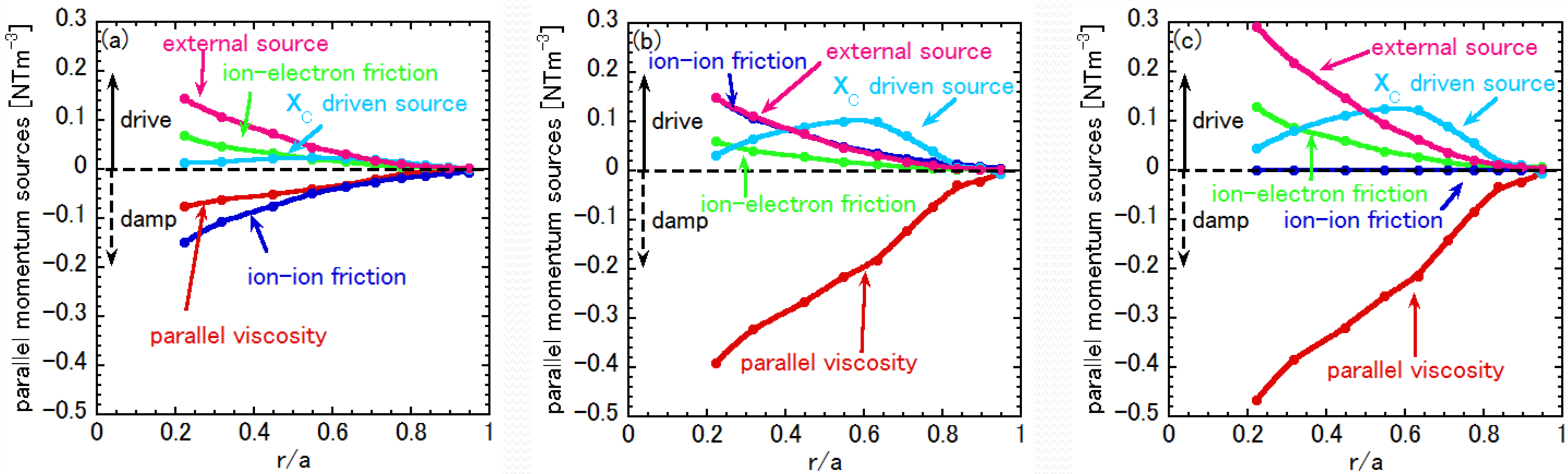
Parallel viscosity

Friction with electron
+ Friction with ion

X driven source

External source

Carbon | Deuterium | Total ion



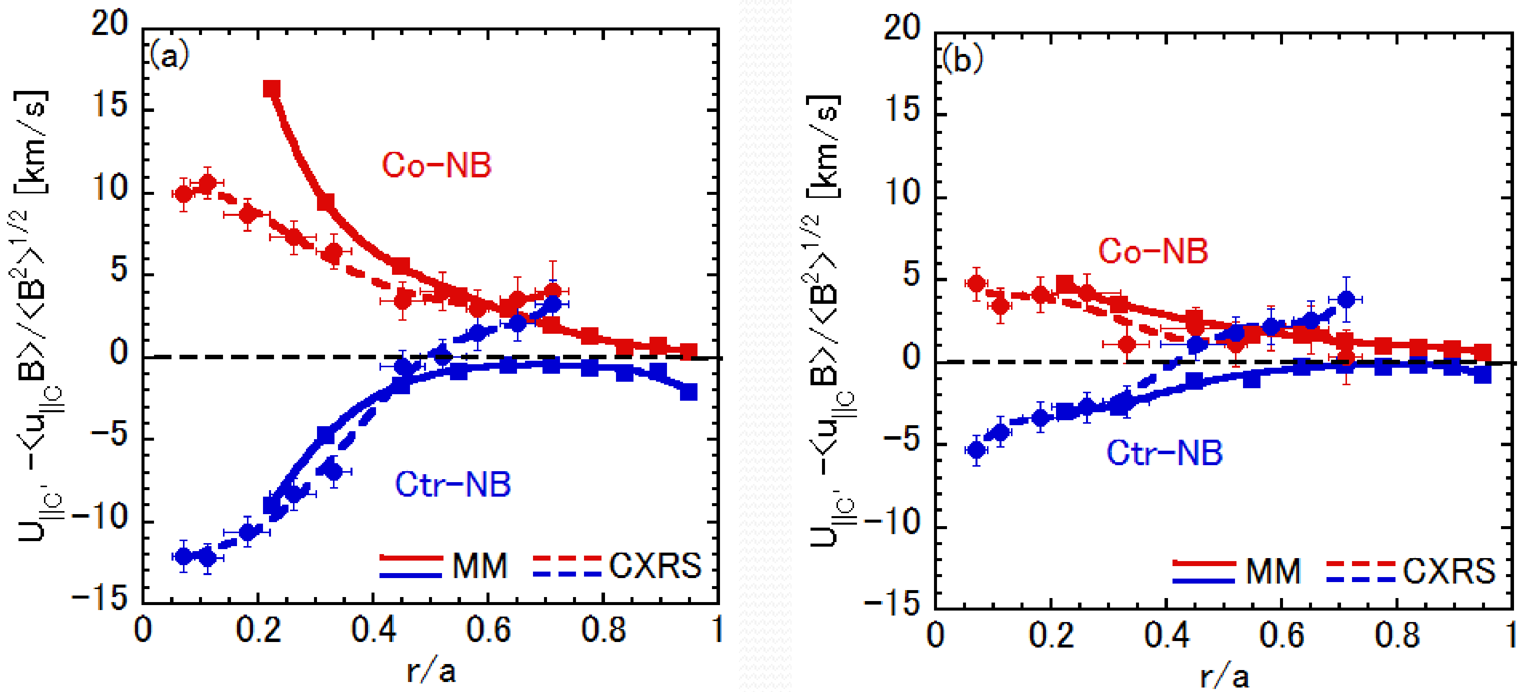
Strong ion-ion friction -> **small difference between $u_{||D}$ and $u_{||C}$**

r/a < 0.5 -> **beam driven flow** is dominant

r/a > 0.5 -> **X driven flow** is dominant

Carbon flow analysis

Std config. High mirror config.



- Experimental results do not contradict with the NC prediction
  -> **No clear evidence of anomalous viscosity**
- Suppression of C^{6+} flow in the high mirror config. is predicted both in experiments and calculation
  -> this suppression is caused by **the strong magnetic ripple**

4:Bootstrap current analysis

Experiments: Controllability of the BS current by **changing the bumpy field component**

Confirmed by the numerical simulation by using the BSC code

Expression of the BS current in the BSC code

$$\boxed{\left\langle j_{\parallel \mathrm{BS}} B \right\rangle \sim -G_{\mathrm{e}}^{(BS)} \left(L_1 \left(\frac{dp_{\mathrm{e}}}{dr} + e_{\mathrm{e}} n_{\mathrm{e}} \frac{d\Phi}{dr} \right) + L_{2\mathrm{e}} n_{\mathrm{e}} \frac{dT_{\mathrm{e}}}{dr} \right) - G_{\mathrm{i}}^{(BS)} \left(L_1 \left(\frac{dp_{\mathrm{i}}}{dr} + e_{\mathrm{i}} n_{\mathrm{i}} \frac{d\Phi}{dr} \right) + L_{2\mathrm{i}} n_{\mathrm{i}} \frac{dT_{\mathrm{i}}}{dr} \right)}$$

($\langle j_{||BS}B \rangle$: The BS current density, $G^{(BS)}$: The geometrical factor of the BS current
L_{ij}: coefficient determined by the friction and the viscosity)

$G^{(BS)}$: the connection formula of the analytical solution

-> E_r dependence of the $G^{(BS)}$ is not included

We estimate the effect of the radial electric field and the friction on the BS current

