第12回若手科学者によるプラズマ研究会
2009年3月17日　原子力機構那珂核融合研究所　ITER会議棟

トロイダルプラズマ・基礎直線プラズマにおける乱流の非線形過程の実験的研究

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東大新領域

本研究は、九州大学伊藤研究室の特別推進研究（16002005）を中心に、東京大学、日本原子力研究開発機構、核融合科学研究所、間の共同研究で実施され、また科学研究費補助金（18760637）の援助を受けて遂行されました。

Plasma Turbulence
Itoh Project 2004-2008

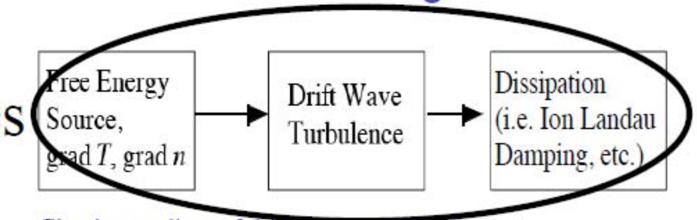
Importance of bispectrum [1,2] in Lagrangian nonlinearity

For example: Meso-scale structures driven by **turbulence Reynolds stress** (e.g. **drift wave-zonal flow systems**)

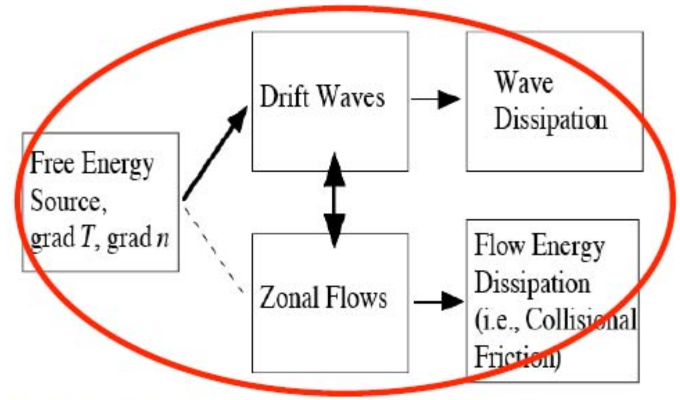
Energy transfer equation

$$\frac{\partial}{\partial t}\left|\bar{v}_\theta\right|^2 + 2\bar{v}_\theta^{\,*}\frac{\partial}{\partial r}\left\langle \tilde{v}_r\tilde{v}_\theta \right\rangle = -\mu\left|\bar{v}_\theta\right|^2$$

bispectral functions.



Classic paradigm of drift wave turbulence



New paradigm of drift wave-zonal flow turbulence

Figure 1. New paradigm for the plasma turbulence.

Quoted from P.H. Diamond, et al., PPCF **47** R35 (2005)

[1] M. Hino, *Spectral Analysis* (Asakura Shoten, Tokyo, 1977).
[2] Y. Kim and E. Powers, IEEE Trans. Plasma Sci. **PS-7**, 120 (1979).

Bispectral functions

Bispectrum, spectrum of third order correlation function

$$B(\omega_1,\omega_2) = \int\int_{-\infty}^{\infty} d\tau_1 d\tau_2 \overline{x(t)y(t+\tau_1)z(t+\tau_2)} e^{i(\omega_1\tau_1+\omega_2\tau_2)}$$

$$= \left\langle X(\omega_1)Y(\omega_2)Z^*(\omega_1+\omega_2)\right\rangle,$$

squared bicoherence

$$\hat{b}^2(\omega_1,\omega_2) = \frac{\left|\left\langle X(\omega_1)Y(\omega_2)Z^*(\omega_1+\omega_2)\right\rangle\right|^2}{\left\langle\left|X(\omega_1)Y(\omega_2)\right|^2\right\rangle\left\langle\left|Z(\omega_1+\omega_2)\right|^2\right\rangle},$$

total bicoherence

$$\sum_{\omega_1+\omega_2=const} \hat{b}^2(\omega_1,\omega_2),$$

biphase

$$\Theta = tan^{-1}\left(\frac{\mathrm{Im}\left(\hat{B}(\omega_1,\omega_2)\right)}{\mathrm{Re}\left(\hat{B}(\omega_1,\omega_2)\right)}\right)$$

[1] M. Hino, *Spectral Analysis* (Asakura Shoten, Tokyo, 1977).
[2] Y. Kim and E. Powers, IEEE Trans. Plasma Sci. **PS-7**, 120 (1979).

