Next workshop
Kyoto Tersra 2016-3-10

Particle simulation of plasma production by electron cyclotron waves in tokamak

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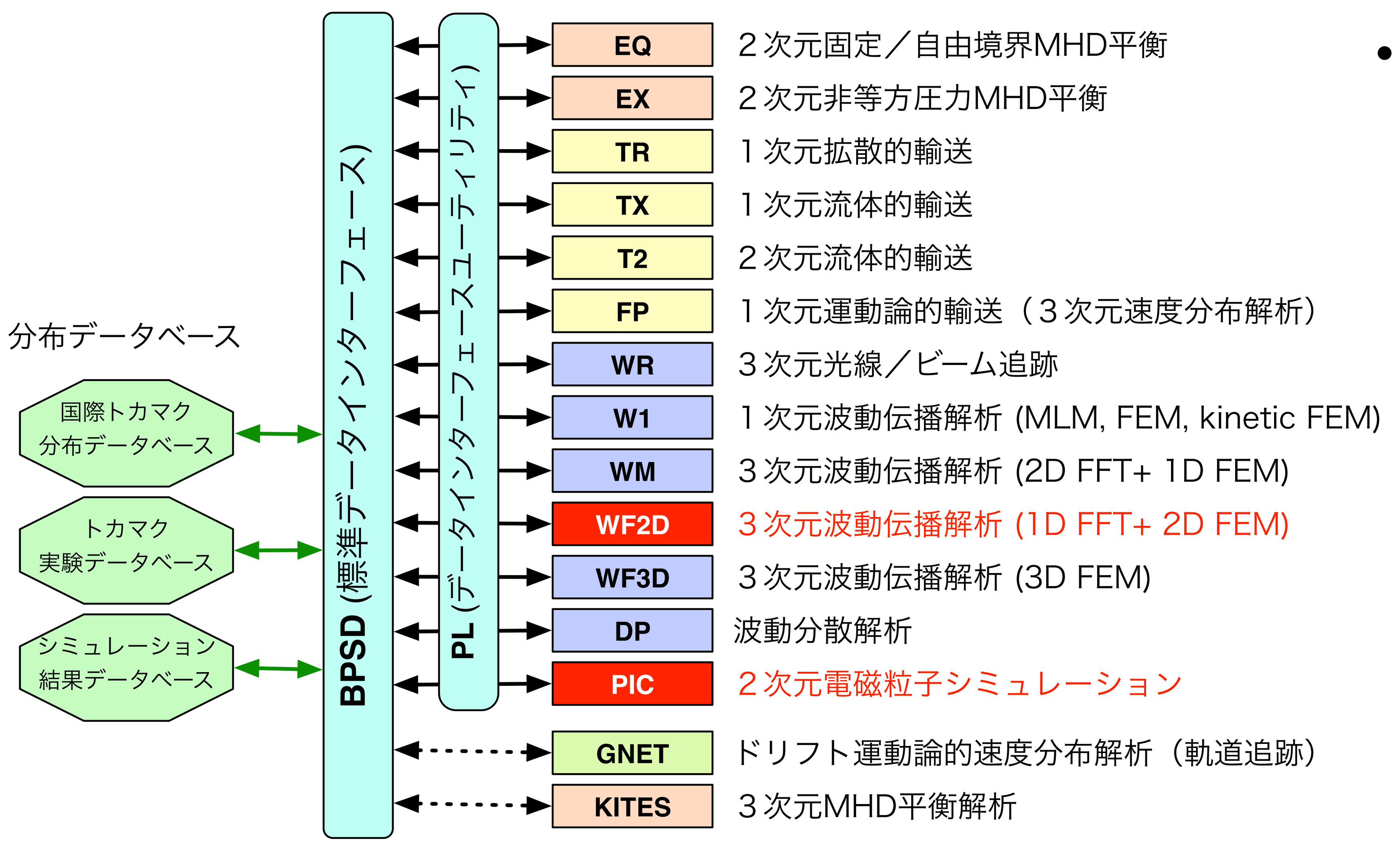
Structure of TASK



The purpose

- **Research purpose**
  - Plasma production has been conducted by electron cyclotron waves in tokamak, though the physics mechanism has not been understood well.
  - Various phenomenons are coupled with each other - electron accelation at the electron cyclotron resonance, ionization by collisions with neutral particles, current drive by toroidal electric field and electron cyclotron waves, suppresion of particle losses by vertical magnetic field, production of closed magnetic configuration, etc.
  - In order to analyze these pheomena self-consistently, we are developing particle simulational modeling.
  - In this presentation, we show the status of two-dimentions electromagnetic particle simulational code and the result of the excitation and propagation of electron cyclotron waves.
- **Analysis using PIC-method** (TASK/PIC)
  - **Analysis : We use configuration in two-dimentional space, and in three-dimentional velocity space**
    - rectangle space, boundary condition in periodic or reflection or absorption
    - distance between grids is order of the Debye length.

