# *Gyrokinetic Analysis of Vortex Structures and Turbulent Transport in Slab Electron Temperature Gradient Turbulence*

*- スラブETG乱流における乱流輸送と渦構造のジャイロ運動論による解析 -*

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第11回 若手科学者によるプラズマ研究会, 3/17-19, 2008

# *Outline*

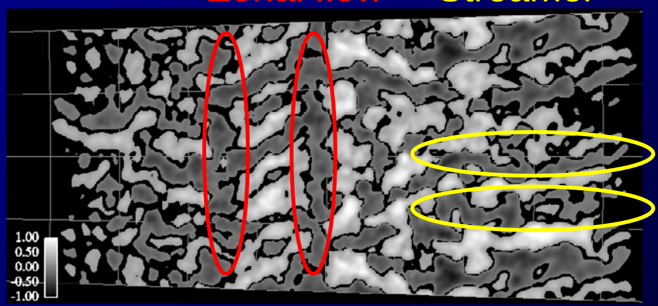
- Background and motivation in this study
- About Gyrokinetic model
- Gyrokinetic Vlasov simulation
  - Linear simulation of slab ETG mode
  - Comparison between slab ETG and ITG turbulence
  - Transport reduction by coherent vortex structure
  - Strong zonal flow in marginally unstable state
- Summary

# *Background and Motivation*

*Anomalous (Turbulent) transport and its reduction by the self-generated zonal flow are key issues for magnetic confinement fusion research.*

- Strong anomaly of electron heat transport
  - Transport experiments observe large electron heat transport compared to ion heat transport ( strong anomaly of the electron turbulent transport ).
  - In ETG turbulence, saturation mechanisms of the turbulence are attracting much interest (e.g. A saturation of radially elongated vortices so called "streamer" ).
- Turbulent transport & vortex structures
  - In ETG turbulence, various vortex structures are formed because of the strong fluctuations caused by the weak zonal flow.

  → Understanding of the relation between turbulent transport and vortex structures is a significant issue.




Y.Idomura et al., NF(2005)

- Gyrokinetic Vlasov simulation for ETG turbulence

- Vortex structures and related transport in slab ETG turbulence are investigated by Vlasov-type simulation with high resolution here.

# *Gyrokinetic models*

# *What is Gyrokinetics ?*

- Vlasov (or Boltzmann) equation is "the first principle" for describing collisionless plasma behavior.

$$\partial_t \mathcal{F}(\boldsymbol{x},\boldsymbol{v},t) + \boldsymbol{v}\cdot\partial_{\boldsymbol{x}}\mathcal{F} + \frac{q}{m}(\bar{\boldsymbol{E}} + \boldsymbol{v}\times\bar{\boldsymbol{B}})\cdot\partial_{\boldsymbol{v}}\mathcal{F} = 0$$

→ However, a microscopic time scale of $\Omega^{-1}$ is included.

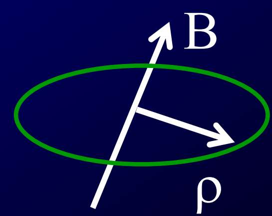
- Deriving a time-averaged kinetic equation eliminated the fast gyro-motion from the original Vlasov equation.

→ For equilibrium distribution $F_0$ : Drift-kinetic equation
( not including finite gyroradius effect )

- Given quiescent fields are often assumed.
- Scale length of $F_0$ is order of minor radius : $L$

Neoclassical transport, BS-current, Particle orbit

→ For perturbed distribution $\delta f$ : Gyrokinetic equation
( including finite gyroradius effect )



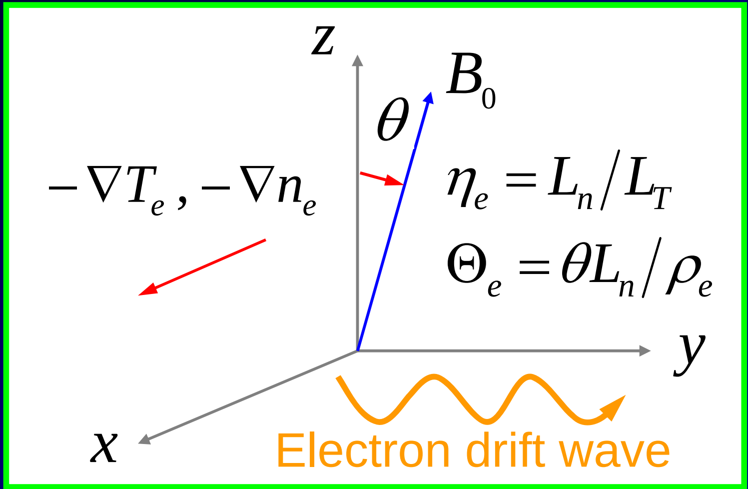

- Fields are determined self-consistently with Maxwell eqs.
- Scale length of $\delta f$ is order of gyroradius : $\rho$

Turbulent transport, Microinstabilities

A. J. Brizard and T. S. Hahm, Rev.Mod.Phys.(2007)

# Simulation model and conditions

- Shearless slab configuration




2-dimension for real space & 1-dimension for velocity space

Normalization with GK orderings

$$\left(\frac{x}{\rho_e}, \frac{y}{\rho_e}, \frac{v_\parallel}{v_e^{th}}, \frac{v_e^{th}}{L_n} t\right) \to (x, y, v_\parallel, t), \quad \left(\frac{L_n}{\rho_e}\frac{v_e^{th}}{n_0}\delta f_e^{(g)}, \frac{v_e^{th}}{n_0} F_M^{\parallel}, \frac{L_n}{\rho_e}\frac{e\delta\phi}{T}\right) \to (\delta f_e^{(g)}, F_M^{\parallel}, \delta\phi)$$

