第21回NEXT研究会
2016年3月10,11日
京都テルサ
京都府京都市

トカマク磁気座標系における流体型輸送コードの開発
Development of the fluid-type transport code on the flux coordinates in tokamaks

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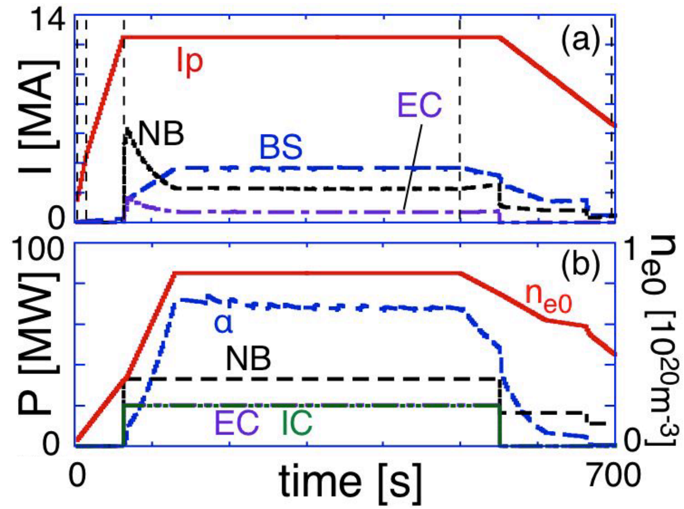
This work was supported by a Grant-in-aid for Young Scientists (B) (No 25820442) from the Japan Society of the Promotion of Science (JSPS).

Introduction

- 1D transport codes are widely used for
  - predictive purpose.
  - operation scenario development.
  - experimental analysis.
- Based on the transport equations:
  - convection-diffusion equations
  - assuming the flux-gradient relationship



[Hayashi, Honda, Fukuyama: ITER Task C19TD26FJ]

- Advantages: Numerically robust, easy-to-use, arbitrary choice of eqs.
- Drawbacks:
  - Transient phenomena are out of scope.
  - Physics derived from code results simply depends upon physics models implemented.

Develop a new transport code based on more fundamental equations.

Two-fluid model: coupling of the fluid variables to Maxwell's equations

Multi-fluid transport code, TASK/TX

TASK/TX is a 1D fluid-type transport code based on two-fluid modeling.

But, original TASK/TX [JCP08] is confined to cylindrical geometry…

No actual equilibria; No poloidal mass enhancement; Obscure rationale behind eqs.

This paper is devoted to

- ✓ deriving governing equations for TASK/TX on **the axisymmetric flux coordinates**
  - **1. Maxwell's equations**
  - **2. Conservation of mass and momentum**
  - 3. Heat transport equations
  - **4. Parallel heat flow equations**
  - 5. Equations for beam ions
  - 6. Equations for neutrals
- ✓ exhibiting numerical results and comparisons with theoretical models.

	TASK/TX	conventional transport codes
Quasi-neutrality	Intrinsically satisfied	Forced
Ambipolarity	Intrinsically satisfied	Forced
Continuity eqs.	Divergence of flux	Flux-gradient assumption

Settings

Small gyroradius expansion

$$\delta \equiv \frac{\rho_s}{L} \ll 1$$

- **0th order**: the unperturbed part: n, T, p, $\boldsymbol{B}$, Φ
- **1st order**: the flow within a flux surface
- **2nd order**: the transport flux across a flux surface

drift ordering: $\boxed{\frac{\partial}{\partial t} \sim \delta\omega_t \qquad u \sim \delta v_t}$

for flows except cross-field flow

transport ordering: $\boxed{\frac{\partial}{\partial t} \sim \delta^2\omega_t \qquad u \sim \delta v_t}$