Formulation (2)

- **Using integer and half-integer grids**
- **Neglecting spatial differentiation to z direction**
- **Evaluating electro-static potential by FFT or Finite difference method**
- **Introducing relativistic effect**
- **Particle division by MPI**
- **Charge and current density on grid**
  - using the second order of spline function from particle postion and velocity
- **Electromagnetic field to each particles**
  - using the first order of spline function from half-integer grid, and the second order of spline function from integer grid

Formulation (3)

- **Finite difference method**

$$([\bar{\phi}]^{n+1/2}_{i+1,j} - 2[\bar{\phi}]^{n+1/2}_{i,j} + [\bar{\phi}]^{n+1/2}_{i-1,j} + [\bar{\phi}]^{n+1/2}_{i,j+1} - 2[\bar{\phi}]^{n+1/2}_{i,j} + [\bar{\phi}]^{n+1/2}_{i,j-1}) = -[\bar{\rho}]^{n+1/2}_{i,j}$$

$$\begin{aligned}
\begin{pmatrix} [\bar{A}_x]^{n+1}_{i+1/2,j} \\ [\bar{A}_y]^{n+1}_{i,j+1/2} \\ [\bar{A}_z]^{n+1}_{i,j} \end{pmatrix} &= \frac{(\Delta\bar{t})^2}{\beta^2} \begin{pmatrix} [\bar{A}_x]^n_{i+3/2,j} + [\bar{A}_x]^n_{i-1/2,j} + [\bar{A}_x]^n_{i+1/2,j+1} + [\bar{A}_x]^n_{i+1/2,j-1} - 4[\bar{A}_x]^n_{i+1/2,j} \\ [\bar{A}_y]^n_{i+1,j+1/2} + [\bar{A}_y]^n_{i-1,j+1/2} + [\bar{A}_y]^n_{i,j+3/2} + [\bar{A}_y]^n_{i,j-1/2} - 4[\bar{A}_y]^n_{i,j+1/2} \\ [\bar{A}_z]^n_{i+1,j} + [\bar{A}_z]^n_{i-1,j} + [\bar{A}_z]^n_{i,j+1} + [\bar{A}_z]^n_{i,j-1} - 4[\bar{A}_z]^n_{i,j} \end{pmatrix} \\
&+ (\Delta\bar{t})^2 \begin{pmatrix} [\bar{j}_x]^n_{i+1/2,j} \\ [\bar{j}_y]^n_{i,j+1/2} \\ [\bar{j}_z]^n_{i,j} \end{pmatrix} - \Delta\bar{t} \begin{pmatrix} [\bar{\phi}]^{n+1/2}_{i+1,j} - [\bar{\phi}]^{n-1/2}_{i+1,j} - ([\bar{\phi}]^{n+1/2}_{i,j} - [\bar{\phi}]^{n-1/2}_{i,j}) \\ [\bar{\phi}]^{n+1/2}_{i,j+1} - [\bar{\phi}]^{n-1/2}_{i,j+1} - ([\bar{\phi}]^{n+1/2}_{i,j} - [\bar{\phi}]^{n-1/2}_{i,j}) \\ 0 \end{pmatrix} \\
&+ \begin{pmatrix} 2[\bar{A}_x]^n_{i+1/2,j} - [\bar{A}_x]^{n-1}_{i+1/2,j} \\ 2[\bar{A}_y]^n_{i,j+1/2} - [\bar{A}_y]^{n-1}_{i,j+1/2} \\ 2[\bar{A}_z]^n_{i,j} - [\bar{A}_z]^{n-1}_{i,j} \end{pmatrix}
\end{aligned}$$

