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Impact of ion diamagnetic drift effect on MHD stability at edge pedestal of rotating tokamaks

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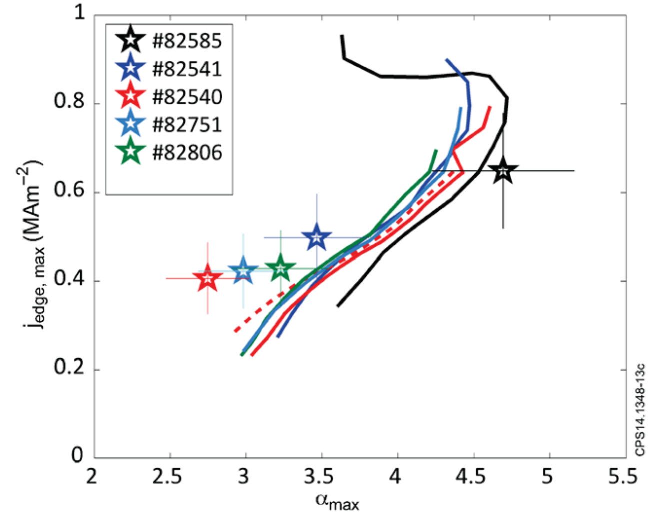
Introduction

- Peeling-ballooning mode (PBM) is the strongest candidate of the trigger of the type-I edge localized mode (ELM). [Snyder PoP2002 etc.]
- Usually, the toroidal mode number (n) of the mode is intermediate (~ 30), but the results in JT-60U and JET-ILW imply higher-n modes sometimes trigger type-I ELM. [Aiba NF2011, Giroud PPCF2015]
- Theoretically, such high-n modes are stabilized by an ion diamagnetic drift (ω_{*i}) effect.[Tang NF1980 etc.]

What causes the discrepancy between theory and experiment?

→ Rotation is a candidate of the key parameter.



PBM stability diagram in JET-ILW (n is up to 50), [Giroud PPCF2015]

Drift MHD model

To analyze the PBM stability with ω_{*i} effect in rotating plasmas, we use the drift MHD model derived by Hazeltine and Meiss. [Hazeltine and Meiss, Plasma Confinement]

$$\frac{DN}{Dt} + N\nabla\cdot\boldsymbol{V} = 0, \qquad \left.\frac{DP}{Dt}\right|_{MHD} + \Gamma P\nabla\cdot\boldsymbol{V}_{MHD} = 0,$$

$$\boldsymbol{E} + \boldsymbol{V}_{MHD}\times\boldsymbol{B} + \frac{1}{eN}\nabla P = 0, \qquad m_i N\left(\frac{D\boldsymbol{V}_E}{Dt} + \left.\frac{D}{Dt}\right|_{MHD}(V_\parallel \boldsymbol{b})\right) = \boldsymbol{J}\times\boldsymbol{B} - \nabla P.$$

$$\boldsymbol{V} = \boldsymbol{V}_{MHD} + \boldsymbol{V}_{pi} = \boldsymbol{V}_E + V_\parallel \boldsymbol{b}, \qquad \boldsymbol{V}_{pi} = \frac{\boldsymbol{B}\times\nabla p_i}{eZNB^2}, \qquad \boldsymbol{V}_E = \frac{\boldsymbol{E}\times\boldsymbol{B}}{B^2},$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + (\boldsymbol{V}\cdot\nabla), \qquad \left.\frac{D}{Dt}\right|_{MHD} = \frac{\partial}{\partial t} + (\boldsymbol{V}_{MHD}\cdot\nabla)$$

N: number density, $\boldsymbol{V}$: velocity, P: pressure, Γ: heat capacity ratio,

$\boldsymbol{E}$: electric field, $\boldsymbol{B}$: magnetic field, $\boldsymbol{J}$: plasma current, $\boldsymbol{b} \equiv \boldsymbol{B}/B$,

e: elementary charge, m_i: ion mass, Z: effective charge, p_i: ion pressure

Approximations used for simplifying the drift MHD model

We simplify the model with Frieman-Rotenberg formalism.

Approximations:

1. In Faraday's law, non-ideal term is neglected.

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times \boldsymbol{E} = -\nabla \times \left(\boldsymbol{V}_{MHD} \times \boldsymbol{B} + \frac{1}{eN} \nabla P \right)$$

   This approximation can be justified when

   a. Rotation is enough slow compared to ion thermal velocity.
   b. Density N or temperature T is constant in a plasma
   c. Functional form of N is proportional to that of T.

   (Details are in Appendix)

2. Magnetic field varies slowly $\nabla \times (\boldsymbol{b}/B) \ll 1$.

   This helps to change the continuity equation as follows.

$$\left. \frac{DN}{Dt} \right|_{MHD} + N \nabla \cdot \boldsymbol{V}_{MHD} = 0$$

Simplified drift MHD equation

Also, by assuming the incompressibility $\nabla \cdot \boldsymbol{\xi} = 0$, the flute approximation $(\boldsymbol{B} \cdot \nabla)\boldsymbol{\xi} \ll 1$ and $T_i = T_e$, we can derive the following equation [Aiba submitted to PPCF].
($T_i(T_e)$: ion (electron) temperature).