Expression of the BS current by the moment method

Parallel momentum balance

$$\left[-\begin{bmatrix}\mathbf{M}_a & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \mathbf{M}_N\end{bmatrix}+\left\langle B^2\right\rangle\begin{bmatrix}\mathbf{\Lambda}_{aa} & \cdots & \mathbf{\Lambda}_{aN}\\ \vdots & \ddots & \vdots\\ \mathbf{\Lambda}_{Na} & \cdots & \mathbf{\Lambda}_{NN}\end{bmatrix}\right]\cdot\begin{bmatrix}\mathbf{U}_a\\ \vdots\\ \mathbf{U}_N\end{bmatrix}=\begin{bmatrix}\mathbf{N}_a & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \mathbf{N}_N\end{bmatrix}\cdot\begin{bmatrix}\mathbf{X}_a\\ \vdots\\ \mathbf{X}_N\end{bmatrix}-\begin{bmatrix}\mathbf{Z}_a\\ \vdots\\ \mathbf{Z}_N\end{bmatrix}\left\langle BE_{\parallel}\right\rangle$$

Flow vector in proton plasma

$$\begin{bmatrix}\mathbf{U}_{\mathrm{e}}\\ \mathbf{U}_{\mathrm{H}}\end{bmatrix}=[\mathbf{G}][\mathbf{X}]=\begin{bmatrix}\mathbf{G}_{\text{e-e}} & \mathbf{G}_{\text{e-H}}\\ \mathbf{G}_{\text{H-e}} & \mathbf{G}_{\text{H-H}}\end{bmatrix}\cdot\begin{bmatrix}\mathbf{X}_{\mathrm{e}}\\ \mathbf{X}_{\mathrm{H}}\end{bmatrix}$$

$$\left([\mathbf{G}]=\left[-\begin{bmatrix}\mathbf{M}_{\mathrm{e}} & 0\\ 0 & \mathbf{M}_{\mathrm{H}}\end{bmatrix}+\left\langle B^2\right\rangle\begin{bmatrix}\mathbf{\Lambda}_{\mathrm{ee}} & \mathbf{\Lambda}_{\mathrm{eH}}\\ \mathbf{\Lambda}_{\mathrm{He}} & \mathbf{\Lambda}_{\mathrm{HH}}\end{bmatrix}\right]^{-1}\cdot\begin{bmatrix}\mathbf{N}_{\mathrm{e}} & 0\\ 0 & \mathbf{N}_{\mathrm{H}}\end{bmatrix}\right)$$

If friction can be ignored

$$\frac{\left\langle u_{\parallel a0}B\right\rangle}{\left\langle B^2\right\rangle}\sim-\frac{N_a(K)}{M_a(K)}\left(X_{a1}-X_{a2}\right)=\frac{G^{(BS)*}}{e_a\left\langle B^2\right\rangle}\left(X_{a1}-X_{a2}\right)$$

$$G^{(BS)*}=-N_{a11}/M_{a11}$$

$$\left\langle j_{\parallel\mathrm{BS}}B\right\rangle=\sum_a e_a n_a\left\langle u_{\parallel a0}B\right\rangle \qquad \frac{dJ_{BS}}{ds}=\frac{\left\langle j_{\parallel\mathrm{BS}}B\right\rangle}{\left\langle B^2\right\rangle}2\pi\frac{d\varphi}{ds}$$

