Effects of Trapped Fast Ions on the Interchange Mode

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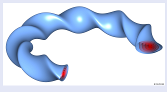
On the basis of the kinetic energy principle, effects on trapped fast (energetic) ions on the ideal interchange mode in helical systems are investigated. Approximating the spatial profile of the ideal interchange mode is given, we introduce an extended dispersion relation of the ideal interchange mode.

Numerical analyses show that the ideal interchange mode is destabilized by trapped energetic ions and has a finite rotation frequency due to the precession drift of the trapped energetic ions. We also derive and analyze an eigenvalue equation of the ideal interchange mode with trapped energetic ions.

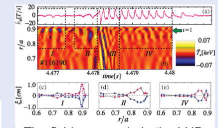
Introduction

- In magnetic confinement fusion devices, energetic particles are produced by external healing and thermonuclear fusion.
- Fast (energetic) ion losses become anomalously large when energetic ion-induced MHD instabilities are excited, such as the Alfven eigenmode, the energetic particle mode, and the fishbone mode. *[Heidbrink(2008)PoP, McGuire(1983)PRL]*
- In tokamaks, the fishbone mode is associated with the internal kink mode or the resistive interchange mode, which are destabilized by the trapped energetic ions. *[Chen(1984)PRL, Coppi(1989)PRL, Hao(2011)PRL]*
- The fishbone mode has been also observed in helical systems, such as the Compact Helical Sytem (CHS) and the Large Helical Device (LHD). *[Toi(2000)NF, Du(2015)PRL]*
- In helical systems, physical mechanism of the fishbone mode is not clarified, but the mode may be linked to the precursor interchange mode.

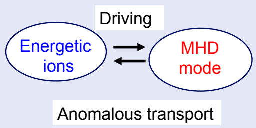
- The ideal MHD instability with kinetic effects has been analyzed by a kinetic energy principle. *[Kruskal(1958)PF, Van Dam(1982)PF, Marchand(1980)PF, Cheng(1992)PR, Konies(2000)PoP, Kolesnichenko(2002)PoP]*
- We revisited the kinetic energy principle to describe the ideal interchange mode stability in the presence of the trapped energetic ions, based on a reduced MHD equation with a bounce-averaged drift-kinetic equation for the trapped energetic ions. The interchange mode is destabilized by the trapped energetic ions even if the ideal interchange mode is stable. *[Nishimura(2015)PoP]*
- In this study, we also examine the effect of the trapped energetic ions on the unstable ideal interchange mode stability and the radial mode structure.

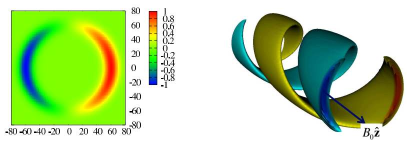


Magnetic surfaces in a helical system for (l,N)=(2,10)



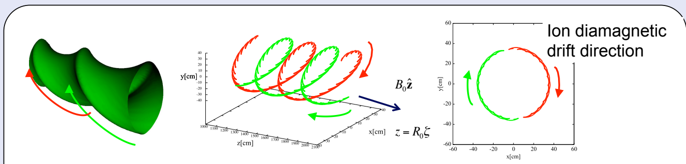
The fishbone mode in the LHD





The rotating interchange mode

Wave-particle resonant interaction



Precession drifts of trapped energetic ions

Kinetic Energy Principle

MHD momentum equation : $\rho_i \frac{\partial \mathbf{v}_{MHD}}{\partial t} = -\nabla\tilde{p} + \frac{1}{c}\left(\tilde{\mathbf{J}}\times\mathbf{B}_0 + \mathbf{J}_0\times\tilde{\mathbf{B}}\right) - \nabla\cdot\tilde{P}_h$

Energetic ion pressure tensor perturbation : $\tilde{P}_h = \tilde{p}_{h\perp}\left(I - \mathbf{b}_0\mathbf{b}_0\right) + \tilde{p}_{h//}\mathbf{b}_0\mathbf{b}_0 \quad \tilde{p}_{h//} = \int m_i v_{//}^2 \tilde{f} d^3\mathbf{v} \quad \tilde{p}_{h\perp} = \int \frac{m_i v_\perp^2}{2}\tilde{f} d^3\mathbf{v}$

Bounce-averaged drift-kinetic equation of energetic ions : $\frac{\partial \tilde{f}}{\partial t} + \left(\mathbf{v}_l + \mathbf{v}_m\right)\cdot\nabla\tilde{f} = -\mathbf{v}_E\cdot\nabla f_0 - (2-\alpha)K\left(\mathbf{v}_E\cdot\vec{\kappa}\right)\frac{\partial f_0}{\partial K}$

Assumption

Displacement vector : $\vec{\xi}$ $\quad \partial\vec{\xi}/\partial t = \mathbf{v}_{MHD} \quad \vec{\xi} \propto \exp(-i\omega t)$

Low- β , incompressible limit, $p_{h//0}, p_{h\perp 0} << p_0$, , no perturbation at the plasma-vacuum boundary