Simulation model

$$\frac{\partial}{\partial t}\delta f_{ek}^{(g)} + i\Theta_e k_y v_\parallel \delta f_{ek}^{(g)} - \sum_{k'+k''=k} \boldsymbol{b}\cdot(\boldsymbol{k}' \times \boldsymbol{k}'')\delta\psi_{k'}\delta f_{ek''}^{(g)}$$
$$= -ik_y\delta\psi_k F_M^{\parallel}\left[1 + \frac{\eta_e}{2}(v_\parallel^2 - 1 - b_k) - \Theta_e v_\parallel\right] + \mathcal{C}_\parallel(\delta f_{ek}^{(g)})$$
$$\text{where, } \mathcal{C}_\parallel(\delta f_{ek}^{(g)}) = \nu_e \frac{\partial}{\partial v_\parallel}\left[\frac{\partial}{\partial v_\parallel} + v_\parallel\right]\delta f_{ek}^{(g)}$$

Lenard-Bernstein collision operator

- Gyrokinetic Vlasov code(3D) for slab ETG turbulence

Toroidal 5D-GKV code: T.-H.Watanabe et al., NF(2006)

- Spectral methods + Runge-Kutta-Gill integration + MPI parallelization in v-space.
- (Mode number (kx,ky), velocity grid number) = (129,257,1024) or (129,257,2048)

- Periodic boundary conditions for x and y directions
- Parameters: $\Theta = \{0.5, 1.0, 2.5\}$, $\eta = \{6, 10\}$, $\nu = 1.25\text{E}-3$ ( fixed )

# Gyrokinetic entropy balance

- Entropy balance derived from GKE.

$$\boxed{\frac{d}{dt}\{\delta S + W_p\} = \eta_e Q_e + D_c}$$

Generation of fine-scale fluctuation by ballistic mode (phase mixing)

Heat transport

Smoothing of fine-scale fluctuation by collisional dissipation

- The definition of each term

$$\begin{cases} \delta S = \sum_k \int dv_\parallel \frac{|\delta f_{ek}^{(g)}|}{2F_M} \\ W_p = \sum_k \frac{-e}{2T_e} \Lambda_k^e |\delta\phi_k|^2 \\ \eta_e Q_e = \sum_k \frac{\boldsymbol{q}_\perp}{T_e} \cdot (-\nabla \ln T_e), \quad \left( \boldsymbol{q}_\perp = \frac{n_0}{2} \sum_k \mathrm{Re}[\boldsymbol{v}_k^{\mathbf{E}} T_{e-k}] \right) \\ D_c = \sum_k \int dv_\parallel \frac{\delta f_{e-k}^{(g)}}{F_M} C_\parallel [\delta f_{ek}^{(g)}] \end{cases}$$

( Fluctuation entropy variable )

( Potential energy )

( Turbulent heat flux )

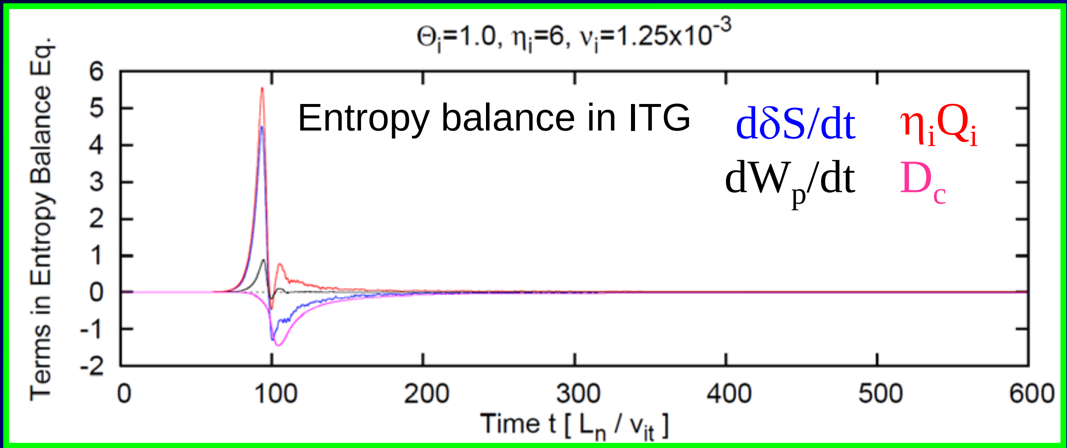
( Collisional dissipation )

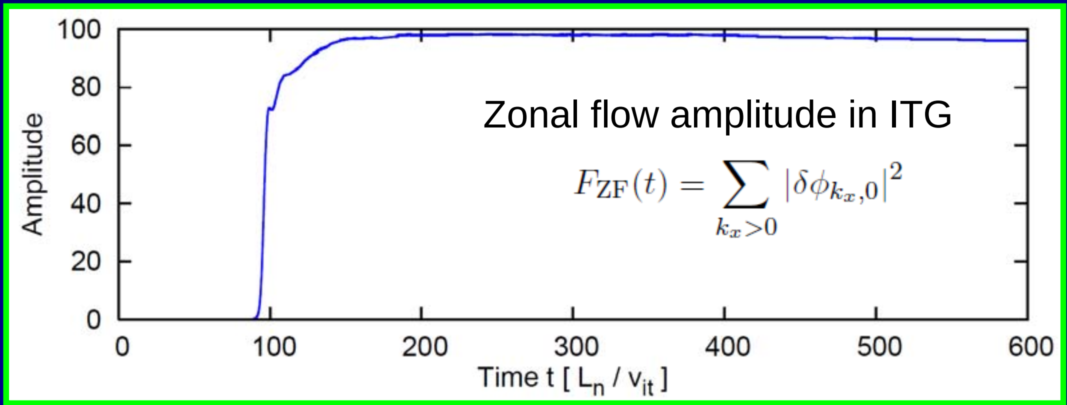
In the statistical steady state with dissipation: $\boxed{\eta Q = -D_c}$