$G^{(BS)}$ only

Full matrix

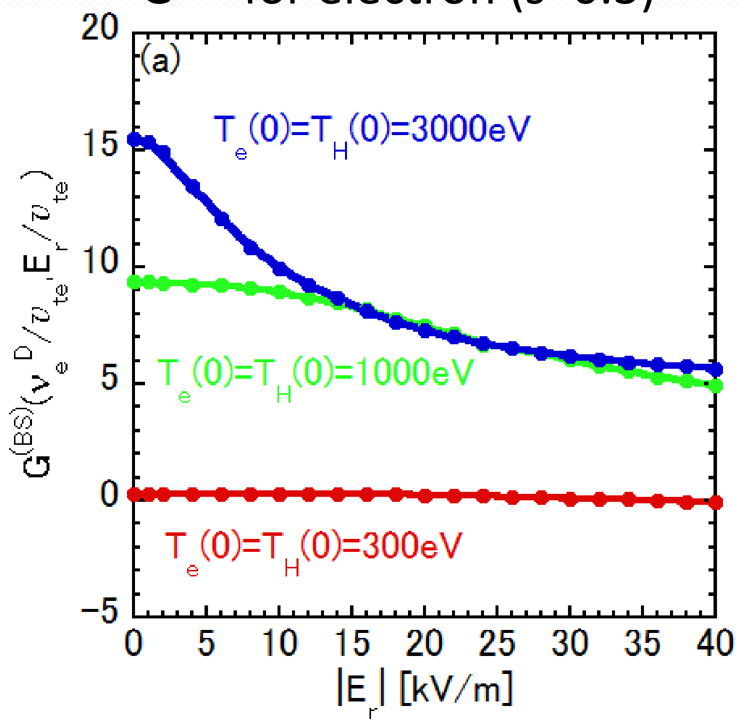
Diagonal (set $\mathbf{G}_{\text{e-H}}=\mathbf{G}_{\text{H-e}}=0$)

Estimate the friction

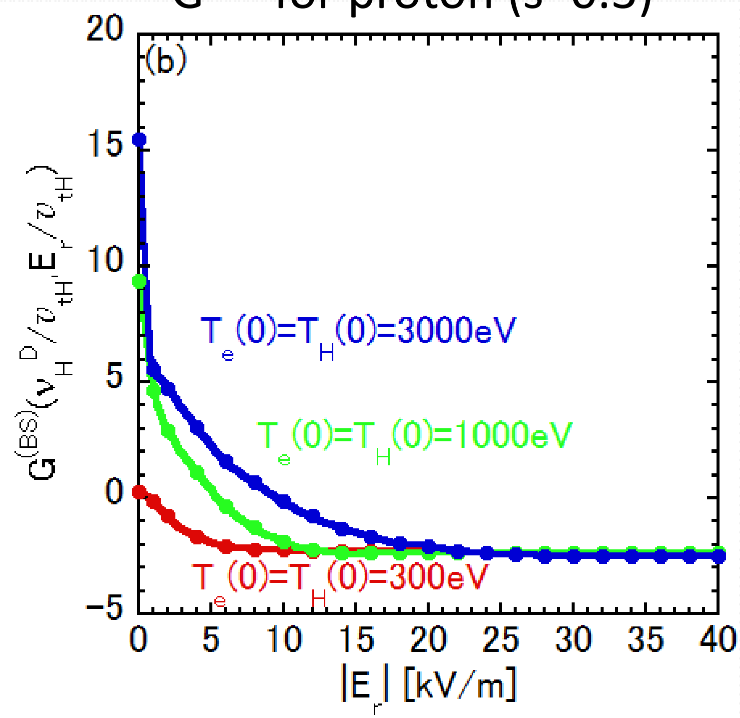
E_r dependence of the $G^{(BS)}$

Plasma parameter $n_e(s) = n_i(s) = 1.5\times10^{19}(1-s)\mathrm{m}^{-3}$ $T_e(s) = T_i(s) = T_0(1-s)\mathrm{eV}$

$G^{(BS)}$ for electron (s=0.3)



$G^{(BS)}$ for proton (s=0.3)



