The effect of magnetic shaping on zonal flow damping in a toroidal global gyrokinetic simulation

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Global gyrokinetic (GK) global simulation is considered to be an essential tool to understand micro-scale instability and associated turbulent transport phenomena including the profile stiffness / resilience and transport barrier formation. While many GK codes already exist, most of them rely on constrainning hypotheses such as a circular section (despite the D-shape of actual tokamak such as ITER) to simplify the equation systems. Our 3D full toroidal GK Vlasov simulation code, GKNET, has been upgraded, with the addition to its real space field solver of a new high accuracy ZF solver, based on a diagonalisation of the ZF equation. In addition to being more rigorous near the center of the poloidal plane compared to those local approximations, a solver based, this method allows for accurate results on low resolution grids. This upgraded code was used to study GAM damping in elliptic and both positive and negative D-shaped configurations. While the influence of elongation had been partially studied, we introduce new results on the influence of triangularity on the damping rate, showing an asymmetry allowing negative triangularities to damp the ZF faster.

Gyrokinetics formalism

The Vlasov-Poisson system is expressed as:

\[ \frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + q \mathbf{E} \times \mathbf{B} \cdot \nabla f = 0 \quad (\text{Vlasov equation}) \]

\[ \Delta \Phi = -\frac{\nabla \cdot (\mathbf{E} \times \mathbf{B})}{\mu_0} \quad (\text{Poisson equation}) \]

where \( f \) is the distribution function describing the plasma's configuration in the 6-dimensional phase space and \( \mathbf{E} \) is the electric field.

The particles' behavioural trajectories around the magnetic field lines can be described through the following characteristics: for each particle with speed \( v \), the magnetic moment \( \mathbf{p} = q \mathbf{v} \) with \( q \) the charge, the gyromagnetic ratio \( \gamma = \frac{q}{m} \), and the angle \( \alpha \) of the gyration

\[ (\mathbf{v}, v, p, \alpha) \rightarrow (\mathbf{R}, v, p, \alpha) \]

As the gyration radius is very small compared to the characteristic scale of the equilibrium structures, the variables can be averaged in \( \alpha \) (over these circles) reducing the number of dimensions to 5. However, this transformation yields values in the Poisson equation due to the 6th term whose computation can be difficult.

Derivation of the model

The gyrokinetic Vlasov equation which describes the evolution of the guiding center distribution \( f_c \) of the species concerned is derived using Hamiltonian mechanics as:

\[ \frac{\partial f_c}{\partial t} + \mathbf{v} \cdot \nabla f_c + \frac{q}{2m} \nabla \Phi \times \mathbf{B} \cdot \nabla f_c = 0 \quad (\text{gyrokinetic Vlasov equation}) \]

The electrostatic potential \( \Phi \) is given by the GK quasi-neutrality condition reads:

\[ \Delta \Phi = -\nabla \cdot (\mathbf{E} \times \mathbf{B}) \quad (\text{Poisson equation}) \]

and where \( \frac{\partial (\Phi)}{\partial \mathbf{r}} \) denotes the simple gyro-averaging (depending on the variable's initial configuration space) and the term \( \nabla \cdot (\mathbf{E} \times \mathbf{B}) \) is the double averaging, defined as:

\[ \int \frac{\partial \Phi}{\partial \mathbf{r}} f_c(x) \delta_R(R + \mathbf{p} \cdot \mathbf{r}) dR d\mathbf{r} \]

where \( f_c \) is the Maxwellian in \( \mathbf{r} \), slowly varying in \( R \) such \( f_c \) dependence is of the form \( f_c(R, \mathbf{0}, v, \mathbf{0})/f_c(R, \mathbf{0}) \) (i.e. a Gaussian distribution of variance the thermal velocity \( \gamma v_\text{th} \)).

The flux averaged term plays an important role in the damping of zonal flow. The Zonal Flow equation is obtained by computing the flux average of the field equation eq. 2:

\[ \frac{\partial f_c}{\partial t} + \mathbf{v} \cdot \nabla f_c + \frac{q}{2m} \nabla \Phi \times \mathbf{B} \cdot \nabla f_c = 0 \quad (\text{Zonal Flow equation}) \]

Real space gyro-averagings

Simple and double averages in real space

Rather than a theoretical simplification of its expression, the simple averaging is computed by sampling a given number of points \( M \) on circular orbits (black circle and green points on top figure on the right).