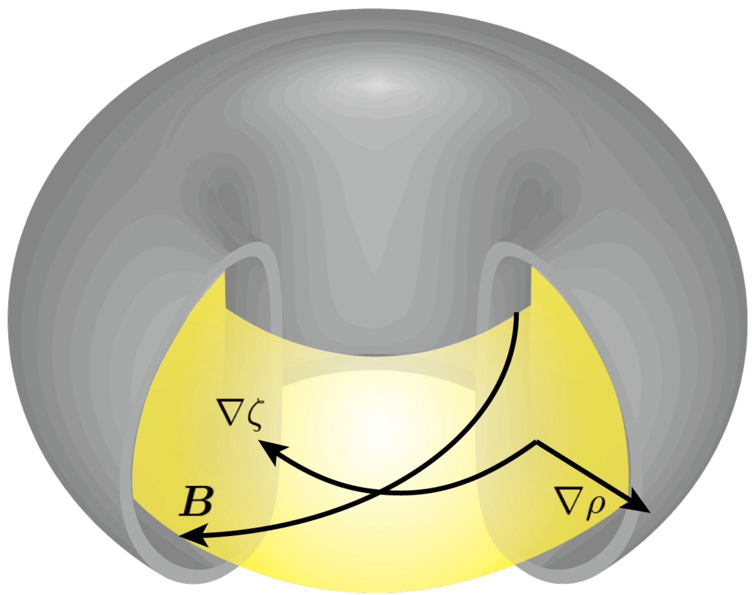
for any other quantities

Equations of motion in 3 directions

- Parallel: Even on the flux coordinates (ρ, θ, ζ), the parallel motion of particles is essential for transport phenomena in toroidal plasmas.
- Toroidal: geometric toroidal direction $\nabla\zeta$
- Radial: perpendicular to $\boldsymbol{B}$ and toroidal directions

$$\boldsymbol{B} \times (R^2\nabla\zeta) = \nabla\psi = \frac{\partial\psi}{\partial\rho}\nabla\rho$$

$$\boxed{\boldsymbol{b},\ \nabla\zeta,\ \nabla\rho}$$



Flux coordinates in a tokamak

Maxwell's equations

Gauss's law (Coulomb gauge): $\nabla \cdot \nabla \Phi = -\dfrac{\sigma}{\epsilon_0}$

Faraday's law: $\boldsymbol{E} = -\nabla \Phi - \dfrac{\partial \boldsymbol{A}}{\partial t}$ + $\boldsymbol{B} = \nabla \times \boldsymbol{A}$

Ampere's law: $\dfrac{1}{c^2}\dfrac{\partial \boldsymbol{E}}{\partial t} = \nabla \times \boldsymbol{B} - \mu_0 \boldsymbol{j}$

Gauss's law

$$0 = \frac{1}{V'}\frac{\partial}{\partial \rho}\left[V'\langle|\nabla\rho|^2\rangle\frac{\partial}{\partial \rho}\Phi\right] + \frac{1}{\epsilon_0}\sum_a e_a n_a \quad \text{(1M)}$$

Ampere's law in the toroidal direction

$$\frac{1}{c^2}\frac{1}{R^2}\frac{\partial \dot{\psi}}{\partial t} = \nabla \cdot \left(\frac{\nabla \psi}{R^2}\right) - \mu_0 j^\zeta$$

cf. Grad-Shafranov equation.

$$\frac{1}{c^2}\frac{\partial \dot{\psi}}{\partial t} = \frac{1}{V'\langle R^{-2}\rangle}\frac{\partial}{\partial \rho}\left[V'\left\langle\frac{|\nabla\rho|^2}{R^2}\right\rangle\frac{\partial \psi}{\partial \rho}\right] - \mu_0\frac{\langle j^\zeta\rangle}{\langle R^{-2}\rangle} \quad \text{(2M)}$$

Ampere's law cf. FSA GS eq.

$$\frac{\partial \psi}{\partial t} \equiv \dot{\psi}(= E_\zeta = RE_t) \quad \text{(3M)}$$