Change in $G^{(BS)}$ by the E_r is predicted

Remarkable change : high temp. and ion at small E_r

Parallel flow and radial flux estimation

Nonlinear change in particle flow on E_r

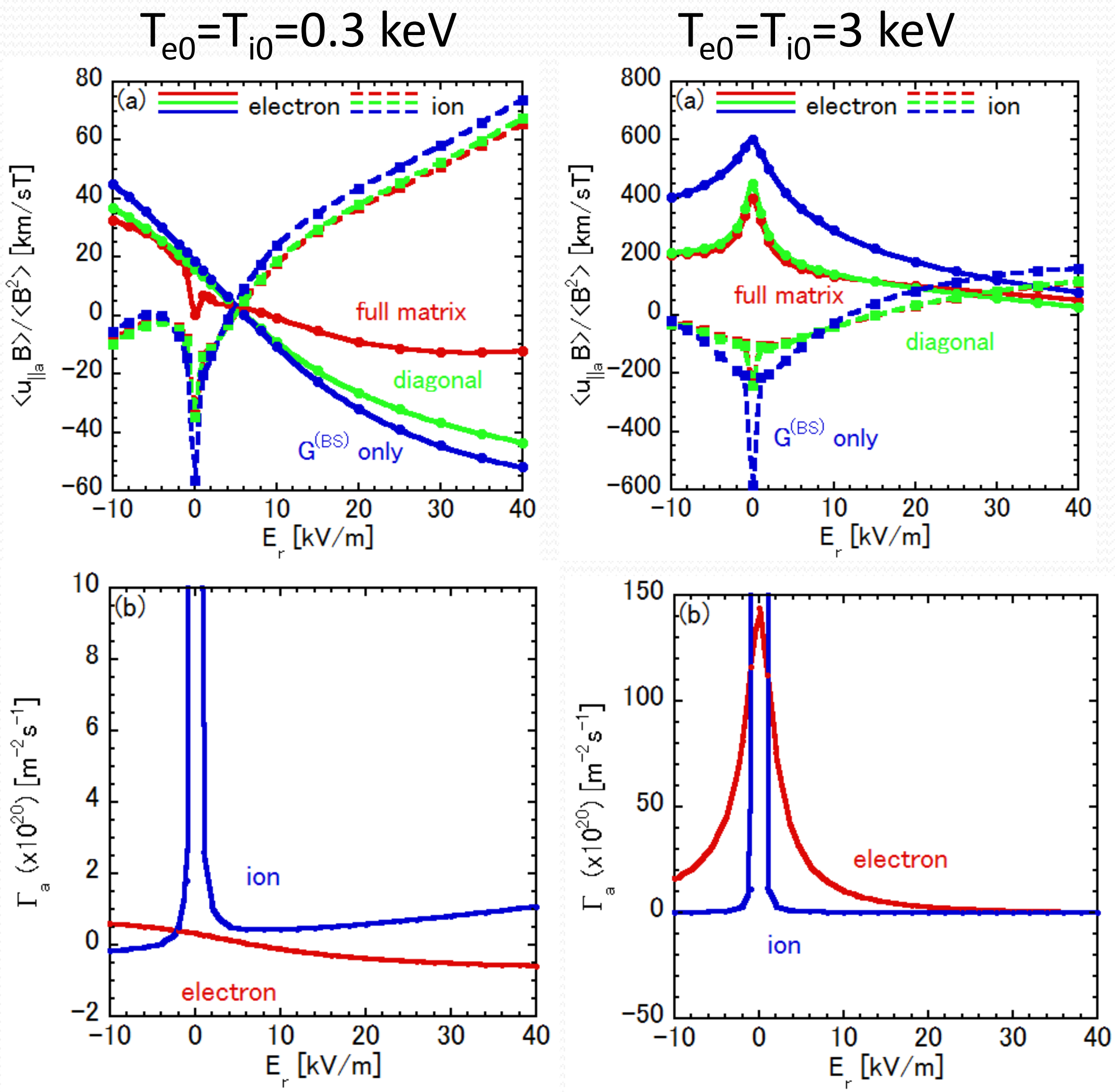
-> Due to **E_r dependence of the $G^{(BS)}$**