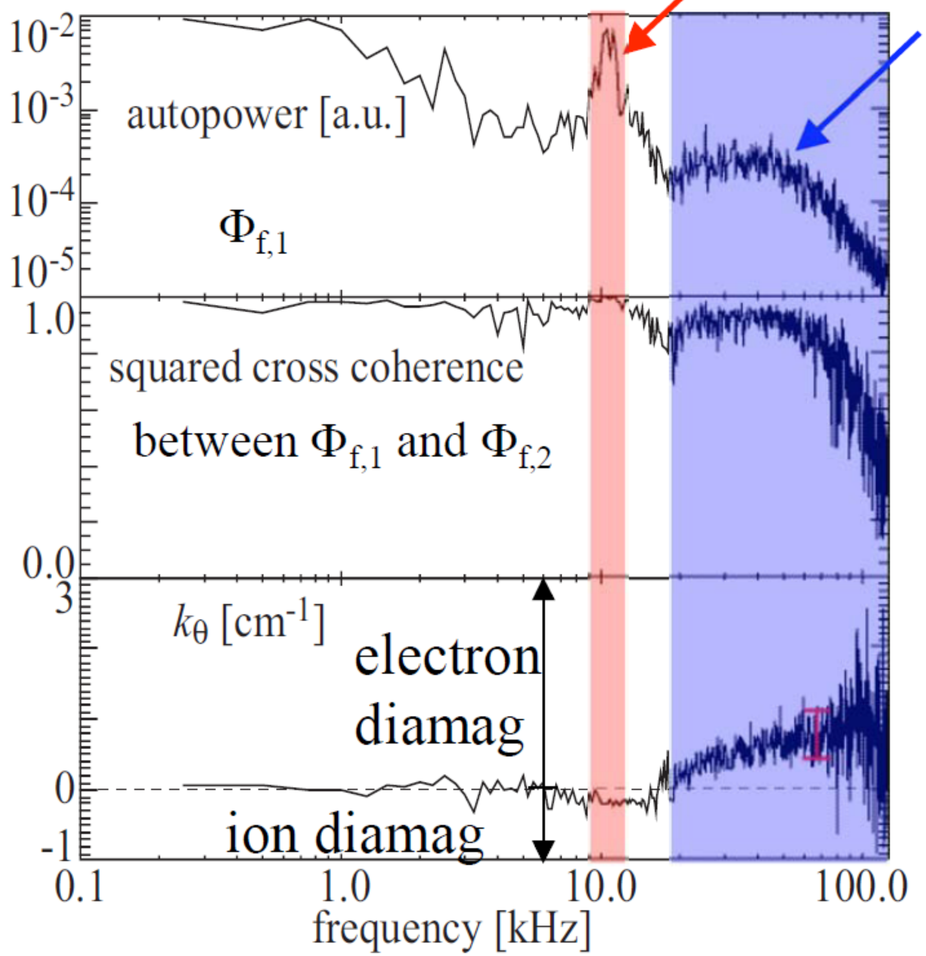
Results of JFT-2M edge plasma

Relationship between geodesic acoustic mode (GAM) and turbulence

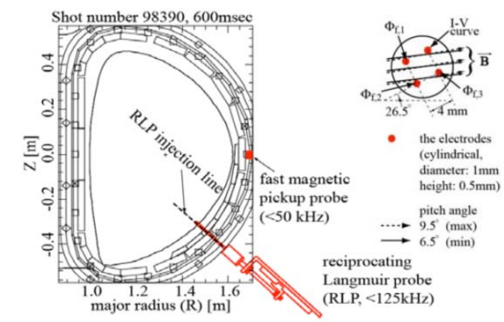
Experimental apparatus on JFT-2M and geodesic acoustic mode

typical spectra

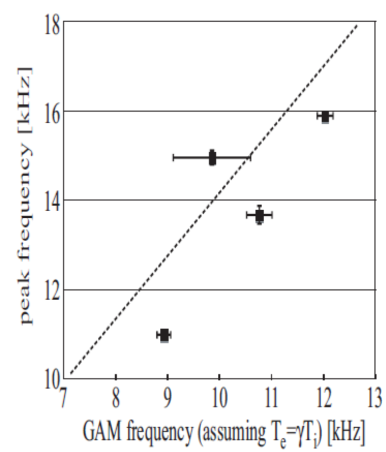
GAM



Broad-band Turbulence (drift waves)