Faraday's law

Ampere's law in the parallel (poloidal) direction

$$\frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}\cdot \boldsymbol{B} = \nabla \times \boldsymbol{B}\cdot\boldsymbol{B} - \mu_0 \boldsymbol{j}\cdot\boldsymbol{B}$$

$$\frac{1}{c^2}\frac{\partial \dot{\psi}_t}{\partial t} = V'\left\langle\frac{|\nabla\rho|^2}{R^2}\right\rangle\frac{\partial}{\partial \rho}\left(\frac{1}{V'\langle R^{-2}\rangle}\frac{\partial \psi_t}{\partial \rho}\right) + \mu_0\frac{\langle \boldsymbol{j}\cdot\boldsymbol{B}\rangle - I\langle j^\zeta\rangle}{\langle B^\theta\rangle} \quad \text{(4M)}$$

$$\frac{\partial \psi_t}{\partial t} \equiv \dot{\psi}_t(= -E_\theta) \quad \text{(5M)}$$

Maxwell's equations of TASK/TX are solved for: $\Phi,\ \dot{\psi},\ \psi,\ \dot{\psi}_t,\ \psi_t$

Continuity equations (conservation of mass)

The FSA continuity equation yields

e: ionization source by NB, i: CX loss by NB

$$\frac{1}{V'}\frac{\partial}{\partial t}(V'n_a) = -\frac{1}{V'}\frac{\partial}{\partial \rho}[V'\underline{n_a(u_a^\rho - u_g^\rho)}] + \nu_{\text{ionize}}\, n_a \sum_{i=1}^{3}\frac{n_{0i}}{n_0} - S_{\text{La}} + S_{\text{CXa}}^{\text{NB}} + \nu_{\text{b}} n_{\text{b}} - \underline{S_{\text{LCi}} - S_{\text{OLi}}}$$

The cross-field particle flux is not approximated as a convection-diffusion form.

Ionization source from neutrals

Loss in the SOL

Slowing down from beam ions

Other sinks

Equations of motion (parallel)

Parallel momentum equations

viscosity friction forces negligible

$$m_a n_a \frac{\partial \langle B u_{a\|}\rangle}{\partial t}\bigg|_\rho = -\underline{\langle \boldsymbol{B}\cdot\nabla\cdot\overleftrightarrow{\pi}_a\rangle} + \underline{\langle \boldsymbol{B}\cdot\boldsymbol{F}_a\rangle} + e_a n_a\langle \boldsymbol{B}\cdot\boldsymbol{E}^A\rangle + \langle \boldsymbol{B}\cdot(\boldsymbol{S}_{\text{ma}} - \cancel{m_a \boldsymbol{u}_a S_{\text{na}}})\rangle$$

Neoclassical moment approach

[Hirshman and Sigmar, NF 1981]

$$\left\langle \boldsymbol{B}\cdot\nabla\cdot\overleftrightarrow{\pi}_a\right\rangle = \hat{\mu}_1^a\left(\langle B u_{a\|}\rangle - \langle BV_{1a}\rangle\right) + \hat{\mu}_2^a\left(\frac{2\langle Bq_{a\|}\rangle}{5p_a} - \langle BV_{2a}\rangle\right) \qquad \langle \boldsymbol{B}\cdot\boldsymbol{F}_a\rangle = \sum_b\left(\ell_{11}^{ab}\langle Bu_{b\|}\rangle - \ell_{12}^{ab}\frac{2\langle Bq_{b\|}\rangle}{5p_b}\right)$$

Diamagnetic flow

$$BV_{1a} = I\omega_a = -I\left(\frac{1}{e_a n_a}\frac{\partial p_a}{\partial \psi} + \frac{\partial \Phi}{\partial \psi}\right) \qquad BV_{2a} = -\frac{I}{e_a}\frac{\partial T_a}{\partial \psi}$$

NOTE: All we have to do is to estimate $\hat{\mu}_i^a$ and ℓ_{ij}^{ab} for neoclassical calculation.