$$\rho_0 \frac{\partial^2 \boldsymbol{\xi}}{\partial t^2} + 2\rho_0 \left(\boldsymbol{V}_{0,MHD} \cdot \nabla\right) \frac{\partial \boldsymbol{\xi}}{\partial t} + \rho_0 \left(\boldsymbol{V}_{0,pi} \cdot \nabla\right) \frac{\partial \boldsymbol{\xi}_\perp}{\partial t} = \boldsymbol{F}_{MHD} + \boldsymbol{F}_{di},$$

$$\boldsymbol{F}_{MHD} = \boldsymbol{J}_0 \times \boldsymbol{B}_1 + (\nabla \times \boldsymbol{B}_1) \times \boldsymbol{B}_0 - \nabla P_1 + \nabla \otimes \left[\rho_0 \boldsymbol{\xi} \otimes (\boldsymbol{V}_0 \cdot \nabla)\boldsymbol{V}_{0,MHD} - \rho_0 \boldsymbol{V}_0 \otimes \left(\boldsymbol{V}_{0,MHD} \cdot \nabla\right)\boldsymbol{\xi}\right],$$

$$\boldsymbol{F}_{di} = \rho_0 \frac{\nabla \cdot (\boldsymbol{\xi} \times \nabla P_0)}{2en_0 B_0^2} (\boldsymbol{B}_0 \cdot \nabla)\boldsymbol{V}_{0,MHD,\perp},$$

$$\nabla \cdot \boldsymbol{\xi} = 0, \qquad \boldsymbol{V}_0 = \boldsymbol{V}_{0,MHD} + \boldsymbol{V}_{0,pi}, \qquad \boldsymbol{V}_{0,pi} = \frac{1}{2eZn_0 B_0^2} \boldsymbol{B}_0 \times \nabla P_0$$

MINERVA code[Aiba CPC2009] is updated to solve this equation.

➡ MINERVA-DI code

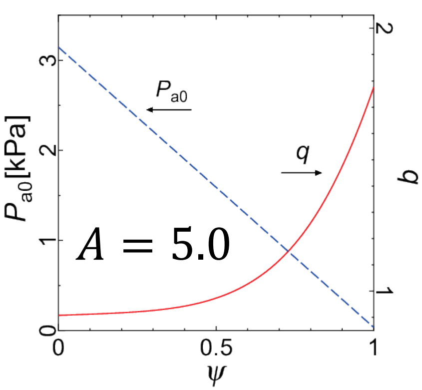
Benchmark test of MINERVA-DI

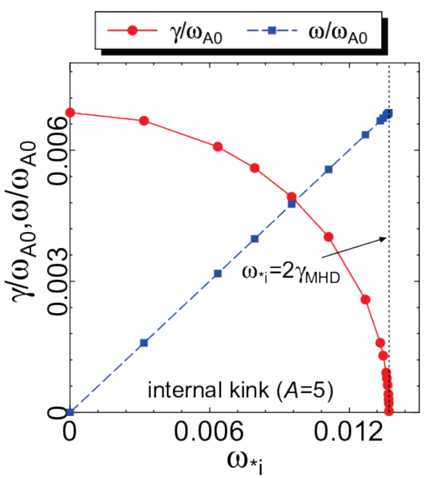
In a static plasma, the growth rate γ and the mode frequency ω can be estimated by the dispersion relation with $\omega_{*i}(=\boldsymbol{k}\cdot\boldsymbol{V}_{0,pi})$ effect as follows.[Tang NF1980 etc.]

$$\gamma + \iota\omega = \iota\frac{\omega_{*i}}{2} \pm \sqrt{\gamma_{MHD}^2 - \frac{\omega_{*i}^2}{4}},$$ γ_{MHD}:γ of ideal MHD mode

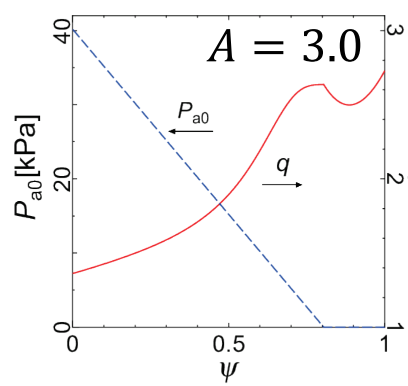
MINERVA-DI shows good agreements with this when internal kink and high-n ballooning stability is analyzed.

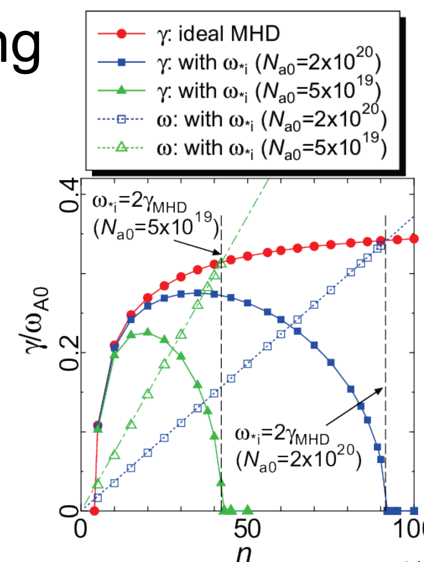
Internal kink mode ($n = 1$)





High-n ballooning mode





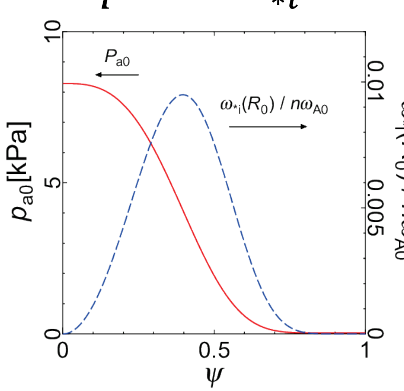
Ballooning mode stability with ω_{*i} is analyzed in rotating tokamaks

- Equlibrium has circular cross-section, and $A = 3.0$.
- Ideal ballooning mode is unstable for $n \geq 12$, but the mode is suppressed by ω_{*i} effect.
- Plasma (hydrogen) density is $N = 2.0 \times 10^{19} [1/m^3]$.
- Rotation profile is determined as follow.