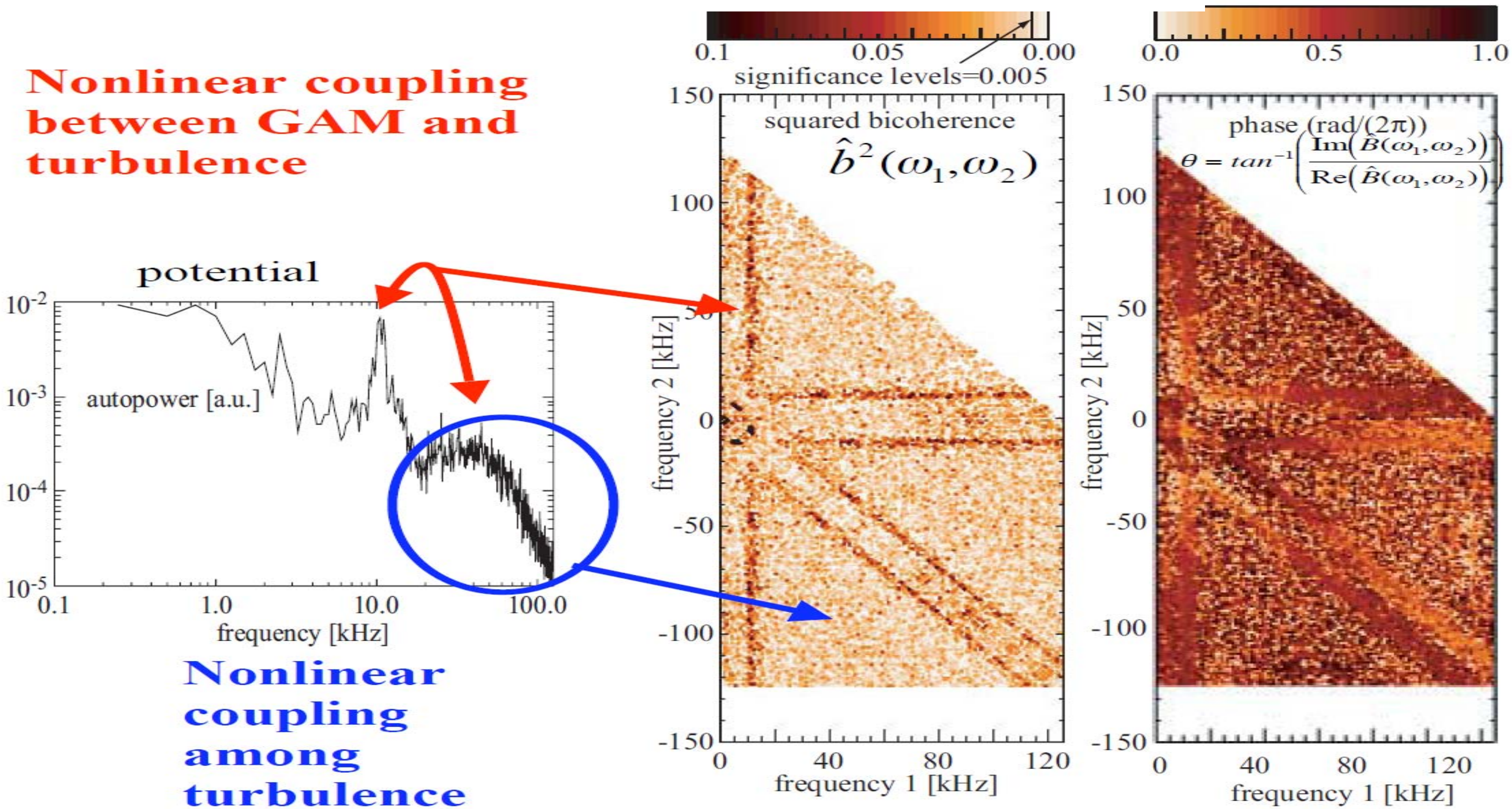


GAM frequency and spectral peak frequency



Results of bicoherence and biphase planes

Nonlinear coupling between GAM and turbulence

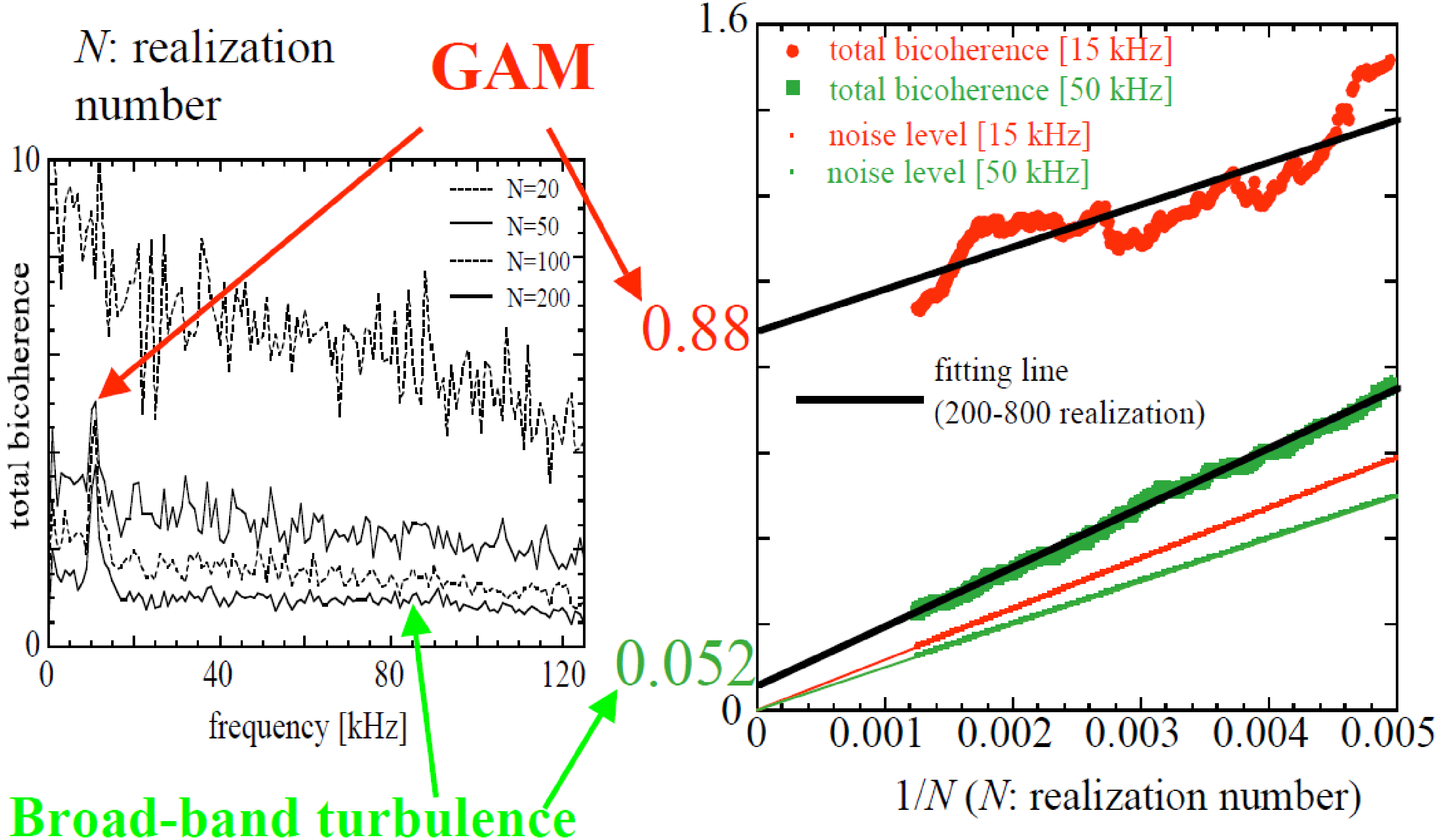


Nonlinear coupling among turbulence

GAM is nonlinearly coupled to the broad-band fluctuations.

The biphase is constant around ~π in the nonlinear coupling between GAM and turbulence.

Convergence property of total bicoherence



Comparison with theoretical prediction

Experimental parameters measured by the RLP and the HIBP are,

M : frequency segments
ρ_s : ion Larmour radius at T_e
k_θ : drift wave poloidal wavenumber
q_x : zonal flow radial wavenumber
$k_\perp = \sqrt{k_\theta + k_x}$
$h(k_\perp \rho_s)$: normalized turbulence deccorelation rate
τ : deccorelation time
ω_* : drift frequency
ϕ_Z : zonal flow potential amplitude
ϕ_d : drift wave potential amplitude

$M = 125$, $\rho_s = 0.2$ cm, $k_\theta \sim 1.5$ cm ^{1}
$q_x \sim 0.5$ cm^{-1}, $k_\perp \sim \sqrt{2} k_\theta$,
$h(k_\perp \rho_s) = \tau^{-1} \omega_*^{-1} \phi^{-1} \sim 0.4$,
$\tau \sim 6$ μsec evaluated by autocorrelation time,
$\omega_* \sim 4 \times 10^5$ rad/sec, and $\phi_Z^2 / \phi_d^2 \sim 1.0$

$$\sum \hat{b}^2(\omega) \sim 4M\left(\frac{1}{h(k_\perp \rho_s)} \frac{q_x k_\perp^2 \rho_s^3}{1 + k_\perp^2 \rho_s^2}\right)^2 \frac{\phi_Z^2}{\phi_d^2} + 3\left(\frac{1}{h(k_\perp \rho_s)} \frac{k_x k_\perp^2 \rho_s^3}{1 + k_\perp^2 \rho_s^2}\right)^2$$

The estimated total bicoherence from theory at GAM of ~0.9 and the ambient fluctuations of ~0.05 are similar to the converged bicoherence.