$$\boxed{m_a n_a \frac{\partial}{\partial t}\langle B u_{a\parallel}\rangle = -\hat{\mu}_1^a \langle B u_{a\parallel}\rangle - \hat{\mu}_2^a \langle B \hat{q}_{a\parallel}\rangle + \hat{\mu}_1^a B V_{1a} + \hat{\mu}_2^a B V_{2a} + \sum_b \ell_{11}^{ab}\langle B u_{b\parallel}\rangle - \sum_b \ell_{12}^{ab}\langle B \hat{q}_{b\parallel}\rangle + \underbrace{e_a n_a \langle B E_\parallel\rangle}_{\text{Electric field acceleration}} - \underbrace{S_{La}}_{\text{Loss in the SOL}} - \underbrace{S_{0a}}_{\text{Neutral friction}} - \underbrace{S_{CXi}}_{\text{CX loss}} - \underbrace{S_{OLi}}_{\text{Other sinks}}}$$

where

$$\langle B \hat{q}_{a\parallel}\rangle \equiv \frac{2\langle B q_{a\parallel}\rangle}{5 p_a}$$

Parallel heat flow equations

$$\begin{cases} m_a n_a \dfrac{\partial \boldsymbol{u}_a}{\partial t} + m_a n_a \boldsymbol{u}_a \cdot \nabla \boldsymbol{u}_a = n_a e_a (\boldsymbol{E} + \boldsymbol{u}_a \times \boldsymbol{B}) + \boldsymbol{F}_a - \nabla p_a \underline{- \nabla \cdot \overset{\leftrightarrow}{\pi}_a} \\ \dfrac{\partial \boldsymbol{Q}_a}{\partial t} = \dfrac{e_a}{m_a}\left[\boldsymbol{E} \cdot \left(\dfrac{5}{2} p_a \overset{\leftrightarrow}{I} + \underbrace{\cancel{\overset{\leftrightarrow}{\pi}_a + m_a n_a \boldsymbol{u}_a \boldsymbol{u}_a}}_{\text{negligible}}\right) + \boldsymbol{Q}_a \times \boldsymbol{B}\right] + \underline{\boldsymbol{G}_a} \underline{- \nabla \cdot \overset{\leftrightarrow}{r}_a} \end{cases}$$

The heat viscosity term

heat flux production rate due to friction,

where

$$\boldsymbol{Q}_a = \boldsymbol{q}_a + \frac{5}{2} p_a \boldsymbol{u}_a + \underbrace{\cancel{\frac{1}{2} m_a n_a u_a^2 \boldsymbol{u}_a + \boldsymbol{u}_a \cdot \overset{\leftrightarrow}{\pi}_a}}_{\text{negligible}}$$

$$\underline{\left\langle \boldsymbol{B} \cdot \nabla \cdot \overset{\leftrightarrow}{\vartheta}_a \right\rangle} = \hat{\mu}_2^a \left(\langle B u_{a\parallel}\rangle - \langle B V_{1a}\rangle\right) + \hat{\mu}_3^a \left(\frac{2\langle B q_{a\parallel}\rangle}{5 p_a} - \langle B V_{2a}\rangle\right)$$

$$\underline{\langle \boldsymbol{B} \cdot \boldsymbol{H}_a\rangle} = \sum_b \left(-\ell_{21}^{ab}\langle B u_{b\parallel}\rangle + \ell_{22}^{ab}\frac{2\langle B q_{b\parallel}\rangle}{5 p_b}\right)$$

$$\boxed{\frac{5}{2} m_a n_a \frac{\partial}{\partial t}\langle B \hat{q}_{a\parallel}\rangle = -\hat{\mu}_2^a \langle B u_{a\parallel}\rangle - \hat{\mu}_3^a \langle B \hat{q}_{a\parallel}\rangle + \hat{\mu}_2^a B V_{1a} + \hat{\mu}_3^a B V_{2a} - \sum_b \ell_{21}^{ab}\langle B u_{a\parallel}\rangle + \sum_b \ell_{22}^{ab}\langle B \hat{q}_{b\parallel}\rangle - S_{La} - S_{0a}}$$

where

$$\langle B \hat{q}_{a\parallel}\rangle \equiv \frac{2\langle B q_{a\parallel}\rangle}{5 p_a}$$