$$\begin{pmatrix} [\bar{E}_x]^{n+1/2}_{i+1/2,j} \\ [\bar{E}_y]^{n+1/2}_{i,j+1/2} \\ [\bar{E}_z]^{n+1/2}_{i,j} \end{pmatrix} = - \begin{pmatrix} [\bar{\phi}]^{n+1/2}_{i+1,j} - [\bar{\phi}]^{n+1/2}_{i,j} \\ [\bar{\phi}]^{n+1/2}_{i,j+1} - [\bar{\phi}]^{n+1/2}_{i,j} \\ 0 \end{pmatrix} - \frac{1}{\Delta\bar{t}} \begin{pmatrix} [\bar{A}_x]^{n+1}_{i+1/2,j} - [\bar{A}_x]^n_{i+1/2,j} \\ [\bar{A}_y]^{n+1}_{i,j+1/2} - [\bar{A}_y]^n_{i,j+1/2} \\ [\bar{A}_z]^{n+1}_{i,j} - [\bar{A}_z]^n_{i,j} \end{pmatrix}$$

$$\begin{pmatrix} [\bar{B}_x]^{n+1/2}_{i,j+1/2} \\ [\bar{B}_y]^{n+1/2}_{i+1/2,j} \\ [\bar{B}_z]^{n+1/2}_{i,j} \end{pmatrix} = \begin{pmatrix} [\bar{A}_z]^{n+1/2}_{i,j+1} - [\bar{A}_z]^{n+1/2}_{i,j} \\ -[\bar{A}_y]^{n+1/2}_{i+1,j} + [\bar{A}_y]^{n+1/2}_{i,j} \\ [\bar{A}_y]^{n+1/2}_{i+1/2,j} - [\bar{A}_y]^{n+1/2}_{i-1/2,j} - [\bar{A}_x]^{n+1/2}_{i,j+1/2} + [\bar{A}_x]^{n+1/2}_{i,j-1/2} \end{pmatrix}$$

Formulation (4)

- **Absorbing boundary condition : Mur's method**

  Approximating the A equations near the absorbing boundary

$$\frac{1}{\beta^2}\bar{\nabla}^2\bar{\boldsymbol{A}} - \frac{\partial^2}{\partial\bar{t}^2}\bar{\boldsymbol{A}} = 0$$

$$\begin{aligned}
0 &= \left(\frac{\partial}{\partial\bar{x}} + \sqrt{\beta^2\frac{\partial^2}{\partial\bar{t}^2} - \frac{\partial^2}{\partial\bar{y}^2}}\right)\left(\frac{\partial}{\partial\bar{x}} - \sqrt{\beta^2\frac{\partial^2}{\partial\bar{t}^2} - \frac{\partial^2}{\partial\bar{y}^2}}\right)\bar{\boldsymbol{A}} \\
&= \left(\frac{\partial}{\partial\bar{y}} + \sqrt{\beta^2\frac{\partial^2}{\partial\bar{t}^2} - \frac{\partial^2}{\partial\bar{x}^2}}\right)\left(\frac{\partial}{\partial\bar{y}} - \sqrt{\beta^2\frac{\partial^2}{\partial\bar{t}^2} - \frac{\partial^2}{\partial\bar{x}^2}}\right)\bar{\boldsymbol{A}}
\end{aligned}$$

Considering $\bar{\boldsymbol{A}}_z$ and only backward wave on x direction Approximating $\frac{1}{\beta^2}\frac{\partial^2}{\partial\bar{y}^2}/\frac{\partial^2}{\partial\bar{t}^2} \ll 1$

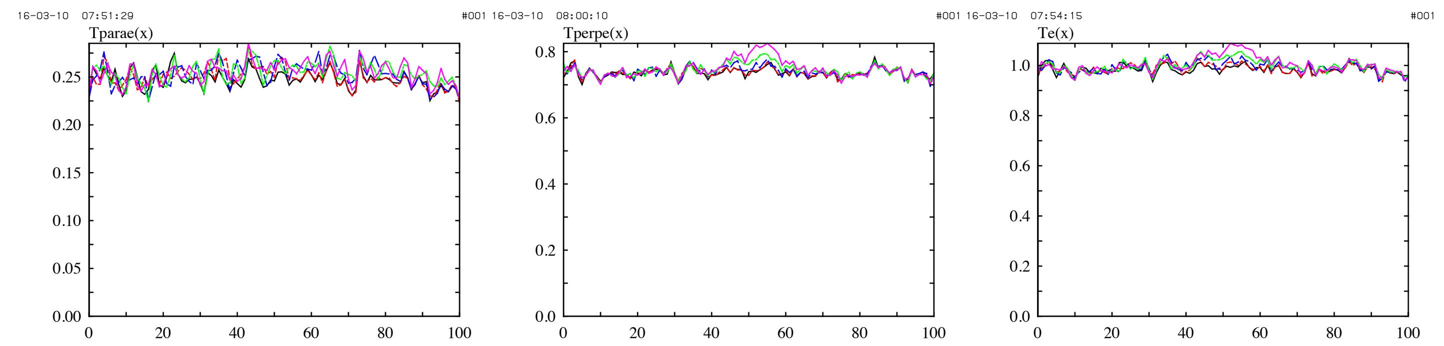
$$\beta\frac{\partial^2\bar{A}_z}{\partial\bar{x}\partial\bar{t}} - \frac{\partial^2\bar{A}_z}{\partial\bar{t}^2} + \frac{1}{2}\frac{\partial^2\bar{A}_z}{\partial^2\bar{y}} = 0$$