Total bicoherence dependence of GAM amplitude

Comparison with theory[6]:

$$\sum \hat{b}^2(\omega) \sim 4M\left(\frac{1}{h(k_\perp \rho_s)} \frac{q_x k_\perp{}^2 \rho_s{}^3}{1+k_\perp{}^2 \rho_s{}^2}\right)^2 \frac{\phi_Z{}^2}{\phi_d{}^2} + 3\left(\frac{1}{h(k_\perp \rho_s)} \frac{k_x k_\perp{}^2 \rho_s{}^3}{1+k_\perp{}^2 \rho_s{}^2}\right)^2$$

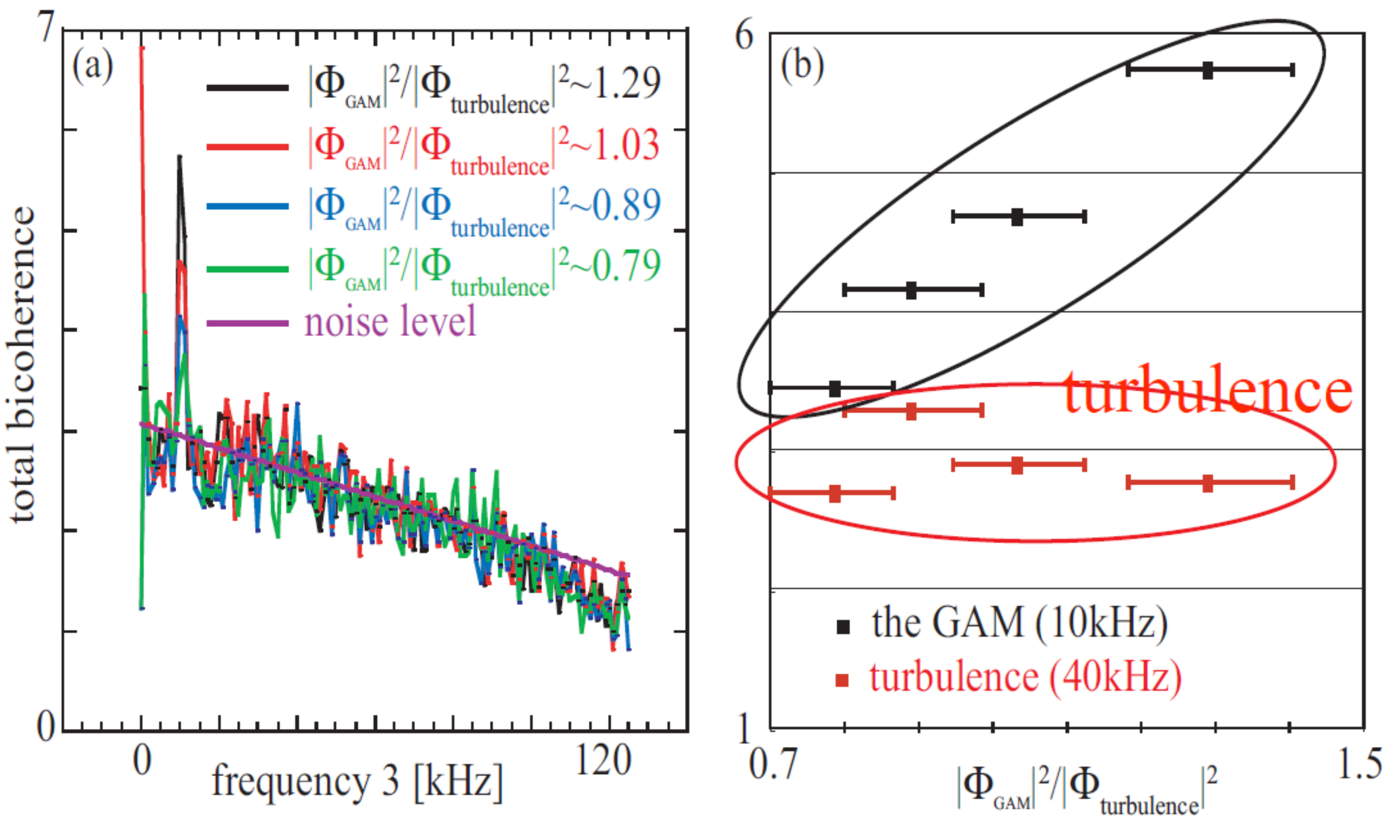
Total bicoherence at GAM is proportional to

$$\left|\phi_{\mathrm{GAM}}\right|^2 / \left|\phi_{\mathrm{drift}}\right|^2$$

[5] Y. Nagashima, et al., PRL 95, 095002 (2005),
[6] Y. Nagashima, et al., PPCF 48, S1 (2006)

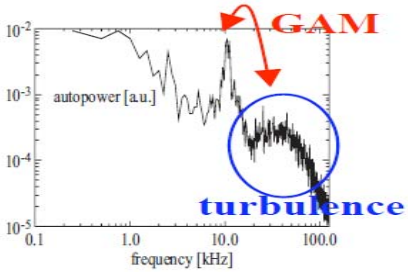
Experiments[5]:

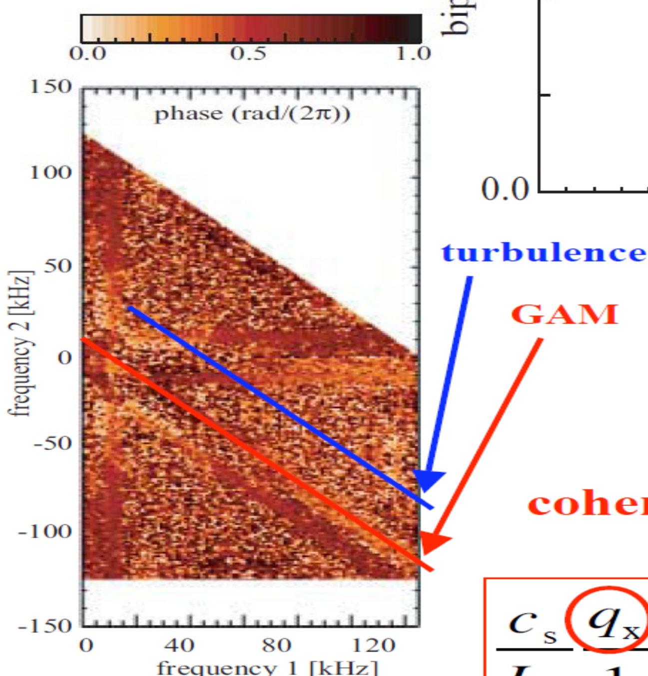
GAM-turbulence

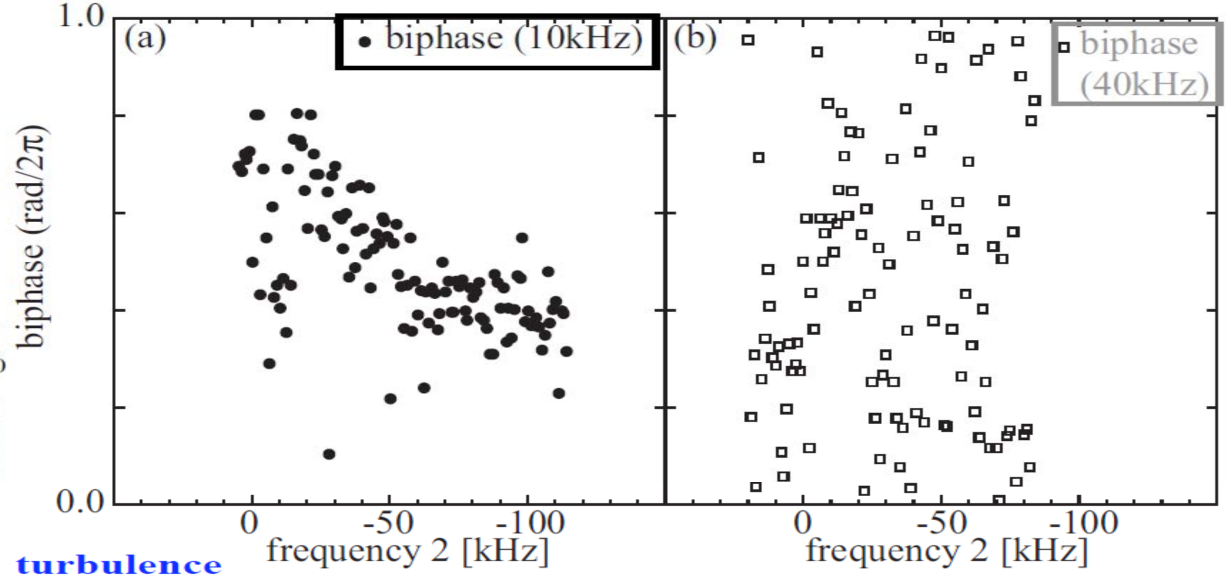


Biphase at GAM and broad-band turbulence

Experiments[6]:







Comparison with theory:

coherent at GAM

$$\frac{c_s}{L_n}\frac{q_x k_y k_\perp^2 \rho_s^4}{1+k_\perp^2\rho_s^2}\phi_Z\phi_d$$

random at broad-band turbulence

$$\frac{c_s}{L_n}\frac{k_x k_y k_\perp^2 \rho_s^4}{1+k_\perp^2\rho_s^2}\left|\phi_d\right|^2$$

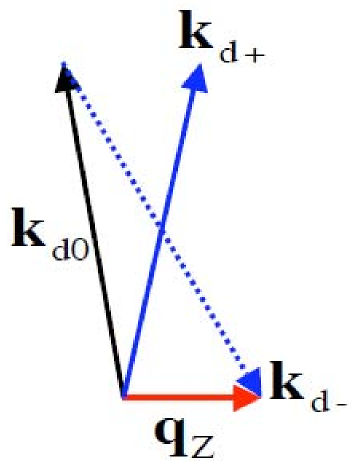
Biphases demonstrate the phase angles of nonlinear interaction terms in drift wave-zonal flow systems.

Proposal of GAM measurement by envelope modulation technique

Y. Nagashima, et al., PPCF 49 1161 (2007)

Nonlinear property of the GAM

Four wave process(parametric-modulational instability)



$$\mathbf{k}_{d+} = \mathbf{k}_{d0} + \mathbf{q}_Z$$

$$\mathbf{k}_{d-} = \mathbf{q}_Z - \mathbf{k}_{d0}$$

Quoted from P.H. Diamond, et al., PPCF **47** R35 (2005)

k_{d0} : Primary drift wave wavenumber

k_{d+}, k_{d-} : Secondary drift wave wavenumbers

q_z : GAM radial wavenumber

Drift wave turbulence is modulated by zonal flows
---> **Envelope of the turbulence has ZF information.**

Drift waves have the condition $\dfrac{\tilde{\phi}}{T_e} \sim \dfrac{\tilde{n}}{n}$

--->Close relationship between density and potential

Envelope of turbulent density fluctuations has information of zonal flows

The envelop of the density fluctuation has the GAM info.