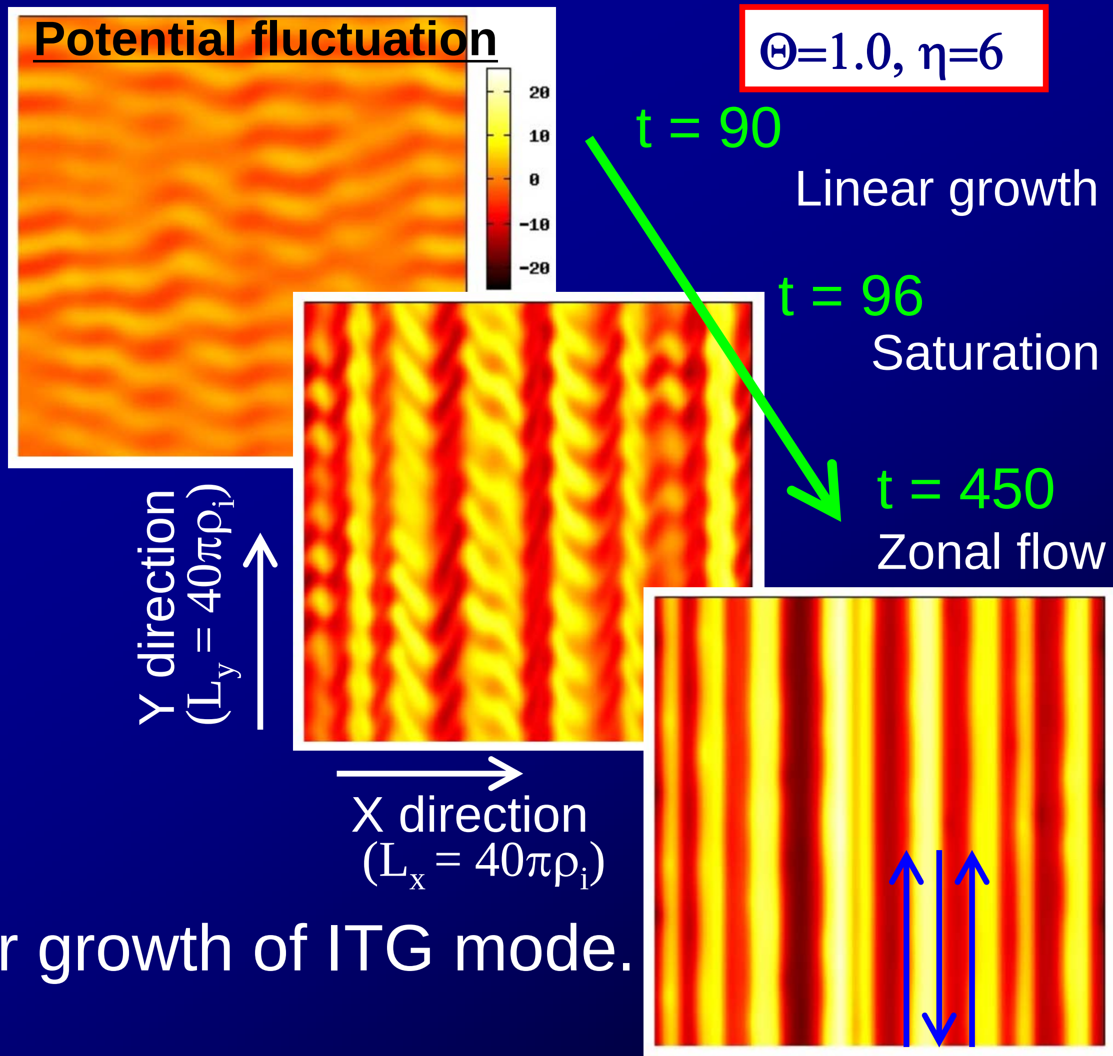
# *Gyrokinetic Vlasov simulation*

# *Slab ITG turbulence*

- Evolution of the entropy balance and the zonal flow amplitude.



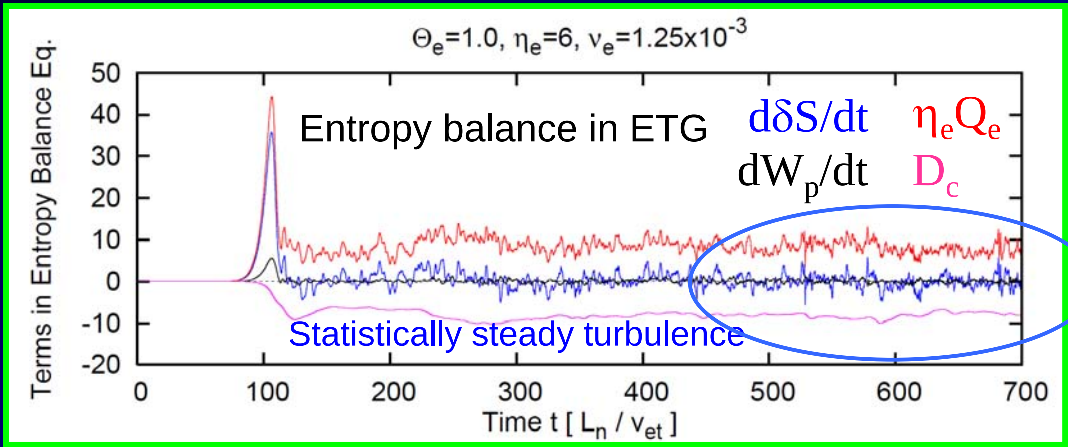

Strong zonal flow saturates the linear growth of ITG mode.