$$\begin{aligned}
[\bar{A}_z]^{n+1}_{0,j} &= -[\bar{A}_z]^{n-1}_{1,j} - \frac{1-\Delta\bar{t}/\beta}{1+\Delta\bar{t}/\beta}\left([\bar{A}_z]^{n+1}_{1,j} + [\bar{A}_z]^{n-1}_{0,j}\right) + \frac{2}{1+\Delta\bar{t}/\beta}\left([\bar{A}_z]^n_{1,j} + [\bar{A}_z]^n_{0,j}\right) \\
&+ \frac{(\Delta\bar{t}/\beta)^2}{2(1+\Delta\bar{t}/\beta)}\left([\bar{A}_z]^n_{1,j+1} - 2[\bar{A}_z]^n_{1,j} + [\bar{A}_z]^n_{1,j-1} + [\bar{A}_z]^n_{0,j+1} - 2[\bar{A}_z]^n_{0,j} + [\bar{A}_z]^n_{0,j-1}\right)
\end{aligned}$$

Time Evolution of Electron Temperature Profile

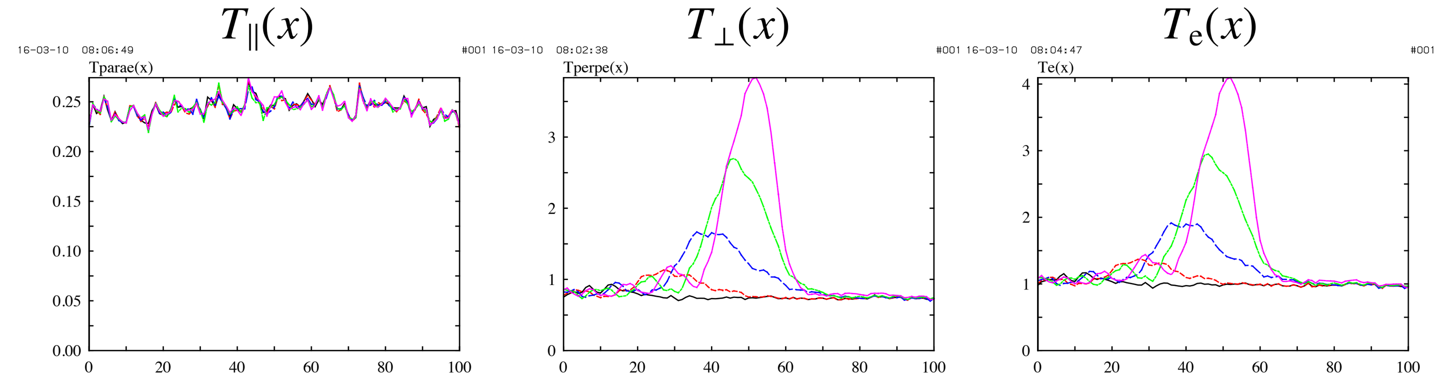
O-mode ($B = 4 \sim 8$)

$T_{\parallel}(x)$ $T_{\perp}(x)$ $T_e(x)$



X-mode ($B = 4 \sim 8$)

$T_{\parallel}(x)$ $T_{\perp}(x)$ $T_e(x)$



Summary

- **We are developing two-dimentional electromagnetic relativistic PIC simulation code TASK/PIC**
  - Final purpose is to analyze plasma production in electron cyclotron resonance
  - At present, the particle motion and the matrix solver are parallelized without region splitting.
  - We have confirmed the excitation and propagation of electromagnetic waves with absorbing boundary.
  - In a magnetized plasma, propagation of O-mode and X-mode propagation and heating near the electron cyclotron resonance are described.
- **Next subject**
  - Developing phisycal model : plasma production by ionization
  - Speeding up calculation : full-scale parallelization, introducing region division, OpenMP
  - Extension to troidal model

Appendix: Formulation (4)