	TASK/TX	conventional transport codes
Parallel flow eqs.	Solved	External neoclassical modules e.g. Matrix Inversion(MI), NCLASS Steady-state assumption

Equation of motion (radial, toroidal)

Radial momentum (force balance) equations

$$0 = -(\langle B^2\rangle\langle R^2\rangle - I^2)\frac{1}{m_a}\frac{\partial}{\partial\rho}n_a T_a - \frac{e_a}{m_a}n_a(\langle B^2\rangle\langle R^2\rangle - I^2)\frac{\partial}{\partial\rho}\Phi + \frac{e_a}{m_a}n_a\frac{\partial\psi}{\partial\rho}I\langle Bu_{a\parallel}\rangle - \frac{e_a}{m_a}n_a\frac{\partial\psi}{\partial\rho}\langle B^2\rangle\langle Ru_{a\zeta}\rangle$$

Toroidal momentum equations

$$\frac{1}{V'}\frac{\partial}{\partial t}(V' m_a n_a\langle Ru_{a\zeta}\rangle) = -\frac{1}{V'}\frac{\partial}{\partial\rho}V'\Big[\langle|\nabla\rho|\rangle v_{a\zeta} m_a n_a\langle Ru_{a\zeta}\rangle + (u_a^\rho - u_g^\rho)m_a n_a\langle Ru_{a\zeta}\rangle$$
$$-\langle|\nabla\rho|^2\rangle\chi_{a\zeta} m_a n_a\frac{\partial}{\partial\rho}\langle Ru_{a\zeta}\rangle + \langle\Pi_a^{\mathrm{res}}\rangle\Big] + \sum_b \ell_{11}^{\mathrm{ab}}\langle Ru_{b\zeta}\rangle - \sum_b \ell_{12}^{\mathrm{ab}}\frac{I}{\langle B^2\rangle}\langle B\hat{q}_{b\parallel}\rangle$$
$$+ e_a n_a\langle RE_\zeta\rangle + \boxed{e_a\frac{\partial\psi}{\partial\rho}n_a u_a^\rho} - S_{\mathrm{L}a} - S_{0a} - S_{\mathrm{CXi}} - S_{\mathrm{OLi}}$$

The magnetic force term produces not only a cross-field particle flux but also a $j\times B$ torque.

	TASK/TX	conventional transport codes
Radial force balance eqs.	Solved	Additional For estimating E_r
Toroidal flow eqs.	Solved	Additional, species-summed For estimating rot, E_r

Particle flux

- Neoclassical particle flux automatically arises due to friction and viscous forces.
  - Intrinsic ambipolarity
- Some model is required to bring about turbulent particle flux.
- We develop the model to describe the forces that impart the turbulent flux.
  - [Sugama and Horton, PoP 1995]
  - Ambipolar assumption and $\frac{\hat{f}_a}{f_a} \sim \frac{e_a \hat{\Phi}}{T_a} \sim \frac{\hat{n}_a}{n_a} \sim \frac{k_\parallel}{k_\perp} \sim \Delta, \quad \boldsymbol{K}_{aj\perp} \sim \Delta, \quad \boldsymbol{K}_{aj\parallel} \sim \Delta^2,$
  - Poloidal shift due to turbulence neglected.

$$\langle \boldsymbol{\Gamma}_a \cdot \nabla\psi \rangle = -\frac{1}{e_a}\langle R^2 \nabla\zeta \cdot (\boldsymbol{F}_a + e_a n_a \boldsymbol{E} + \underline{\boldsymbol{K}_{a1}}) \rangle$$