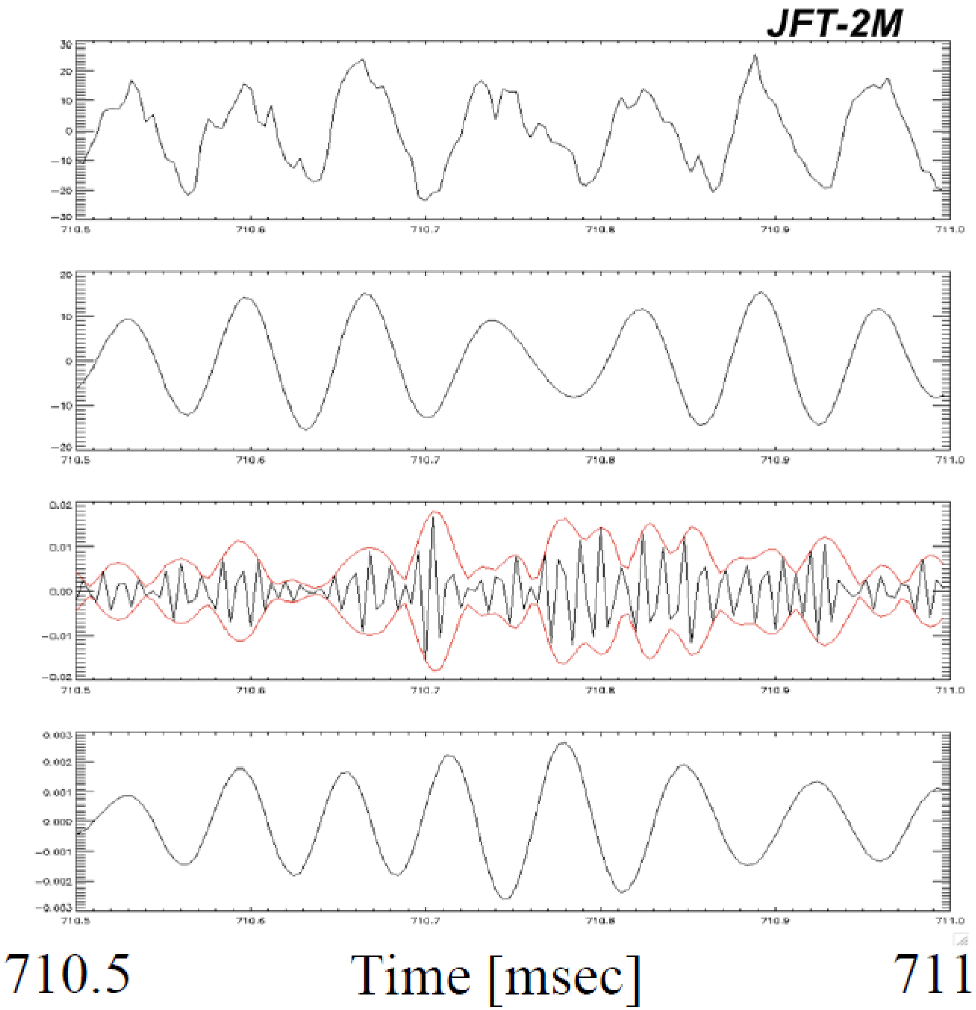
$\phi(t)$ Raw data

$\phi(t)$ Band-pass (12<f<18 kHz, GAM frequency)

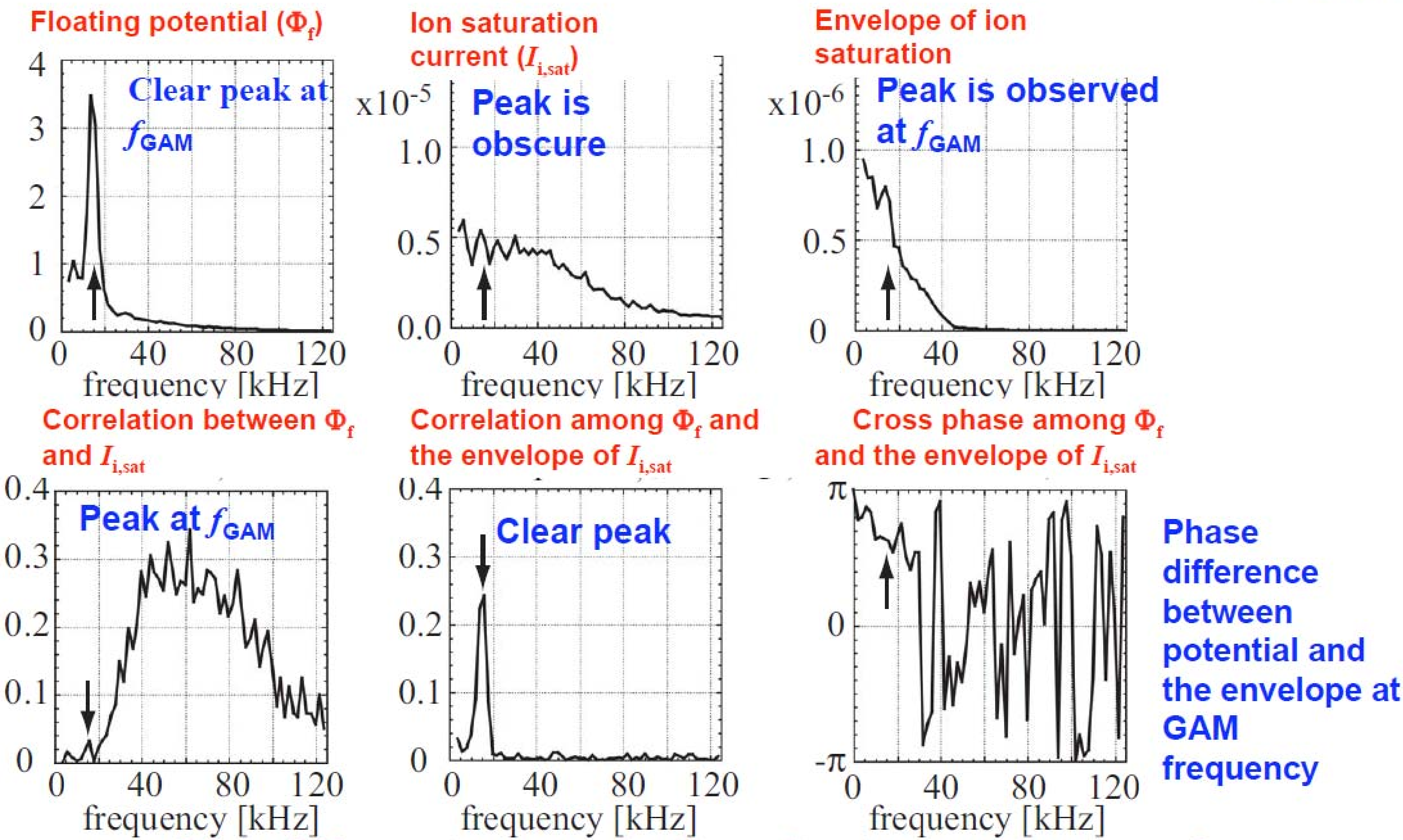
$n(t)$ Black: high-pass (80 kHz <f, frequency of turbulence)
Red: the envelope of black signals by Hilbert transform

$$Hf(\tau) = \int_{\mathbb{R}} \frac{f(t)}{t-\tau} dt$$

$n(t)$ Band-pass (12<f<18 kHz, GAM frequency)