The double averaging being the composition of 2 simple averages, the averaging is here performed over \( M^2 \) “secondary samples” sampled on “secondary” circular orbits centered on each “primary point” (the blue and red points on the grey circles on the same figure). The integration over \( v \), (i.e. in radius), is computed as a weighted sum of double averages for given radius, the weights and radii being computed numerically to minimize the error.

The error of the 2 to 3-D interpolations and of the sampling on the circles can be estimated theoretically. In the latter case, for a given mode \( k_z \) in Cartesian coordinates, using \( M \) points will yield an error of the order of \( 2M_2(k_z)/M_3(k_z) \) for \( M \) odd and \( 2M_2(k_z)/M_3(k_z) \) for \( M \) even.

Resolution of the ZF equation by diagonalisation

To study the ZF equation eq. 3, expanding an arbitrary flux function \( \Phi \) around the magnetic axis, we can derive the parameterisation of the D-shaped magnetic flux surfaces.

\[ \frac{\partial f_c}{\partial t} + \mathbf{v} \cdot \nabla f_c + \frac{q}{2m} \nabla \Phi \times \mathbf{B} \cdot \nabla f_c = 0 \quad (\text{Zonal Flow equation}) \]

where \( \alpha \) is the elongation, \( \delta \) is the triangularity, \( \Delta \) is the Shafranov shift and \( \mathbf{p} \) is a linear vector of flux surfaces (simply equivalent to the radius in circular cases) ranging from 0 to \( R_0 \). Using these coordinates, we can establish the equations verified by eigenfunctions:

\[ M_1(\alpha) \frac{\partial f_c}{\partial t} + M_2(\alpha) \frac{\partial f_c}{\partial \mathbf{r}} + M_3(\alpha) \frac{\partial f_c}{\partial R} = 0 \quad (\text{Zonal Flow equation}) \]

where the coefficients \( G_{1,2,3} \) are \( O(1+\delta+\Delta^2) \). The neglect near the first order derivative leads to a usual Fourier solution but is inaccurate near the center of the poloidal section. The solutions read:

\[ f_c(\alpha) = (1-C_2\delta^2)\mathcal{A}_f \omega_n + \frac{\alpha}{\alpha_{\text{crit}}}(C_2+G_2^2)\mathcal{A}_f \omega_n \]

with the eigenvalue \( \lambda = \frac{\alpha}{\alpha_{\text{crit}}} \) for the ZF equation eq. 3, where \( \mathcal{A}_f \) being the kth zero of \( J_1 \). Using this result, the ZF equation is finally solved by projecting the RHS onto an eigenbasis:

\[ \mathbf{f}_c = \sum_k \lambda_k f_{c,k} \quad (\text{with eigenfunctions}) \]

As the eigenvalue decrease very rapidly, eigenvalues can be restricted to the first few eigenvalues, resulting in very small linear equation systems to solve.

Influence of the elongation and triangularity on the ZF damping

Elongation \( \kappa \) is found to enhance the convergence while increasing the limit of this convergence mostly correlates with the results built on the toroidal level by Y. Xiao (Phys. Plasmas 13, 082507 (2006)) and numerical ones by P. Angulo (Phys. Plasmas 15, 062306 (2008)).

In addition, a theoretical paper by Z. Guo (Phys. Plasmas, 17, 092501 (2010)) also suggest that the damping rate should increase with elongation, although the formula proposed can only be used for tendencies and not for numerical comparison.

Summary and conclusion

Our full toroidal GK Vlasov code, GKNET, has been upgraded, by introducing to its real space field solver of a new ZF solver which can accurately solve the ZF equation based on a diagonalisation of the ZF equation. This new solver allows for accurate numerical results on very low resolution grids.

With this new solver, GAM damping tests were performed to study the influence of the shape of the magnetic field on the residual ZF level, oscillation frequency and damping rate. Results in the influence of the elongation \( \kappa \) of the section confirm numerical and theoretical works found in the literature. Additionally, while the triangularity is found to be of little influence on the residual ZF and oscillation frequency, it is found to strongly enhance the damping of the ZF, with an asymmetry favouring negative triangularities over positive ones.

Future plans

As the ZF solver was developed with the aim of studying GAM damping, it requires a constant temperature over the section whereas this condition was not necessary in the pre-existing code. This study will be continued to attempt to generalise the diagonalisation of the ZF equation and if possible towards theoretical formulas for the GAM damping parameters based on this approach. Later GKNET will also be used to study the shaping effects on linear ITG/TEM growth rates and in particular the effects of negative-D shapes.