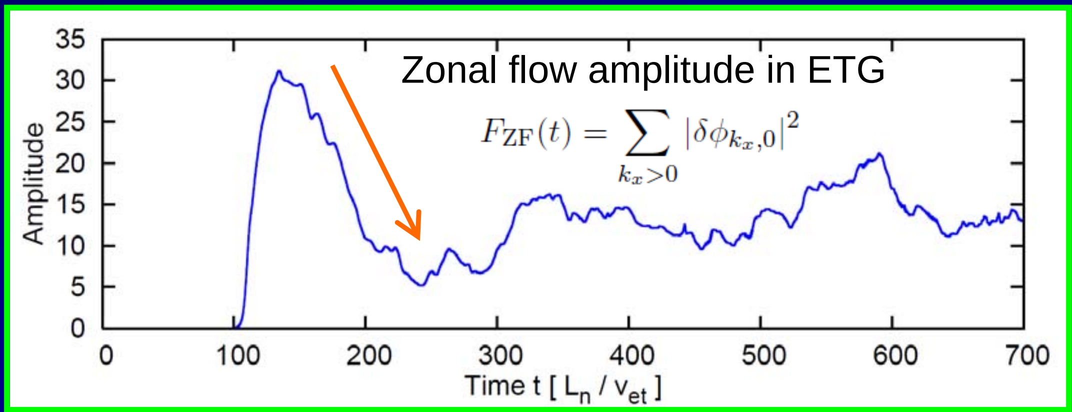
In slab ITG turbulence,

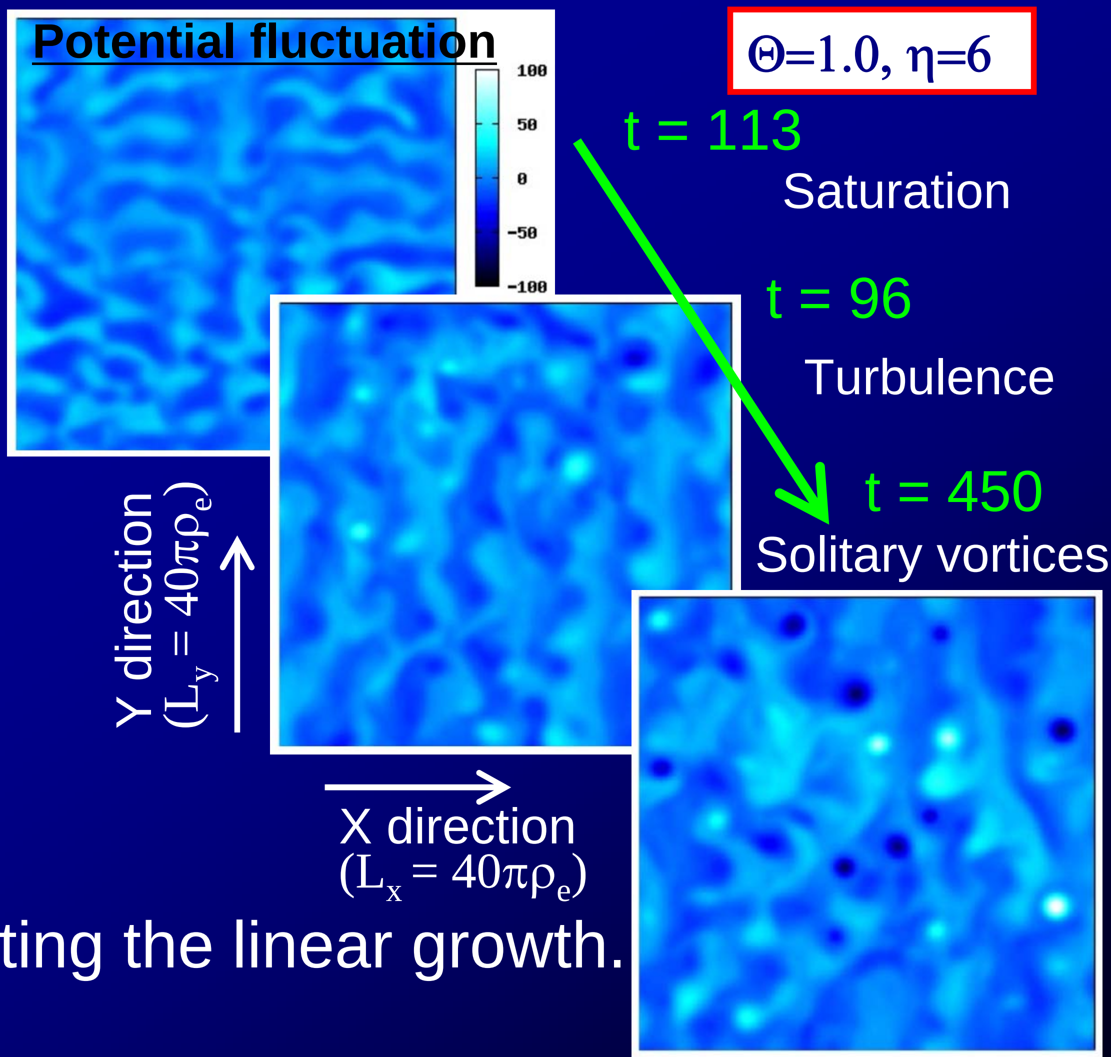
the turbulent transport is completely suppressed by steady zonal flow.

# Slab ETG turbulence

- Evolution of the entropy balance and zonal flow amplitude.