Adding this turbulent forces in the toroidal equation gives rise to

$$= \underbrace{-\frac{I}{e_a \langle B^2 \rangle}(\langle \boldsymbol{B} \cdot \nabla \cdot \overleftrightarrow{\pi}_a \rangle + \langle B K_{a1\parallel} \rangle)}_{\text{BP}} \underbrace{- \frac{I}{e_a}\left\langle \frac{F_{a\parallel}}{B}\left(1 - \frac{B^2}{\langle B^2 \rangle}\right)\right\rangle}_{\text{PS}}$$
$$+ \frac{1}{e_a}\left\langle \frac{\boldsymbol{B} \times \nabla\psi}{B^2} \cdot \boldsymbol{F}_a \right\rangle - I n_a \left\langle \frac{E_\parallel}{B}\left(1 - \frac{B^2}{\langle B^2 \rangle}\right)\right\rangle + \left\langle n_a \frac{\boldsymbol{E} \times \boldsymbol{B}}{B^2} \cdot \nabla\psi \right\rangle$$
$$- \frac{I}{e_a}\left\langle \frac{K_{a1\parallel}}{B}\left(1 - \frac{B^2}{\langle B^2 \rangle}\right)\right\rangle + \boxed{\frac{1}{e_a}\left\langle \frac{\boldsymbol{K}_{a1} \times \boldsymbol{B}}{B^2} \cdot \nabla\psi \right\rangle}$$

this perpendicular (radial) particle flux!

- The following expression of ***K*** yields the quasilinear particle flux.

$$\langle R K_{a1\zeta} \rangle = Z_a e \underbrace{\langle |\nabla V|^2 \rangle D_\mathrm{e}^\mathrm{QL}}_{\text{Diffusivity (amplitude)}} n_a \frac{\partial\psi}{\partial V}\left[(1 - \underbrace{C_\mathrm{T}}_{}) \frac{1}{n_\mathrm{e}}\frac{\partial n_\mathrm{e}}{\partial V} + \underbrace{C_\mathrm{T}}_{\text{Thermodiffusive pinch}} \frac{1}{n_\mathrm{e} T_\mathrm{e}}\frac{\partial n_\mathrm{e} T_\mathrm{e}}{\partial V} + \frac{\partial\psi}{\partial V}\underbrace{C_\mathrm{p}}_{\text{Other pinch term}}\right]$$

$$\langle \boldsymbol{\Gamma}_\mathrm{e}^\mathrm{QL} \cdot \nabla\psi \rangle = -n_\mathrm{e} \langle |\nabla\psi|^2 \rangle D_\mathrm{e}^\mathrm{QL}\left(\frac{1}{n_\mathrm{e}}\frac{\partial n_\mathrm{e}}{\partial\psi} + C_\mathrm{T}\frac{1}{T_\mathrm{e}}\frac{\partial T_\mathrm{e}}{\partial\psi} + C_\mathrm{p}\right)$$: QL particle flux

Heat transport equations (conservation of energy)

Terms and sources:

Conservation of momentum and energy due to collisions: $\sum_a \boldsymbol{F}_a = 0 \qquad \sum_a (Q_a + \boldsymbol{F}_a \cdot \boldsymbol{u}_a) = 0$

CGL stress tensor: $\langle \nabla \cdot \overset{\leftrightarrow}{\pi}_i \cdot \boldsymbol{u}_i \rangle = 0 \quad \langle \boldsymbol{u}_i \cdot \nabla \cdot \overset{\leftrightarrow}{\pi}_i \rangle = \hat{u}_{i\theta} \langle \boldsymbol{B} \cdot \nabla \cdot \overset{\leftrightarrow}{\pi}_i \rangle + \omega_i \langle R^2 \nabla \zeta \cdot \nabla \cdot \overset{\leftrightarrow}{\pi}_i \rangle = \hat{u}_{i\theta} \langle \boldsymbol{B} \cdot \nabla \cdot \overset{\leftrightarrow}{\pi}_i \rangle$

Heat flux closure: $\boldsymbol{q}_i = -n_i \chi_i \nabla T_i + p_i \boldsymbol{V}_i^{\mathrm{hp}}$

Energy exchange term between i and j : $Q_{ij} = \frac{3}{2} n_i \frac{T_j - T_i}{\tau_{ij}}$ where $\tau_{ij} = \frac{3\pi (2\pi)^{1/2} \epsilon_0^2 m_i m_j}{n_j e^4 Z_i^2 Z_j^2 \ln \Lambda_{ij}} \left(\frac{T_i}{m_i} + \frac{T_j}{m_j} \right)^{3/2}$