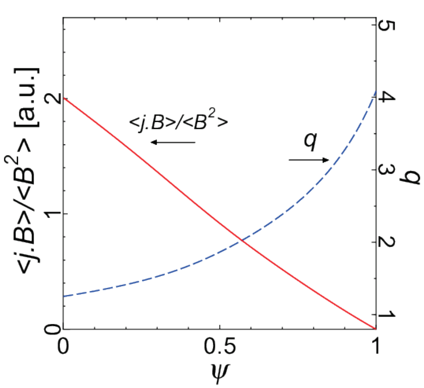
$$\Omega_\phi = C_* \omega_{*i,peak} \quad (0 \leq \psi \leq \psi_{peak})$$
$$= C_* \omega_{*i}(\psi, R_0) \quad (\psi_{peak} \leq \psi \leq 1)$$

Here $\omega_{*i,peak} = 9.51 \times 10^{-3} \omega_{A0}$ at $\psi = \psi_{peak}$, ω_{A0} is the toroidal Alfven frequency on axis.

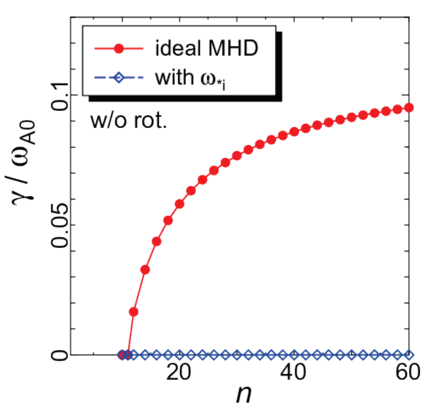
p and ω_{*i}



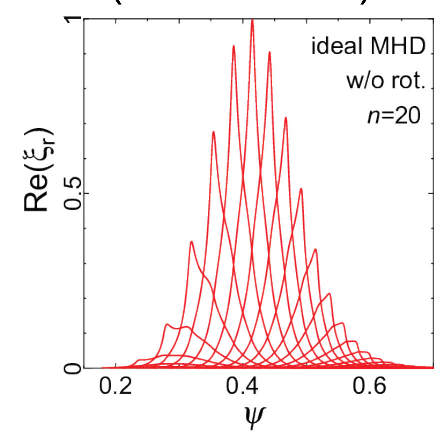
$\langle \boldsymbol{j} \cdot \boldsymbol{B} \rangle$ and q



Growth rate

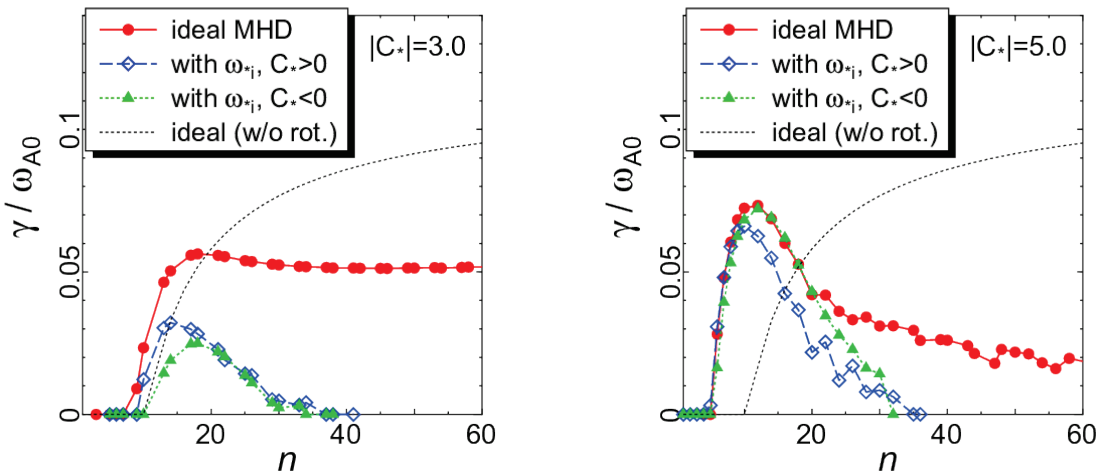


Eigen function (ideal mode)

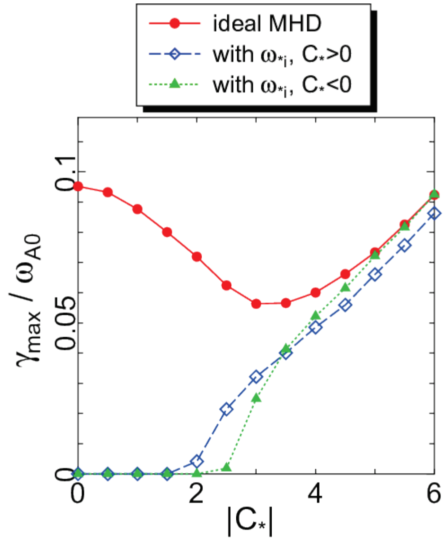


Plasma rotation can cancel ω_{*i} stabilizing effect on ballooning mode

Dependence of γ on n (left: $|C_*| = 3.0$, right: $|C_*| = 5.0$)



Dependence of γ on $|C_*|$

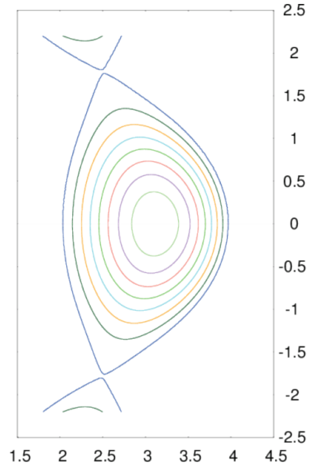


Plasma rotation changes ballooning mode stability as follows.