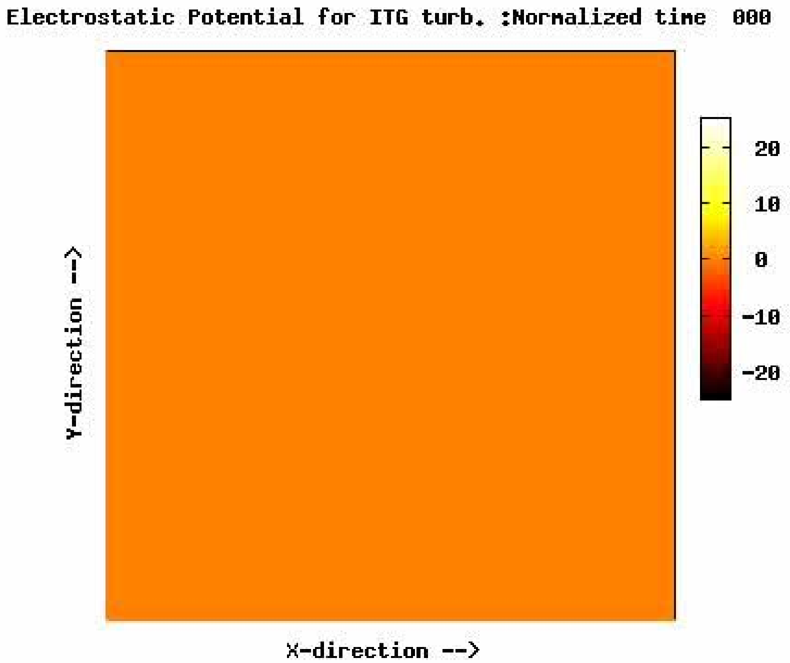



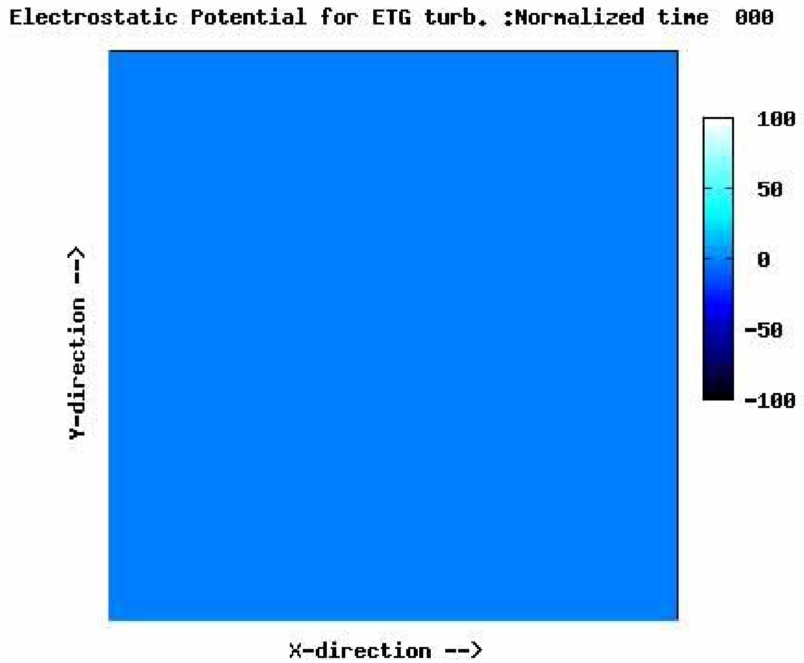

Zonal flow damps quickly after saturating the linear growth.

In slab ETG turbulence,

the steady turbulent transport is driven with many solitary vortices.

# Evolutions of potential fluctuation




Slab ITG turbulence
with Strong zonal flow

Slab ETG turbulence
with solitary vortices

# *Various vortex structure in slab ETG turbulence*

- The levels of turbulent transport and vortex structures depend on the parameter Θ and η in slab ETG turbulence.

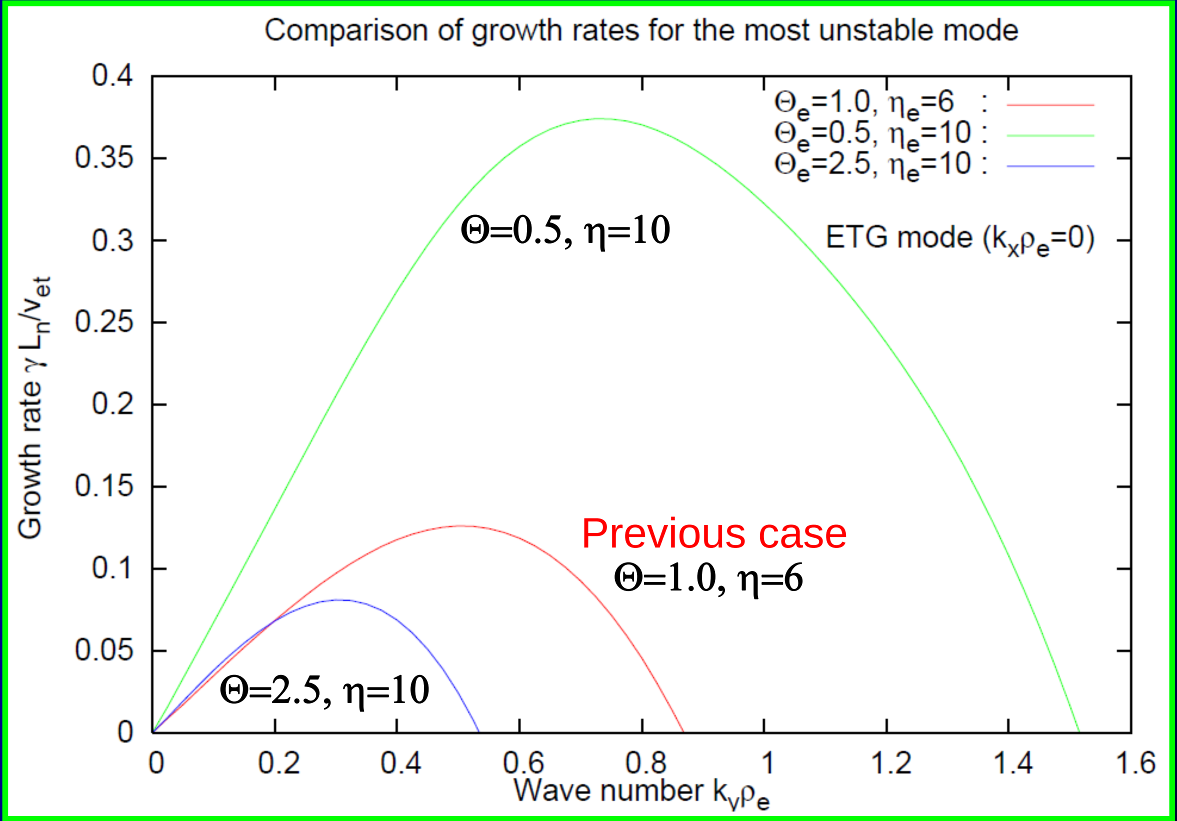
Transport and vortex structures in other 2 types of parameters were investigated by nonlinear simulation of slab ETG turbulence with high resolution.