- **Buneman-Boris method**

  Lorentz factor γ and the transformaition verocity $\bar{\boldsymbol{u}}$ are

$$\gamma^n = \frac{1}{\sqrt{1-\beta^2[\bar{\boldsymbol{v}}^n]^2}}$$

$$\bar{\boldsymbol{u}}^n = \gamma^n\bar{\boldsymbol{v}}^n$$

Finite difference form of equation of motion

$$\frac{\bar{\boldsymbol{u}}^{n+1} - \bar{\boldsymbol{u}}^n}{\Delta\bar{t}} = \bar{\boldsymbol{E}}^{n+1/2} + \frac{\bar{\boldsymbol{u}}^{n+1} + \bar{\boldsymbol{u}}^n}{2\gamma^{n+1/2}} \times \bar{\boldsymbol{B}}^{n+1/2}$$

$$\bar{\boldsymbol{u}}^- = \bar{\boldsymbol{u}}^n + \frac{\bar{q}\Delta\bar{t}}{2\bar{m}}\bar{\boldsymbol{E}}^{n+1/2}$$

$$\bar{\boldsymbol{u}}^+ = \bar{\boldsymbol{u}}^{n+1} - \frac{\bar{q}\Delta\bar{t}}{2\bar{m}}\bar{\boldsymbol{E}}^{n+1/2}$$

we obtain

$$\bar{\boldsymbol{u}}^+ = \bar{\boldsymbol{u}}^- + \frac{\bar{q}\Delta\bar{t}}{2\bar{m}\gamma^{n+1/2}}(\bar{\boldsymbol{u}}^+ + \bar{\boldsymbol{u}}^-) \times \bar{\boldsymbol{B}}^{n+1/2}$$

Model equations (1)

- **Equation of motion**

$$m\frac{d\boldsymbol{v}}{dt} = q(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})$$

- **Gauss's law for scalar potential ϕ**

$$-\nabla^2\phi = \frac{\rho}{\epsilon_0}$$

- **Time evolution equations for vector potential A**

$$\nabla^2\boldsymbol{A} + \mu_0\boldsymbol{j} - \mu_0\epsilon_0\frac{\partial\nabla\phi}{\partial t} - \mu_0\epsilon_0\frac{\partial^2\boldsymbol{A}}{\partial t^2} = 0$$

- **Equaitons of electric field E, magnetic field B**

$$\boldsymbol{E} = -\nabla\phi - \frac{\partial\boldsymbol{A}}{\partial t}$$

$$\boldsymbol{B} = \nabla\times\boldsymbol{A}$$

Model equations (2)

- **Normalization**
  - **Normalizing physical quantity**
  - **Using the electron plasma frequency ω_{pe}, distance Δ between grids as normalization unit**
  - **Defining electron (ion) average density in each cells as n_0 : $\omega_{pe}^2 = n_0e^2/m_e\epsilon_0$**

$$\bar{\boldsymbol{x}} = \frac{\boldsymbol{x}}{\Delta},\quad \bar{t} = \omega_{pe}t,\quad \bar{q} = \frac{q}{e},\quad \bar{m}_p = \frac{m_p}{m_e}$$

$$\bar{\boldsymbol{v}} = \frac{\boldsymbol{v}}{\omega_{pe}\Delta},\quad \bar{\omega} = \frac{\omega}{\omega_{pe}},\quad \bar{\boldsymbol{j}} = \frac{\boldsymbol{j}}{qn_0\omega_{pe}\Delta},\quad \bar{\rho} = \frac{\rho}{qn_0},$$

$$\bar{\phi} = \frac{q}{m_e\omega_{pe}^2\Delta^2}\phi,\quad \bar{\boldsymbol{A}} = \frac{q}{m_e\omega_{pe}\Delta}\boldsymbol{A},\quad \bar{\boldsymbol{E}} = \frac{q}{m_e\omega_{pe}^2\Delta}\boldsymbol{E},\quad \bar{\boldsymbol{B}} = \frac{q}{m_e\omega_{pe}}\boldsymbol{B}$$

  - **Normalizing the equations** : defining $\beta \equiv \omega_{pe}\Delta/c = (\Delta/\lambda_{De})(v_{te}/c)$

$$\bar{m}_p\frac{d\bar{\boldsymbol{v}}}{d\bar{t}} = \bar{q}(\bar{\boldsymbol{E}} + \bar{\boldsymbol{v}}\times\bar{\boldsymbol{B}})$$

$$-\bar{\nabla}^2\bar{\phi} = \bar{\rho}$$

$$\frac{1}{\beta^2}\bar{\nabla}^2\bar{\boldsymbol{A}} + \bar{\boldsymbol{j}} - \frac{\partial}{\partial\bar{t}}(\bar{\nabla}\bar{\phi}) - \frac{\partial^2}{\partial\bar{t}^2}\bar{\boldsymbol{A}} = 0$$