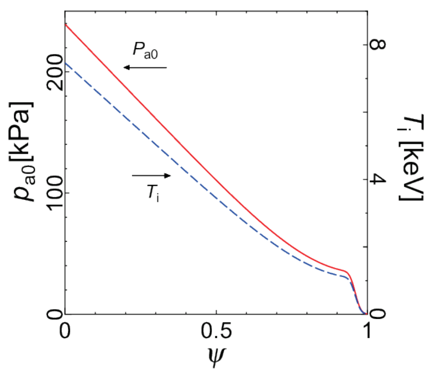
- Intermediate-n modes become unstable, though high-n modes are stabilized.
- Ballooning mode becomes unstable even when ω_{*i} effect is taken into account.
- Maximum γ of ballooning mode with ω_{*i} effect is converged to that of ideal ballooning mode.

PBM stability with ω_{*i} is analyzed in a rotating shaped tokamak

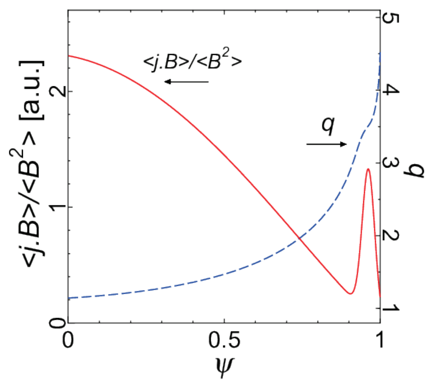
Contours of ψ



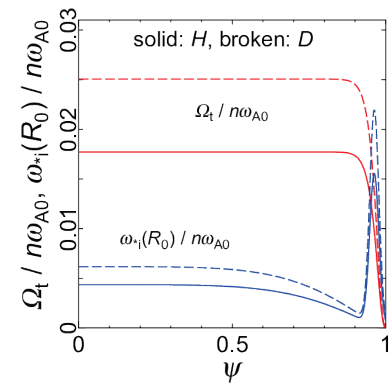
p and T_i



$\langle \boldsymbol{j} \cdot \boldsymbol{B} \rangle$ and q



Ω_ϕ and ω_{*i}



- PBM stability is analyzed in D-shaped (double-null) plasma.
- Density is assumed as $N = 1.0 \times 10^{20}[1/m^3]$.
- Rotation profile is determined as follow.

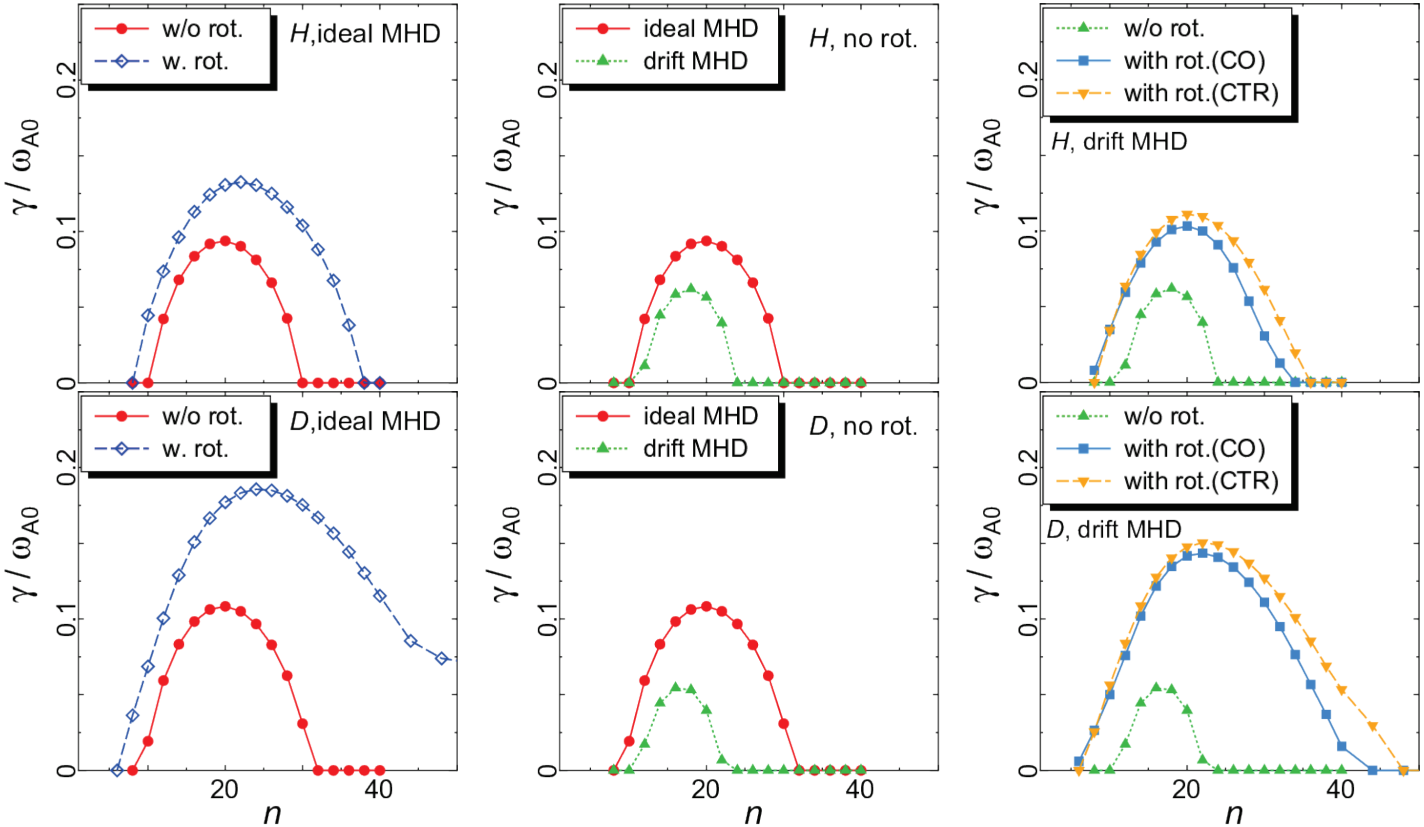
$$\Omega_\phi[\text{krad/s}] = 49.5(1-\psi^{48})^4 + 0.5.$$

- By changing bulk ion species from hydrogen H to deuterium D, Ω_ϕ/ω_{A0} and ω_{*i}/ω_{A0} is increased by factor of $\sqrt{2}$.
- PBM stability is analyzed in both H and D plasmas.