**Θ=0.5, η=10**

- Unstable modes spreads with higher growth rate. (green line)

**Θ=2.5, η=10**

- Marginally unstable state with lower growth rate. (blue line)

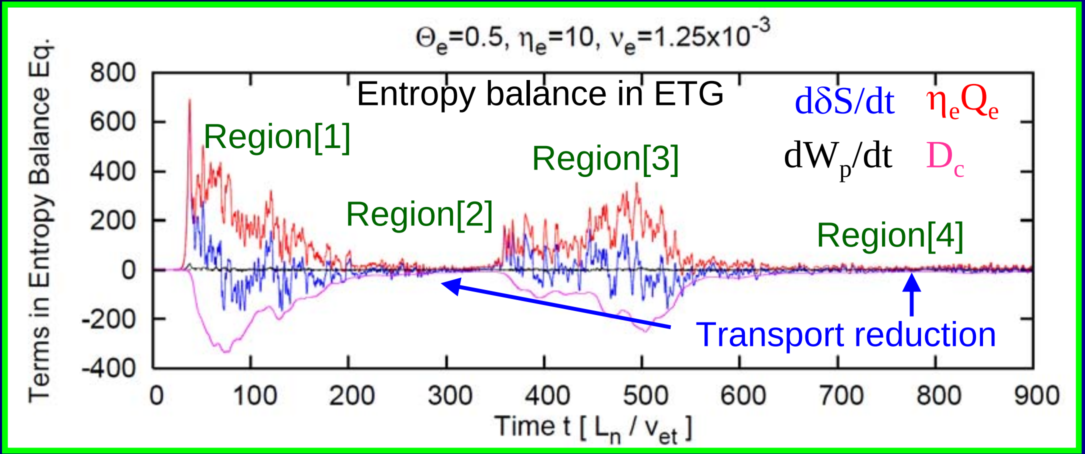


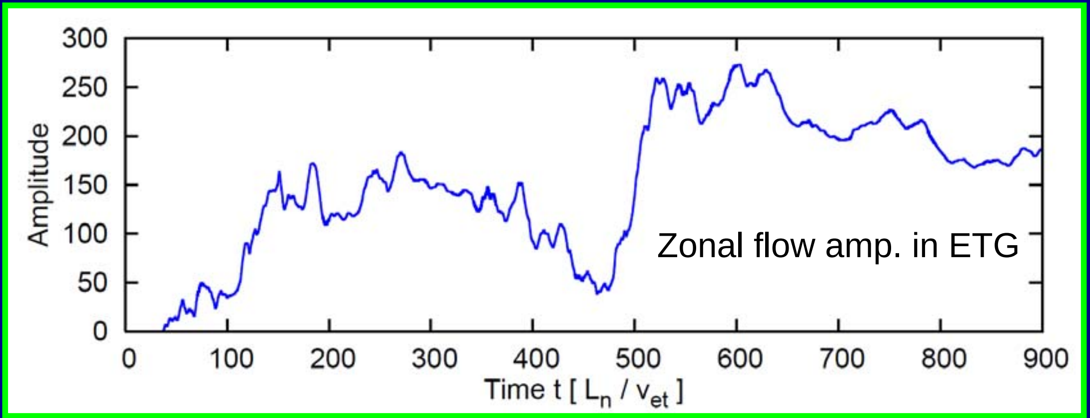

<u>Various vortex structures can be generated in slab ETG turbulence not only the zonal flow similar to slab ITG turbulence.</u>

# Turbulent vortex & coherent vortex

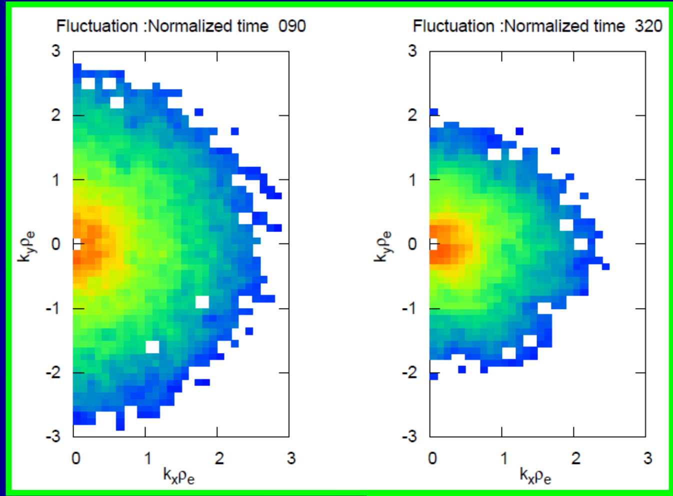
- In high unstable case, the turbulent transport reduction occurs in region[2] and [3] although the saturation peak level is much high.

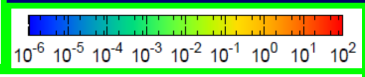
$\Theta$=0.5, $\eta$=10



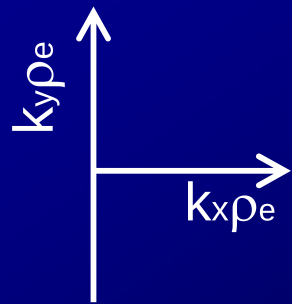



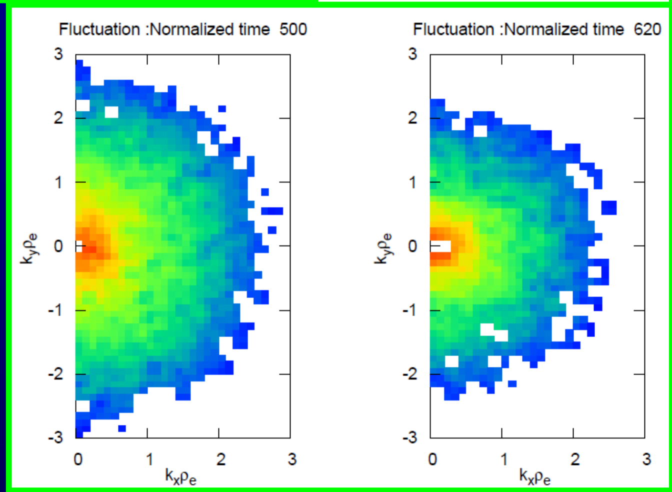

Region[1] Region[2]