Kinetic energy principle : $\delta I + \delta W_{MHD} + \delta W_K = 0$

$$\delta I = -\frac{\omega^2}{2}\int \rho_i \left|\vec{\xi}\right|^2 dV \quad \delta W_{MHD} = \frac{1}{2}\int\left[\frac{1}{4\pi}\left|\mathbf{Q}\right|^2 - 2\left(\vec{\xi}_\perp\cdot\nabla p_0\right)\left(\vec{\xi}_\perp^*\cdot\vec{\kappa}\right) - \frac{j_{//0}}{c}\vec{\xi}_\perp^*\cdot\mathbf{b}_0\times\mathbf{Q}\right]dV$$

$$\delta W_K = \frac{1}{2}\int\vec{\xi}_\perp^*\cdot\vec{\kappa}\left(\tilde{p}_{h\perp} + \tilde{p}_{h//}\right)dV + \frac{1}{2}\int\vec{\xi}_{//}^*\cdot\mathbf{b}_0\cdot\nabla\cdot\tilde{P}_h dV \quad \mathbf{Q} = \nabla\times\left(\vec{\xi}_\perp\times\mathbf{B}_0\right)$$

Approximation

$B_0 = B\left[1 - \varepsilon_t\cos\theta + \varepsilon_h\cos\left(l\theta - N\zeta\right)\right]$ $\quad \varepsilon_t = r/R_0$ $\quad \varepsilon_h$: helical ripple rate ($\varepsilon_h = 0$ in tokamaks) $\quad (r,\theta,\zeta)$: toroidal coordinate

Reduced MHD model based on stellarator expansion ordering ($\varepsilon_t \sim \varepsilon_h^2 \sim \beta$) :

$$\vec{\xi}_\perp \approx \frac{c}{i\omega B}\nabla\tilde{\phi}\times\vec{\zeta} \approx \xi\hat{\mathbf{r}} \propto \exp\left[i\left(m\theta - n\zeta - \omega t\right)\right] \quad \vec{\kappa} = \mathbf{b}_0\cdot\nabla\mathbf{b}_0 \approx \kappa_h\hat{\mathbf{r}} \quad \kappa_h = -\frac{Nr\iota}{2lR_0^2}(4-s)$$

Deeply trapped energetic ions : $\tilde{p}_{h//} << \tilde{p}_{h\perp}$ $\quad$ Slowing-down distribution : $f_0 \propto K^{-3/2}$

Curvature at trapping region : $\vec{\kappa} \approx -\varepsilon_h'$ $\quad$ Precession drift frequency : $\Omega_{ds} \approx \frac{mcK_{\max}}{eBr}\frac{d\varepsilon_h}{dr}$

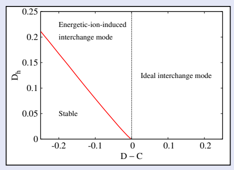
Approximate form of kinetic energy principle : $\alpha_I\Omega^2 = C - D + D_h\Omega\ln\left(1 - \frac{1}{\Omega}\right)$

$$\Omega = \frac{\omega}{\Omega_{ds}} \quad \alpha_I = \frac{L_s^2\Omega_{ds}^2}{v_A^2} \quad C = \frac{\int_{-\infty}^{\infty}x^2\left|\xi'\right|^2 dx}{\int_{-\infty}^{\infty}\left|\xi\right|^2 dx} \quad D = \left.\frac{\kappa_h p_0' L_s^2}{B^2/8\pi}\right|_{r_s} \quad D_h = \left.\frac{\left(-\varepsilon_h'\right)p_h' L_s^2 F}{B^2/8\pi}\right|_{r_s}$$

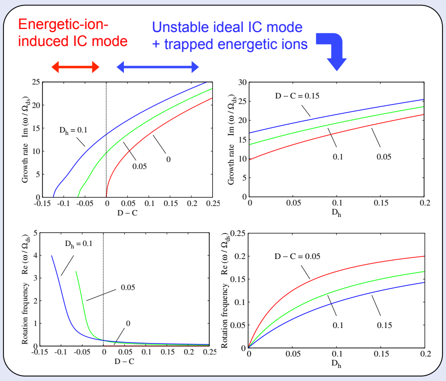
Numerical Analysis

Ideal interchange (IC) mode stability: (i) unstable if $D - C > 0$
(Suydam criterion) (ii) stable if $D - C < 0$

$\alpha_I = 5.4\times10^{-4}$



Stability of boundary is affected by trapped energetic ions. *[Nishimura(2015)PoP]*



- Originally unstable, non-rotating, ideal IC mode is enhanced by trapped energetic ions.
- The mode has finite rotation frequency due to precession drift frequency of trapped energetic ions.
- The direction of the mode rotation frequency is in the precession drift of trapped energetic ions, which is of order the diamagnetic drift frequency.

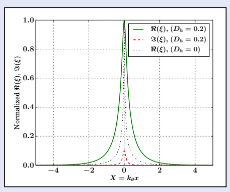
Eigenvalue equation for the ideal interchange mode with trapped energetic ions:

$$\frac{d}{dx}\left[\left(x^2 - \frac{\alpha_I\Omega^2}{k_\theta^2}\right)\frac{d\xi}{dx}\right] + \left[\alpha_I\Omega^2 - k_\theta^2 x^2 + D - D_h\Omega\ln\left(1 - \frac{1}{\Omega}\right)\right]\xi = 0$$

→ Numerically solved by the shooting method.

- Radial mode structure of the ideal interchange mode tends to be radially expanded by trapped energetic ions.
- Such tendency is also observed in experiments. *[Du(2015)PRL]*

$D = 0.42$ $\quad \alpha_I = 5.4\times10^{-4}$



Summary:

- The interchange mode in helical systems is excited by trapped energetic ions even if the ideal interchange mode is stable. The theory explains how to apply the fishbone mode theory in tokamaks to helical systems.
- The unstable interchange mode is also destabilized and forced to rotate due to the precession drift frequency of trapped energetic ions, where the mode structure is radially expanded.

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