Rotation of "heavy" plasma has large impact on PBM stability

Results in H plasma

Results in D plasma



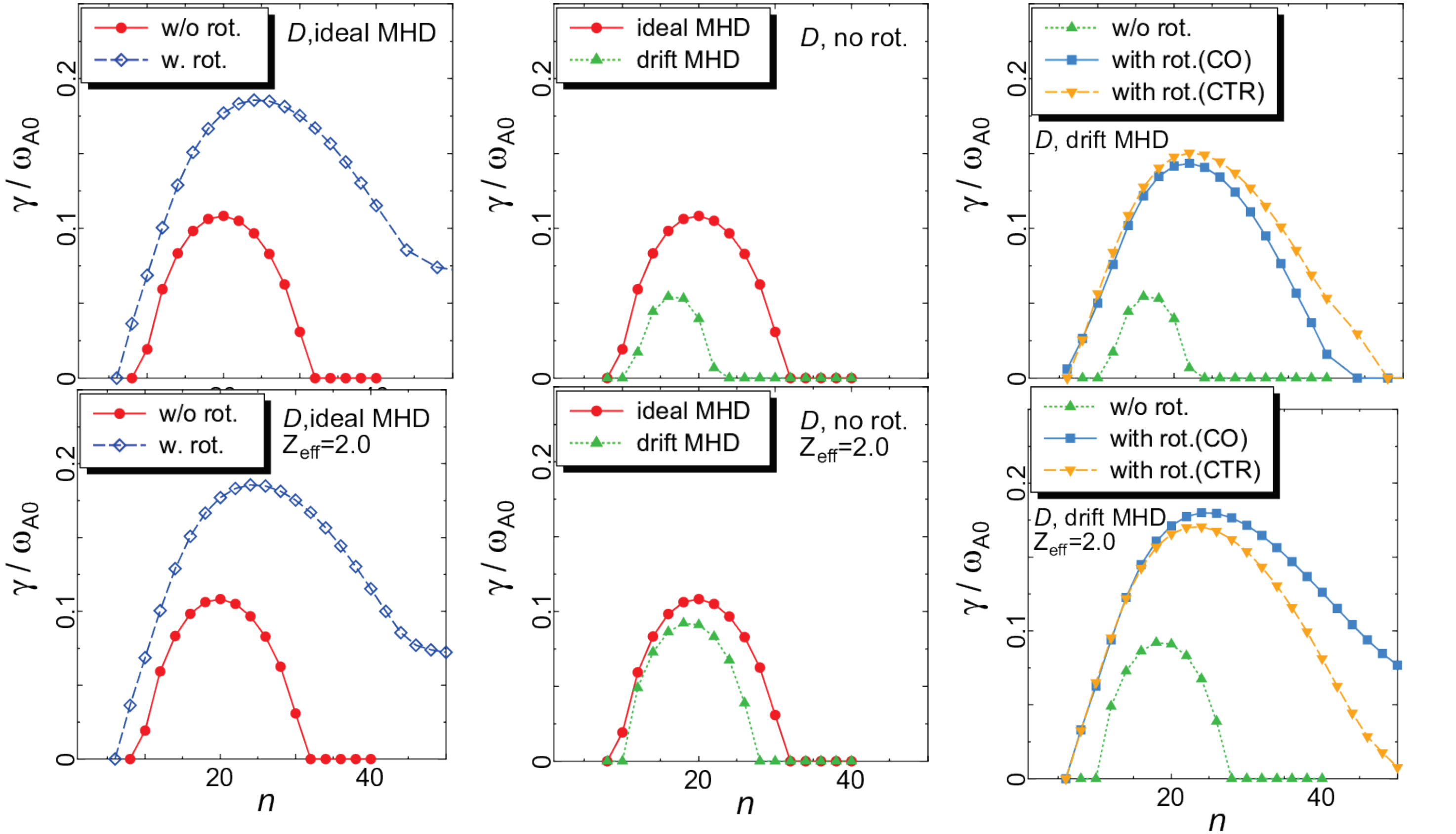
Large Ω_ϕ/ω_{A0} has impact on PBM stability, but impact of ω_{*i} on PBM is not affected by ion species so much,

➡ Plasma effective mass potentially changes (ideal) PBM stability.