Obviously change in $u_{||e}$ among full matrix, diagonal, and $G^{(BS)}$ only

->strong **electron-ion friction**

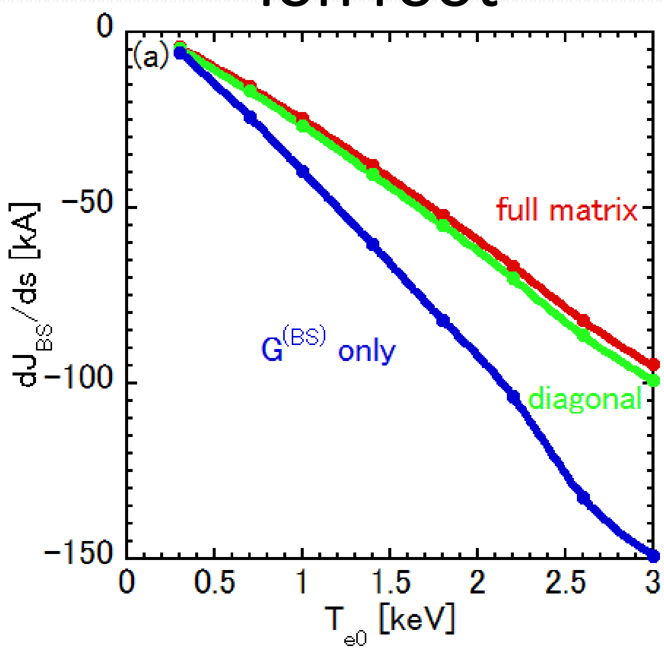
Prompt change in $u_{||a}$ in small E_r region

-> related to the radial NC particle fluxes

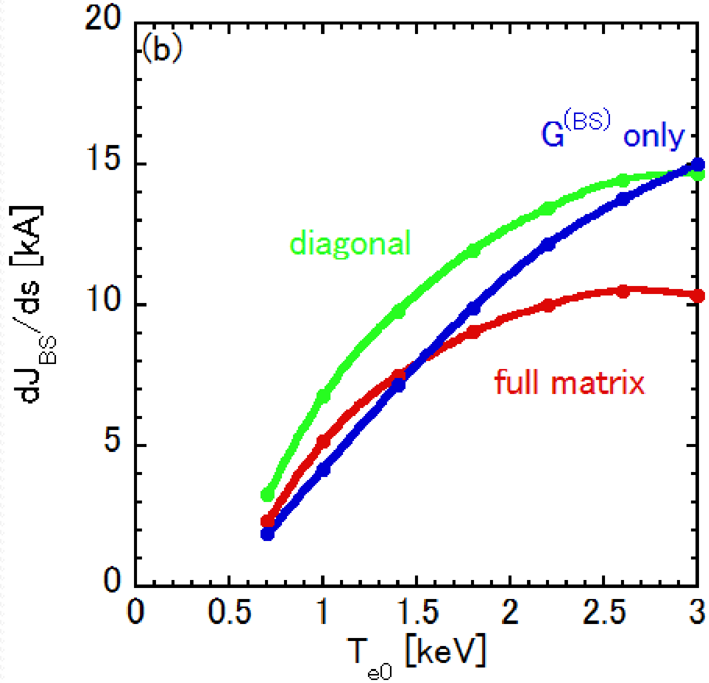


Effects of the friction and the E_r on the BS current

Ion root



electron root



Opposite direction of the BS current in electron roots

Ion root -> diagonal term is dominant

Electron root -> difference between "diagonal" and "full matrix" is predicted

5. Summary

- The moment method estimation is applied to the neoclassical transport and flow analysis in Heliotron J
- Inclusion on the external momentum source term enables us to estimate the neoclassical parallel flow and its mechanism in NB heated plasmas consistently
- Experimentally observed C^{6+} flow does not contradict with the neoclassical prediction and suppression of this due to strong magnetic ripple is shown both in the experiment and calculation
- Effect of the E_r on the BS current through the change in the geometric factor $G^{(BS)}$ and the suppression of the BS current due to electron-ion friction is predicted