Spectra of the turbulence fluctuation



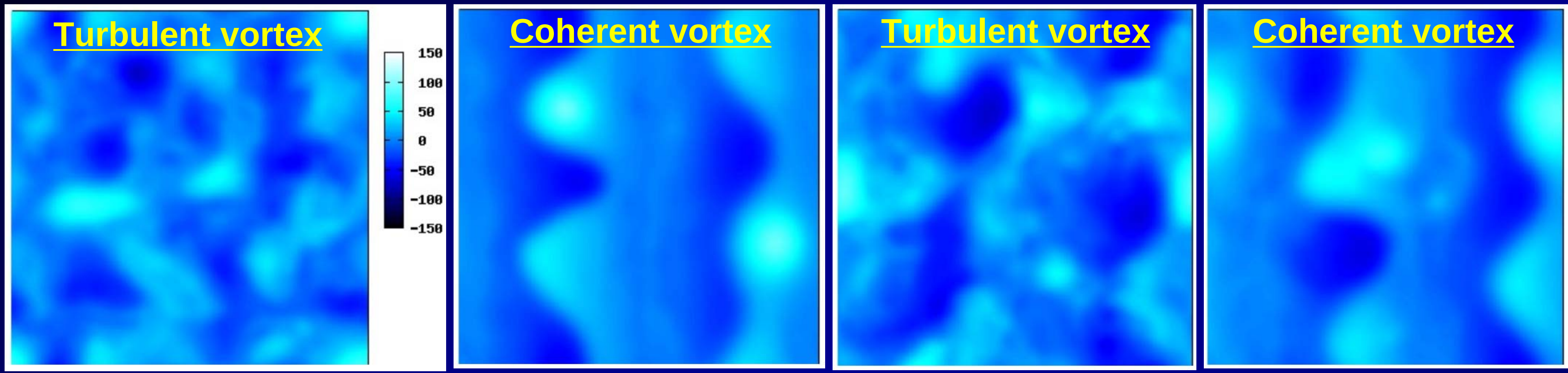

Region[3] Region[4]

<u>These transport reductions correspond to reduction of high wave number component of fluctuation.</u>

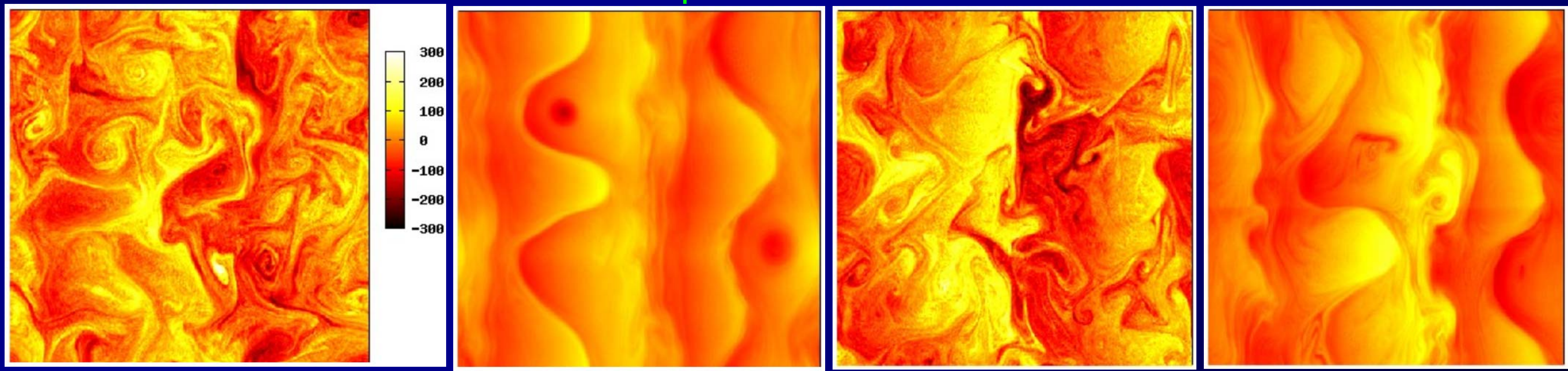
- High k-component reduction and coherent vortex structure

Electrostatic potential fluctuation

( $L_{x,y} = 20\pi\rho_e$ )




Temperature fluctuation




Region[1] Region[2] Region[3] Region[4]

<u>In coherent vortex state, the phase difference of these fluctuations( turbulent transport ) decrease.</u>

$$q_x = \frac{n_0 c}{2B} \sum_k \mathrm{Re}\left[ i k_y \delta\phi_k^* \delta T_k \right]$$

# *Summary*

- Entropy balance in slab ETG turbulence has been evaluated by gyrokinetic Vlasov simulation for the first time. Statistically steady state in slab ETG turbulence with high transport level compared to slab ITG turbulence are found. Further, the transport reduction are found in high unstable parameter region.

- In high unstable case, the turbulent transport reduction are caused as the result of the coherent vortex structure between potential fluctuation and temperature fluctuation.

- Steady dipole vortex structure on zonal flow are found in marginally unstable case. The relation with Hasegawa-Mima dipole vortex solution needs further discussions.

# *Appendix*

# *Quasi-neutrality & Adiabatic response*

- Relation between gyrocenter and particle distribution function

$$\delta f_k = \delta f_k^{(\mathrm{g})} e^{-i\boldsymbol{k}\cdot\boldsymbol{\rho}} + \frac{e\delta\phi_k}{T}\underbrace{[1 - J_0(k_\perp\rho)e^{-i\boldsymbol{k}\cdot\boldsymbol{\rho}}]}_{\text{Polarization effect}} F_M^{\parallel}$$

- Quasi-neutrality condition ( $\delta n_i = \delta n_e$ ) :

$$\int d\boldsymbol{v}\ J_0(k_\perp\rho_i)\delta f_{ik}^{(\mathrm{g})} - \underbrace{n_0\frac{e\delta\phi_k}{T_i}[\ 1 - \Gamma_0(b_k^i)\ ]}_{\text{Ion perpendicular adiabatic motion in ETG}} = \int \underbrace{d\boldsymbol{v}\ J_0(k_\perp\rho_e)\delta f_{ek}^{(\mathrm{g})}}_{\text{Electron parallel adiabatic motion in ITG}} + n_0\frac{e\delta\phi_k}{T_e}[\ 1 - \Gamma_0(b_k^e)\ ]$$

$$\text{where, } \Gamma_0(b_k) = I_0(b_k)e^{-b_k}\ \ (\ I_n\ :\ \text{Modified Bessel function}\ )$$