Heat sources and sinks (excerpt):

$$S_{\mathrm{E}e} = S_{\mathrm{E}e}^{\mathrm{NB}} + S_{\mathrm{E}e}^{\mathrm{NBmom}} + S_{\mathrm{E}e}^{\mathrm{iz}} + S_{\mathrm{E}e}^{\mathrm{RF}} + S_{\mathrm{E}e}^{\mathrm{rad}}$$

$$S_{\mathrm{E}i} = S_{\mathrm{E}i}^{\mathrm{NB}} + S_{\mathrm{E}i}^{\mathrm{NBmom}} - \frac{3}{2} n_i n_{01} \langle \sigma_{\mathrm{CX}} v \rangle (T_i - T_{01}) + S_{\mathrm{E}i}^{\mathrm{iz}} + S_{\mathrm{E}i}^{\mathrm{RF}} + S_{\mathrm{E}i}^{\mathrm{rad}}$$

electrons

$$\frac{3}{2} \frac{1}{V'^{5/3}} \frac{\partial}{\partial t} (V'^{5/3} p_{\mathrm{e}}) \bigg|_\rho = -\frac{1}{V'} \frac{\partial}{\partial \rho} V' \left[-n_{\mathrm{e}} \chi_{\mathrm{e}} \langle |\nabla \rho|^2 \rangle \frac{\partial T_{\mathrm{e}}}{\partial \rho} + p_{\mathrm{e}} \langle |\nabla \rho| \rangle V_{\mathrm{e}}^{\mathrm{hp}} + \frac{5}{2} p_{\mathrm{e}} \langle (\boldsymbol{u}_{\mathrm{e}} - \boldsymbol{u}_g) \cdot \nabla \rho \rangle \right]$$
$$- \langle \boldsymbol{u}_g \cdot \nabla \rho \rangle \frac{\partial p_{\mathrm{e}}}{\partial \rho} - \sum_{a \neq \mathrm{e}} \left(\langle \boldsymbol{u}_a \cdot \nabla \rho \rangle \frac{\partial p_a}{\partial \rho} + \underline{\hat{u}_{a\theta} \langle \boldsymbol{B} \cdot \nabla \cdot \overset{\leftrightarrow}{\pi}_a \rangle} + \langle Q_{a\mathrm{e}} \rangle \right) + \langle \boldsymbol{j} \cdot \boldsymbol{E} \rangle + \langle S_{\mathrm{Ee}} \rangle$$

ion species

$$\frac{3}{2} \frac{1}{V'^{5/3}} \frac{\partial}{\partial t} (V'^{5/3} p_i) \bigg|_\rho = -\frac{1}{V'} \frac{\partial}{\partial \rho} V' \left[-n_i \chi_i \langle |\nabla \rho|^2 \rangle \frac{\partial T_i}{\partial \rho} + p_i \langle |\nabla \rho| \rangle V_i^{\mathrm{hp}} + \frac{5}{2} p_i \langle (\boldsymbol{u}_i - \boldsymbol{u}_g) \cdot \nabla \rho \rangle \right]$$
$$+ \langle (\boldsymbol{u}_i - \boldsymbol{u}_g) \cdot \nabla \rho \rangle \frac{\partial p_i}{\partial \rho} + \underline{\hat{u}_{i\theta} \langle \boldsymbol{B} \cdot \nabla \cdot \overset{\leftrightarrow}{\pi}_i \rangle} + \sum_{a \neq i} \langle Q_{ia} \rangle + \langle S_{\mathrm{E}i} \rangle$$

viscous heating

Numerical implementation

- 24 equations (24 dependent variables) are simultaneously solved.
  - Finite element method (FEM; Petrov-Galerkin) with linear basis functions; V as the radial coordinate.
  - 5 for Maxwell's, 2 for cont., 6 for momentum, 2 for heat transp., 2 for heat flow, 2 for beam, 3 for neut.
  - 2 additional equations connecting $\langle R/u_{a\zeta}\rangle$ and $\langle Ru_{a\zeta}\rangle$ are solved.