Large Z_{eff} reduces impact of ω_{*i} on PBM stability

Results in D plasma ($Z_{eff} = 1.0$)

Results in D plasma ($Z_{eff} = 2.0$)



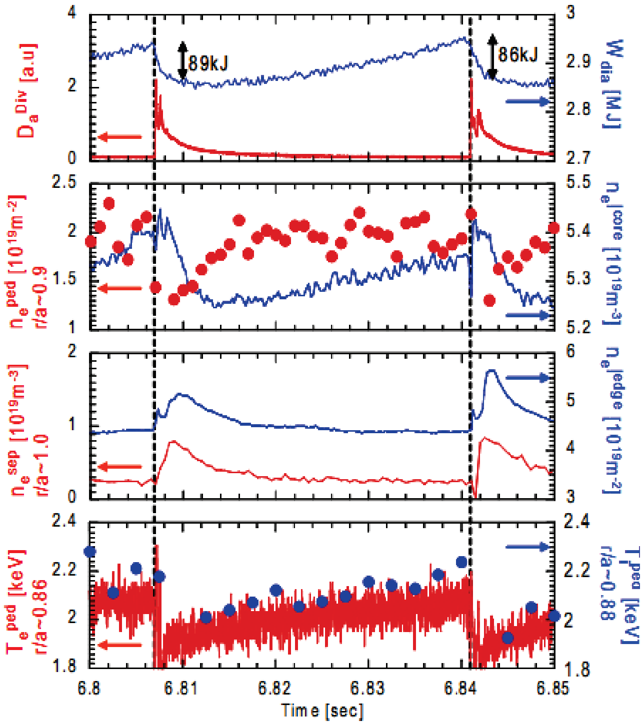
Impact of ω_{*i} on PBM stability depends on Z_{eff} due to $\omega_{*i} \propto Z_{eff}^{-1}$, but ideal MHD stability is independent from Z_{eff}.

➡ Plasma effective charge changes ω_{*i} effect on PBM stability.

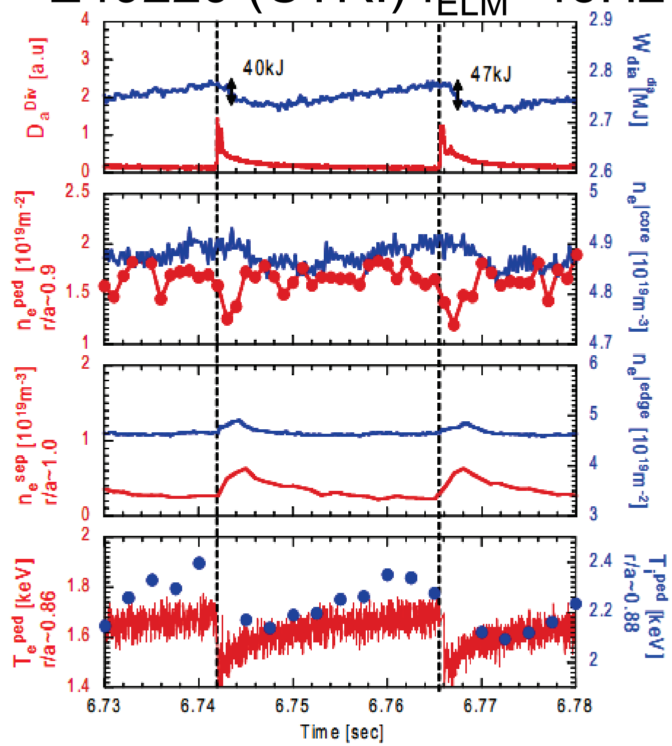
Rotation effect on type-I ELM in JT-60U

The equilibira analyzed numerically are E49228 and E49229; these are type-I ELMy H-mode plasmas [A. Kojima et al., NF, (2009)].