- Different adiabatic response between ITG & ETG :

$$\delta n_k^{(i,e)} = \Lambda_k^{(i,e)}\delta\phi_k$$

Ion (or Electron) adiabatic response

$$\begin{cases} \Lambda_k^i = \dfrac{en_0}{T_e}\left[(1 - \underline{\delta_{k_\parallel,0}}) + \dfrac{T_e}{T_i}(1 - \Gamma_0(b_k^i))\right] & \text{for ITG} : \underline{k_\perp\rho_e \ll 1\ (\Gamma_0(b_k^e) \to 1)} \\ \Lambda_k^e = -\dfrac{en_0}{T_i}\left[1 + \dfrac{T_i}{T_e}(1 - \Gamma_0(b_k^e))\right] & \text{for ETG} : \underline{k_\perp\rho_i \gg 1\ (\Gamma_0(b_k^i) \to 0)} \end{cases}$$

The origin of different zonal flow response between ITG & ETG

# *Linear simulation*

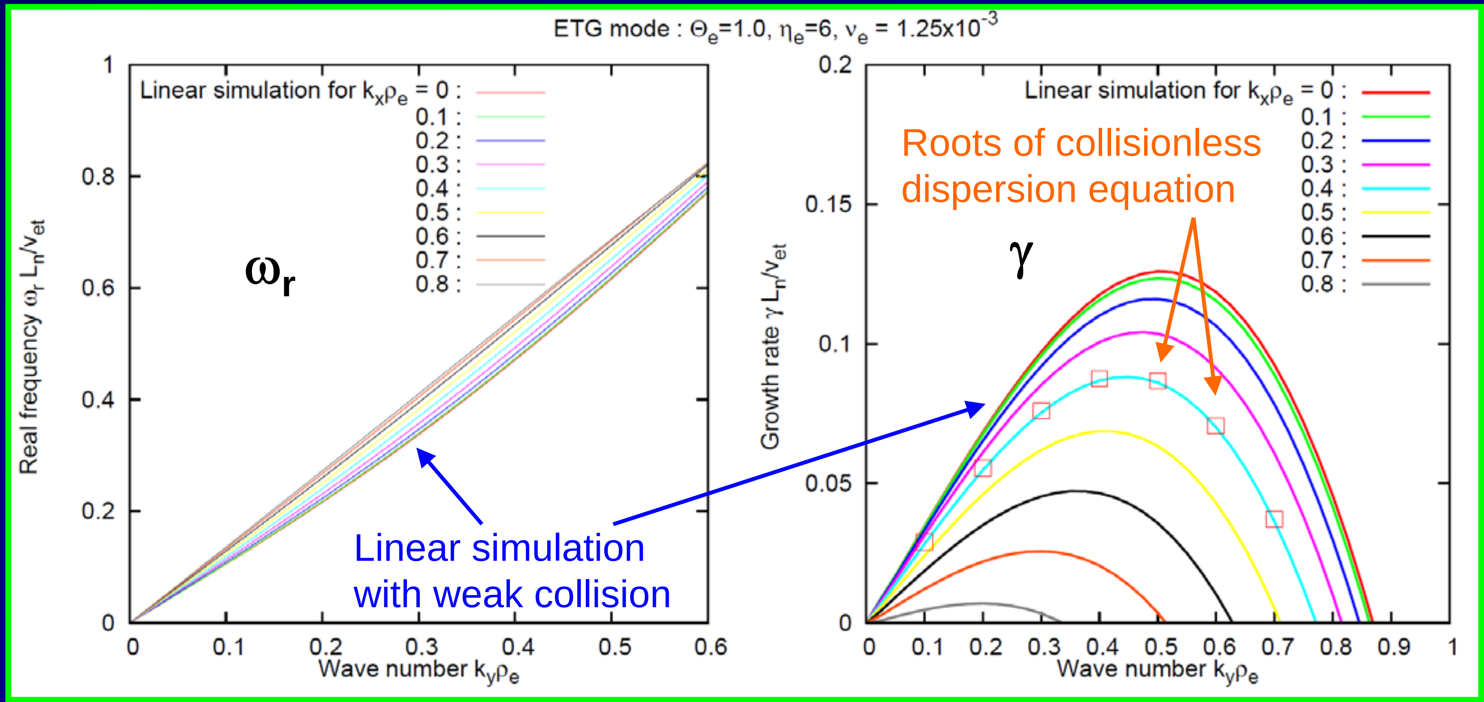
- Linear dispersion equation for ETG mode

$$1+\tau^{-1}-\Gamma_0(b_k^e)+e^{-b_k^e}\left[1-\frac{\eta_e}{\sqrt{2}\Theta_e}\zeta-\left\{\frac{1}{\sqrt{2}\Theta_e}\left(1-\frac{1}{2}(1+b_k^e)\right)-\zeta-\frac{\eta_e}{\sqrt{2}\Theta_e}\zeta^2\right\}Z(\zeta)\right]=0,$$

$$\zeta\equiv\frac{\omega}{\sqrt{2}\Theta_e k_y}=\frac{\omega_r+i\gamma}{\sqrt{2}\Theta_e k_y},\quad Z(z)=i\sqrt{\pi}e^{-z^2}\{1+\mathrm{Erf}(iz)\},\quad z,\ \omega\in\mathbb{C},$$

Plasma dispersion function

- Growth rate and real frequency of the ETG mode for $\Theta=1.0$, $\eta=6$.




The most unstable mode is $k_x\rho_e=0$, $k_y\rho_e=0.5$

・ITG mode and ETG mode are isomorphic except for the sign of real frequency.

<u>Linear simulation results show good agreements with collisionless dispersion relation.</u>