Formulation (1)

- **Time step**
  - $\bar{\boldsymbol{v}}$, $\bar{\boldsymbol{A}}$ - integer mesh in time
  - $\bar{\boldsymbol{x}}$, $\bar{\phi}$, $\bar{\boldsymbol{E}}$, $\bar{\boldsymbol{B}}$ - half-integer mesh in time
- **The flow of PIC-method algorithm**

1. from particle position $\bar{\boldsymbol{x}}$ and velocity $\bar{\boldsymbol{v}}$, evaluate charge density $\bar{\rho}$ and current density $\bar{\boldsymbol{j}}$
2. from $\bar{\rho}$ and $\bar{\boldsymbol{j}}$, evaluate electro static potential $\bar{\phi}$ and vector potential $\bar{\boldsymbol{A}}$
3. from $\bar{\phi}$ and $\bar{\boldsymbol{A}}$, evaluate electric field $\bar{\boldsymbol{E}}$ and magnetic field $\bar{\boldsymbol{B}}$
4. from $\bar{\boldsymbol{E}}$ and $\bar{\boldsymbol{B}}$, update particle position $\bar{\boldsymbol{x}}$ and velocity $\bar{\boldsymbol{v}}$
5. return to 1

Simulation condition

- **Parameter**
  - number of particles for each species(electron, ion), 160000
  - number of grids 100×100
  - constant initial density (electron and ion are in same position.)
  - Maxwellian velocity distribution
  - $\beta = \omega_{pe}\Delta/c = 0.1$
  - time interval $\Delta\bar{t} = 0.02$
- **External action**
  - static magnetic field with linear gradient : $\bar{B}_{ymin}(x=0)$, $\bar{B}_{ymax}(x=100)$
  - wave guide electric field: at $x=0$, from $y=25$ to 75 , $\bar{A}_y = \bar{A}\sin\bar{\omega}\bar{t}$, $\bar{E}_y = -\partial\bar{A}_y/\partial\bar{t} = -\bar{\omega}\bar{A}_y\cos\bar{\omega}\bar{t}$
  - $\bar{A} = 0.5$
  - $\bar{\omega} = 6.0$