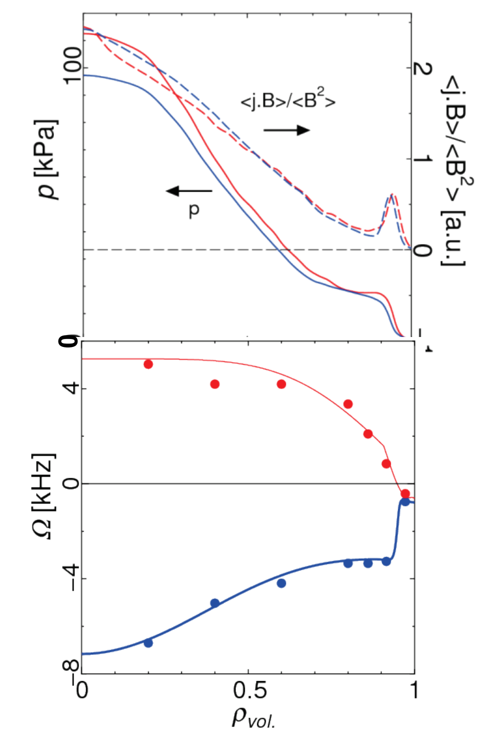
E49228 (CO.) f_{ELM}~37Hz



E49229 (CTR.) f_{ELM}~45Hz



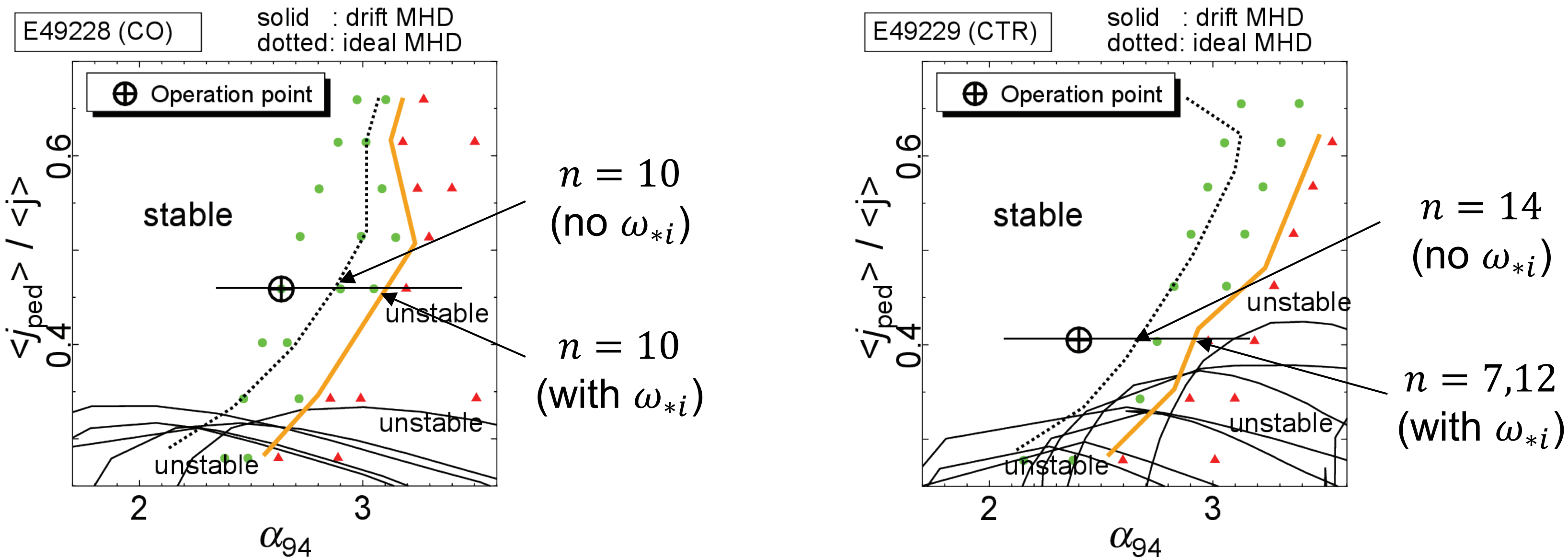
Profiles of *p*, ***j.B***, *Ω*



We compare the results of the numerical stability analysis of these equilibria just before ELM.

Stability boundary is shifted by ω_{*i} effect in static plasmas

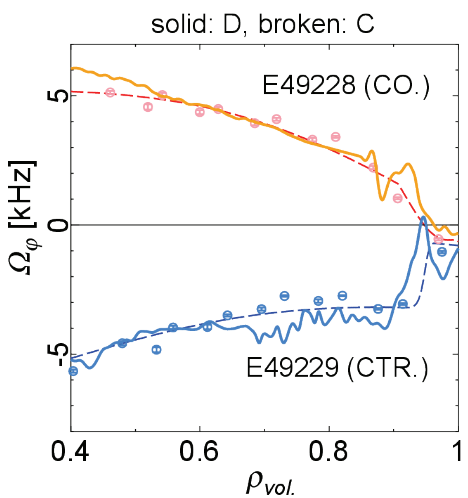
$\langle j_{ped} \rangle - \alpha_{94}$ diagram ($Z_{\rm eff} = 2.0$) (left:CO., right CTR.)



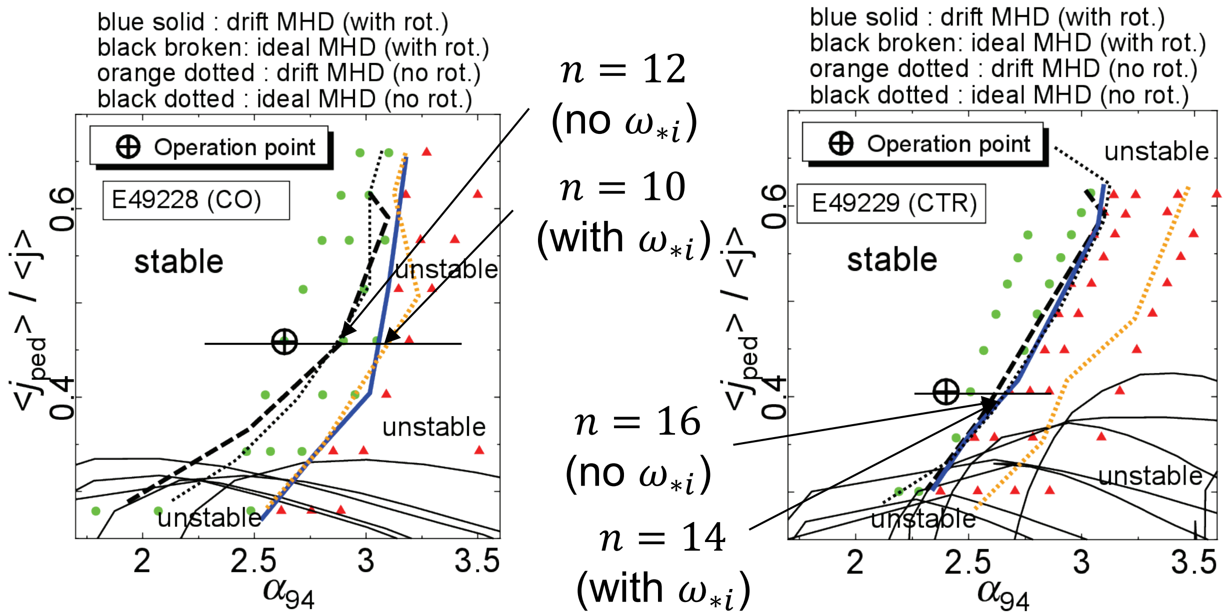
- When the plasma is assumed as static, PBM stability boundary is shifted to larger-α side by ω_{*i} effect as expected.
- Difference between operation point and boundary increases from ~10% to >20% of α_{94} on operation point in E49228, and from ~20% to ~40% in E49229.

Plasma rotation can counteract ω_{*i} effect on PBM stability boundary

Ω_ϕ of C and D



$\langle j_{ped} \rangle - \alpha_{94}$ diagram ($Z_{\rm eff} = 2.0$) (left:CO., right CTR.)



- Rotation profile of deuterium (D) is estimated with TOPICS integrated simulation code. [M. Honda NF2013]
- Rotation has little impact on pedestal stability in E49228.
- Rotation cancels ω_{*i} effect on pedestal stability in E49229, but the stability boundary is still far from operation point.