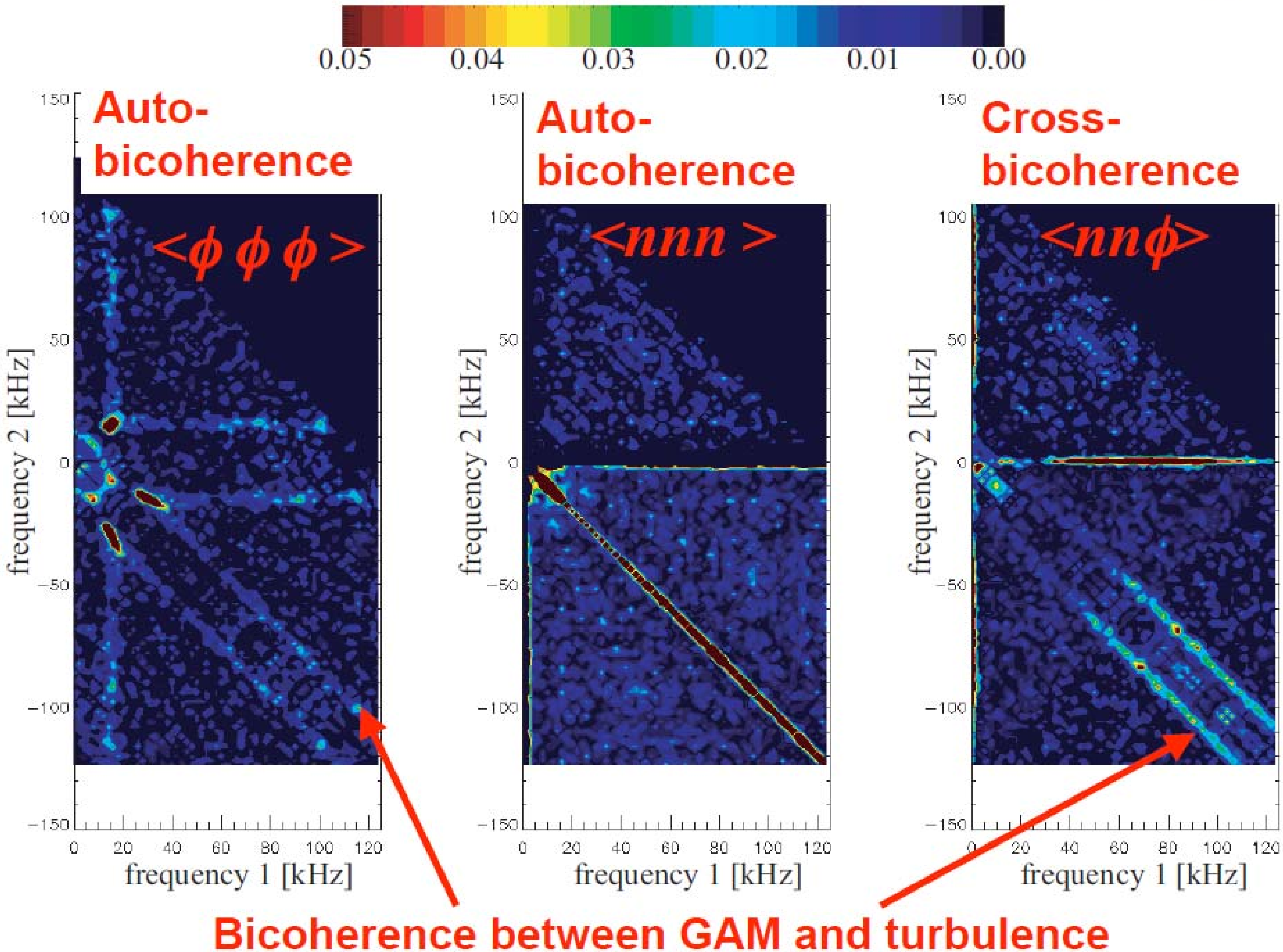


Quadratic spectra (Φ_f, I_{is}, $Env(I_{is})$)



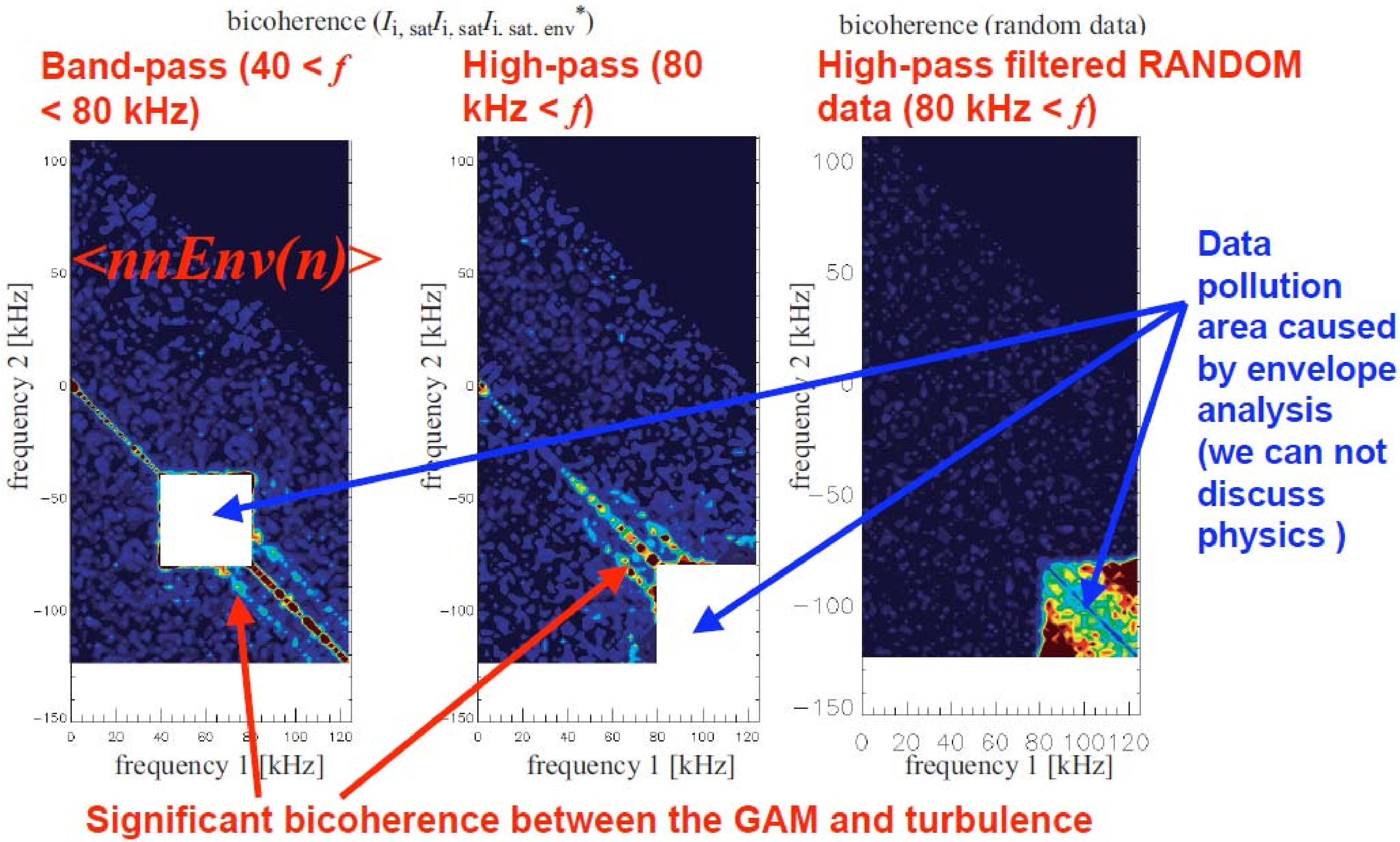
Significant correlation between Φ_f and the envelope of Ii,sat at f_{GAM}