Electromagnetic Wave Propagation

Wave excitation: $\omega = 4$, $B = 6 \sim 8$, no FLR since $\Delta > \rho_e$

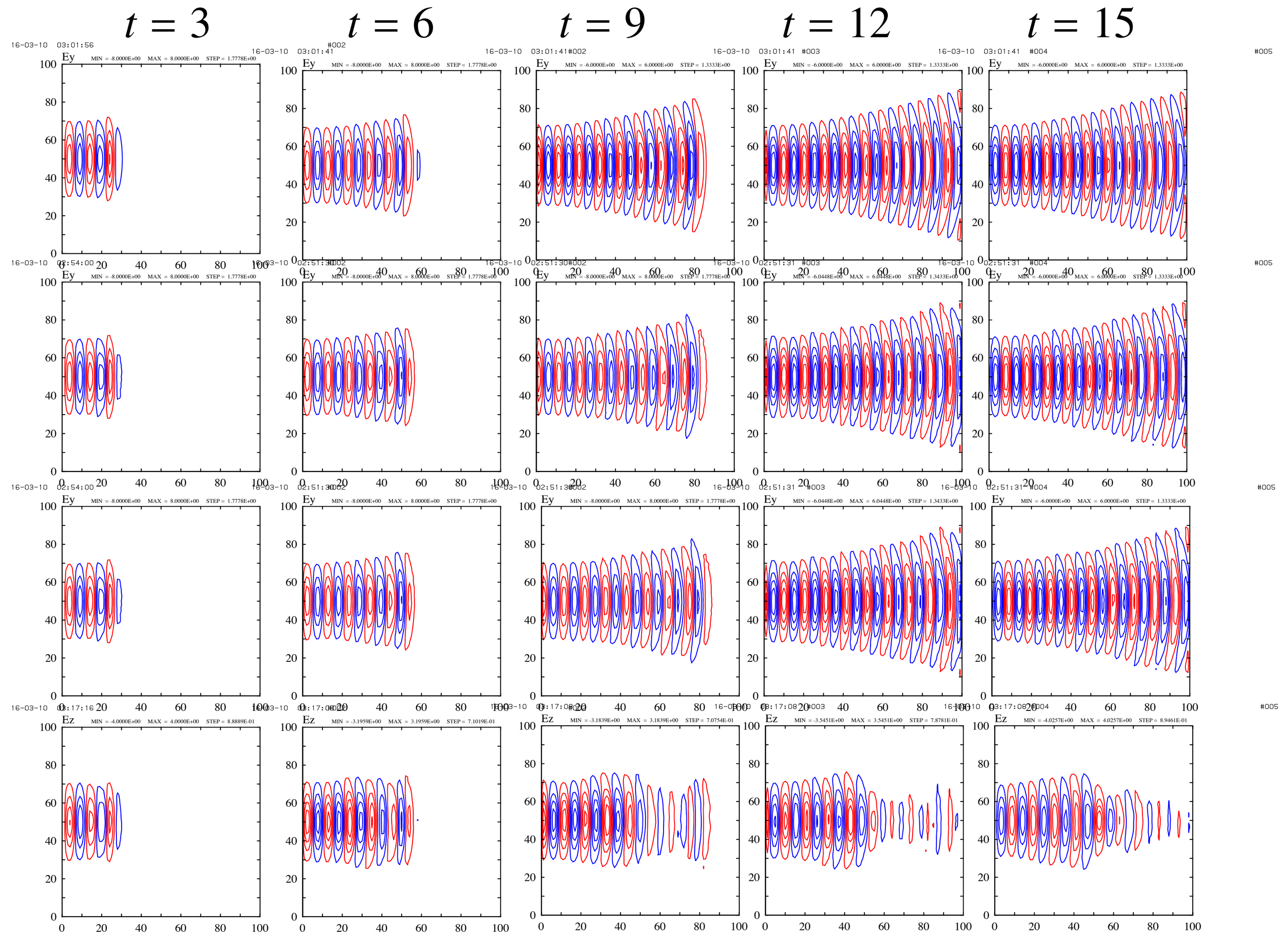
$t=3$ $t=6$ $t=9$ $t=12$ $t=15$

Vaccum

Plasma

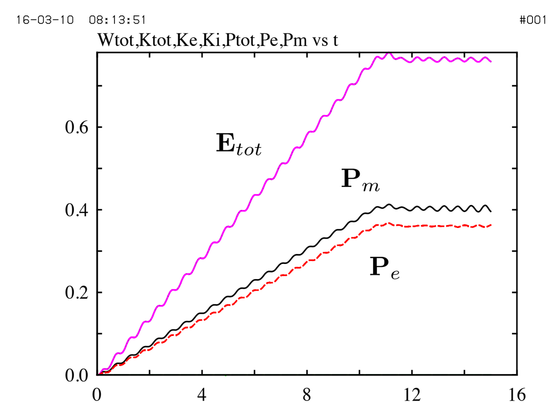
O-mode

X-mode

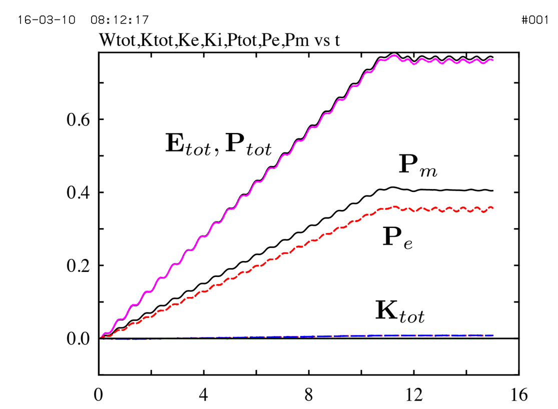


Time Evolution of Energy

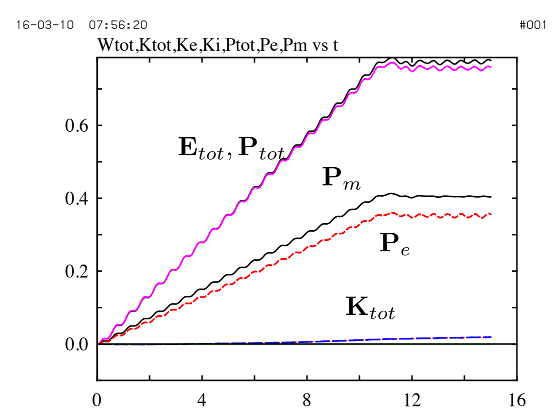
Vacuum



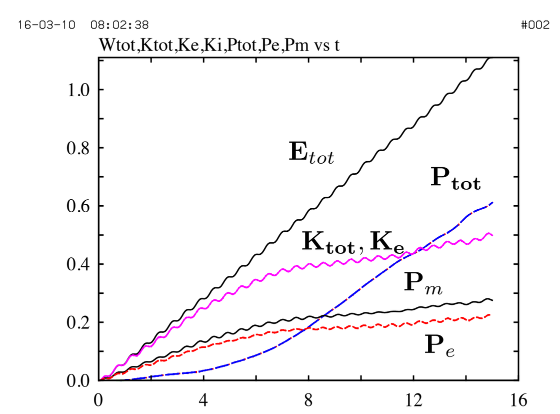
Plasma ($B=0$)



O-mode ($B=6$)



X-mode ($B=6$)



$$\gamma^- = \frac{1}{\sqrt{1+\beta^2[\bar{\boldsymbol{u}}^-]^2}}$$

$$\boldsymbol{T}^{n+1/2} = \gamma^-\frac{\bar{q}\Delta\bar{t}}{2\bar{m}}\bar{\boldsymbol{B}}^{n+1/2}$$

$$\bar{\boldsymbol{u}}^0 = \bar{\boldsymbol{u}}^- + \bar{\boldsymbol{u}}^-\times\bar{\boldsymbol{T}}^{n+1/2}$$

$$\boldsymbol{S}^{n+1/2} = \frac{2\bar{\boldsymbol{T}}^{n+1/2}}{1+[\boldsymbol{T}^{n+1/2}]^2}$$

$$\bar{\boldsymbol{u}}^+ = \bar{\boldsymbol{u}}^- + \bar{\boldsymbol{u}}^0\times\boldsymbol{S}^{n+1/2}$$

$$\gamma^+ = \frac{1}{\sqrt{1+\beta^2[\bar{\boldsymbol{u}}^+]^2}}$$

$$\bar{\boldsymbol{v}}^{n+1} = \gamma^+\bar{\boldsymbol{u}}^+ + \frac{\bar{q}\Delta\bar{t}}{2\bar{m}}\bar{\boldsymbol{E}}^{n+1/2}$$