Poloidal rotation can affect edge MHD stability

Difference of the mode frequency $n\omega$ from the Doppler-shifted frequency **k.v** is essential for destabilizing edge MHD modes.[Aiba NF2011].

$$\boldsymbol{k}\cdot\boldsymbol{v} = -\imath n\Omega_\phi + \imath m\Omega_\theta$$

Ω_ϕ : toroidal rotation freq., Ω_θ : poloidal rotation freq.
n : toroidal mode number, m : poloidal mode number

For the Fourier harmonics which satisfy m-nq=0, $m\Omega_\theta \sim n\Omega_\phi$ even when $v_\theta=(nr/mR)v_\phi$ ~$0.1v_\phi$ if q=3 and R/r ~3.3.

$$= \imath\Omega_{\parallel}(\boldsymbol{k}\cdot\boldsymbol{B}/B) - \imath n\Omega_t$$

$\Omega_{\parallel}$: freq. parallel to magnetic field,
Ω_t : freq. in toroidal direction

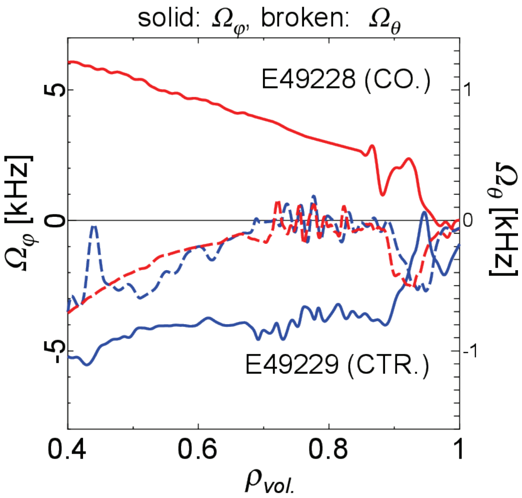
Near rational surfaces, **k.B**~0. → Ω_t will be important.

MINERVA(-DI) can identify the poloidal rotation effect on MHD stability (at present, poloidal rotation effect on equilibrium is neglected.)

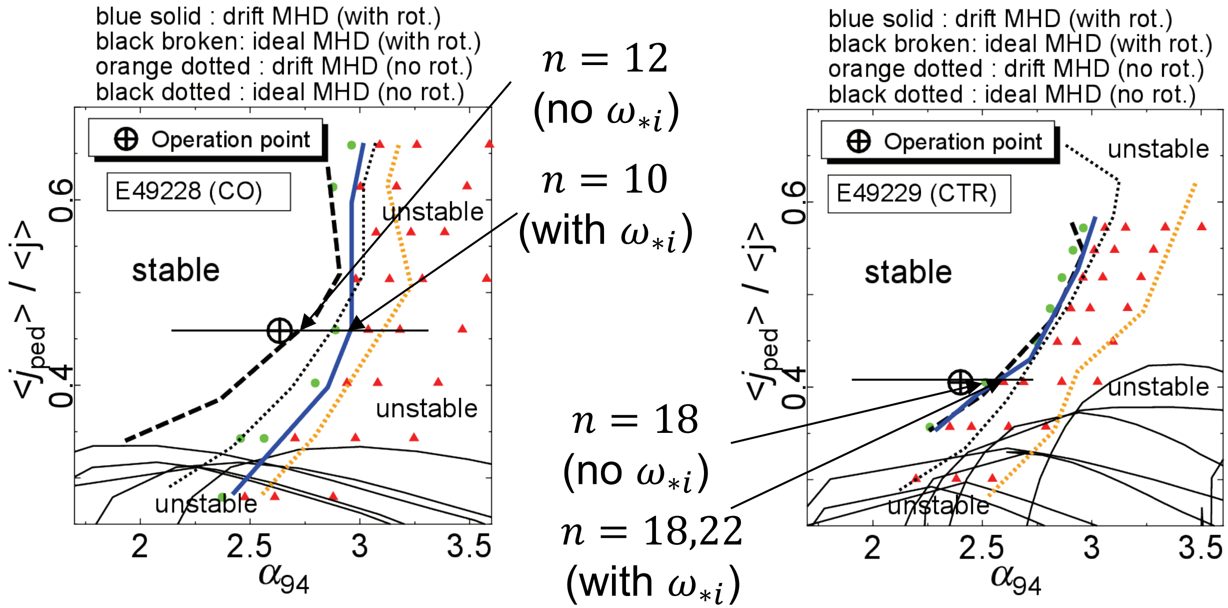
→ Re-evaluate stability diagram with not only Ω_ϕ but also Ω_θ.

PBM stability boundary can be predicted by counting Ω_ϕ and Ω_θ

Ω_ϕ and Ω_θ of D



$\langle j_{ped}\rangle - \alpha_{94}$ diagram ($Z_{\mathrm{eff}} = 2.0$) (left:CO., right CTR.)



- In E49228, poloidal (+toroidal) rotation moves the ideal MHD boundary near operation point, but ω_{*i} effect shifts the boundary far from the point.
- In E49229, ω_{*i} effect is negligible as the toroidally rotating case, and the boundary becomes closer operation point.

Summary

- Linearized drift MHD equation was derived with Frieman-Rotenberg formalism.
- MINERVA-DI code was developed to solve the drift MHD equation.
- It is found that ion diamagnetic drift (ω_{*i}) effect on ballooning/peeling-ballooning(PBM) stability can be canceled by plasma rotation.
- Effective mass/charge have potential to change PBM stability when rotation and ω_{*i} are counted.
- Cancellation of ω_{*i} effect by rotation plays an important role on PBM stability boundary in JT-60U.
- PBM stability analysis in JET ITER-like metallic wall is on going, and the result will be reported in near future.