$$\boxed{n_a\left\langle\frac{u_{a\zeta}}{R}\right\rangle=\frac{1}{\langle R^2\rangle}n_a\langle Ru_{a\zeta}\rangle-\frac{2\hat{q}^2}{1+2\hat{q}^2}\frac{1}{\langle R^2\rangle}n_a\langle Ru_{a\zeta}\rangle+\frac{2\hat{q}^2}{1+2\hat{q}^2}\frac{1}{I}n_a\langle Bu_{a\|}\rangle}$$

where $\hat{q}^2\equiv\frac{I^2}{2\langle B_p^2\rangle}\left(\left\langle\frac{1}{R^2}\right\rangle-\frac{1}{\langle R^2\rangle}\right)$

- Only for geometric quantities (metrics), do we have to use an equilibrium solver:
  - $\langle R^2\rangle, \langle R^{-2}\rangle, \partial V/\partial\rho, \langle|\nabla V|^2/R^2\rangle, \langle|\nabla V|\rangle, \langle|\nabla V|^2\rangle, \langle R^{-1}\rangle$
  - Any other quantities which the code entails can be internally evaluated.
  - In the following simulations, large aspect ratio plasmas of circular cross section are assumed.
- Boundary conditions (BCs)
  - Number of BCs depends upon the order of the highest order derivative of the dependent variables.

	axis	wall
Zero Dirichlet cond.	$\dot{\psi}_t, n_a u_a^V$	Φ, n_{02}, n_{03}
Zero Neumann cond.	$\Phi, \psi, n_a, n_a\langle Ru_{a\zeta}\rangle, n_a T_a, n_{01}, n_{02}, n_{03}$	$n_a, n_a\langle Ru_{a\zeta}\rangle, n_a T_a$

  - Neumann cond. at wall: $\left\langle\frac{|\nabla V|^2}{R^2}\right\rangle\frac{I_{\rm vac}}{4\pi^2}$ for $\dot{\psi}_t$, $2\pi I_{\rm p}$ for $\dot{\psi}$, $\langle|\nabla V|\rangle\Gamma^{\rm puff}$ for n_{01}

Summary

TASK/TX : 1D fluid-type transport code on the axisym. flux coordinates

$\Phi,\ \dot{\psi}_t,\ \dot{\psi},\ \psi,\ \psi_t,\ n_a,\ n_a u_a^V,\ n_\mathrm{b}\langle Bu_{\mathrm{b}\|}\rangle,\ n_a\langle Ru_{a\zeta}\rangle,\ n_a T_a,\ \langle B\hat{q}_{a\|}\rangle,\ n_a\langle u_{a\zeta}/R\rangle,\ n_\mathrm{b},\ n_\mathrm{b} u_\mathrm{b}^V,\ n_\mathrm{b}\langle Bu_{\mathrm{b}\|}\rangle,\ n_\mathrm{b}\langle Ru_{\mathrm{b}\zeta}\rangle,\ n_\mathrm{b}\langle u_{\mathrm{b}\zeta}/R\rangle,\ n_{01},\ n_{02},\ n_{03}$

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Quasi-neutrality	Intrinsically satisfied	Forced
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Continuity eqs.	Divergence of flux	Flux-gradient assumption
Radial force balance eqs.	Solved	Additional (for E_r)
Parallel flow eqs.	Solved	External neoclassical modules
Toroidal flow eqs.	Solved	Additional

- Numerical simulations show several features:
  - The particle flux is generated compatible with the neoclassical theory and the QL model.
  - Comparison of the parallel current shows good agreement.
  - TX and MI give the same parallel flows in a steady state.

Future work:

- ✧ **Submit the paper to the journal**
- ✧ **Including impurity species, coupling of TASK/TX and an equilibrium solver**
- ✧ **LH transition in the light of E_r**