$$\frac{d\bar{\boldsymbol{r}}^{n+1}}{d\bar{t}} = \frac{\bar{\boldsymbol{r}}^{n+3/2} - \bar{\boldsymbol{r}}^{n+1/2}}{\Delta\bar{t}} = \bar{\boldsymbol{v}}^{n+1}$$

$$\bar{\boldsymbol{r}}^{n+3/2} = \bar{\boldsymbol{r}}^{n+1/2} + \bar{\boldsymbol{v}}^{n+1}\Delta\bar{t}$$

$$\gamma^- = \frac{1}{\sqrt{1+\beta^2[\bar{\boldsymbol{u}}^-]^2}}$$

$$\boldsymbol{T}^{n+1/2} = \gamma^-\frac{\bar{q}\Delta\bar{t}}{2\bar{m}}\bar{\boldsymbol{B}}^{n+1/2}$$

$$\bar{\boldsymbol{u}}^0 = \bar{\boldsymbol{u}}^- + \bar{\boldsymbol{u}}^-\times\bar{\boldsymbol{T}}^{n+1/2}$$

$$\boldsymbol{S}^{n+1/2} = \frac{2\bar{\boldsymbol{T}}^{n+1/2}}{1+[\boldsymbol{T}^{n+1/2}]^2}$$

$$\bar{\boldsymbol{u}}^+ = \bar{\boldsymbol{u}}^- + \bar{\boldsymbol{u}}^0\times\boldsymbol{S}^{n+1/2}$$

$$\gamma^+ = \frac{1}{\sqrt{1+\beta^2[\bar{\boldsymbol{u}}^+]^2}}$$

$$\bar{\boldsymbol{v}}^{n+1} = \gamma^+\bar{\boldsymbol{u}}^+ + \frac{\bar{q}\Delta\bar{t}}{2\bar{m}}\bar{\boldsymbol{E}}^{n+1/2}$$

$$\frac{d\bar{\boldsymbol{r}}^{n+1}}{d\bar{t}} = \frac{\bar{\boldsymbol{r}}^{n+3/2} - \bar{\boldsymbol{r}}^{n+1/2}}{\Delta\bar{t}} = \bar{\boldsymbol{v}}^{n+1}$$

$$\bar{\boldsymbol{r}}^{n+3/2} = \bar{\boldsymbol{r}}^{n+1/2} + \bar{\boldsymbol{v}}^{n+1}\Delta\bar{t}$$