Auto/cross-bicoherence (Φ_f, I_{is})



Bicoherence of $nn\Phi_f$ is higher than that of $\Phi_f\Phi_f\Phi_f$

Cross-bicoherence (I_{is}, $Env(I_{is})$)



Bicoherence between turbulence and the GAM is observed _only by using density fluctuation data_

This research is applicable to GAM spectroscopy (S.-I. Itoh, et al. PPCF 49 L7 (2007))

Bispectral analysis in linear plasmas

Analysis of momentum transfer

Energy transfer function **T** by use of $E_{r,ZV}$ [7]

$$T = \sum_{k_{r,Z3}=k_{r,d1}+k_{r,d2}} k_{r,Z3}\,\mathrm{Im}\left\langle \tilde{u}_{r,d1}\tilde{u}_{\theta,d2}U_{\theta,Z3}^{\;*}\right\rangle$$

$$= \sum_{k_{r,Z3}=k_{r,d1}+k_{r,d2}} k_{r,Z3}\,\mathrm{Im}\left\langle \frac{\tilde{E}_{\theta,d1}\tilde{E}_{r,d2}}{B^2}E_{r,Z3}^{\;*}\right\rangle$$

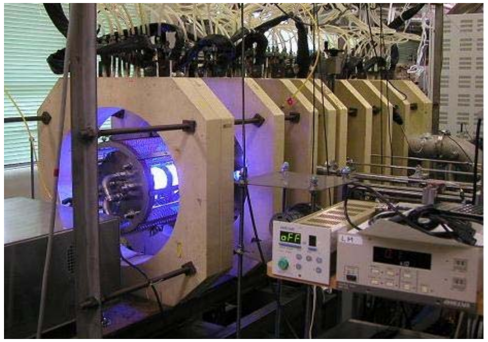
$$\propto \sum_{k_{r,Z3}=k_{r,d1}+k_{r,d2}} k_{r,Z3}\,\mathrm{Im}\left\langle \tilde{E}_{\theta,d1}\tilde{E}_{r,d2}E_{r,Z3}^{\;*}\right\rangle$$

[7] G.R. Tynan, et al.,
Phys. Plasmas **8** (2001) 2691

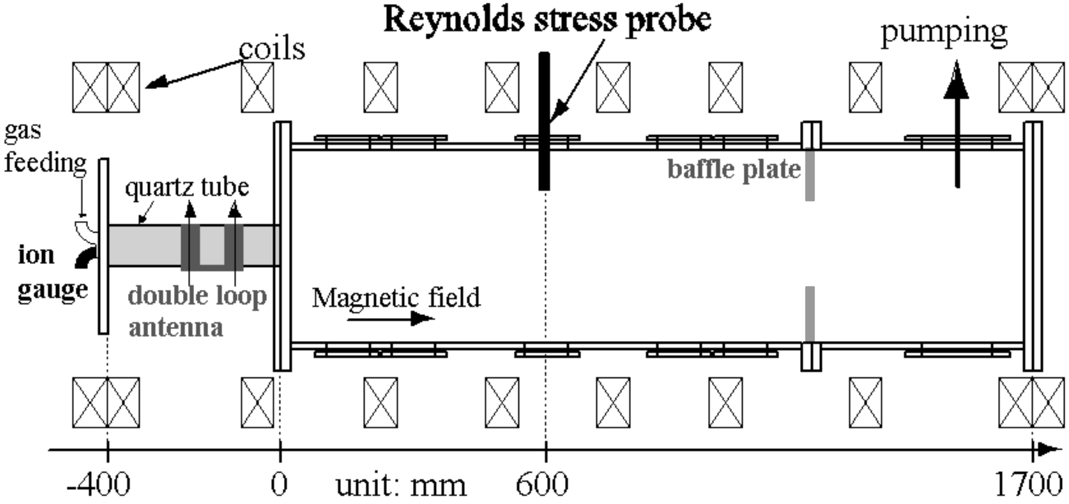
The analysis reflects local interaction of waves.
-->more precise analysis than potential analysis

Experimental devices

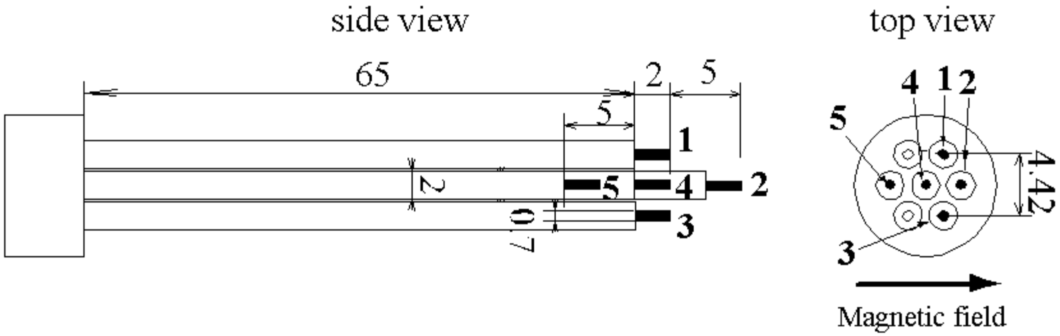
The Large Mirror device (LMD [8]) [8] Y. Saitou, et al., Phys. Plasmas **14** (2007) 072301



Helicon source (2kW, 7MHz) Ar gas



The Reynolds stress probe

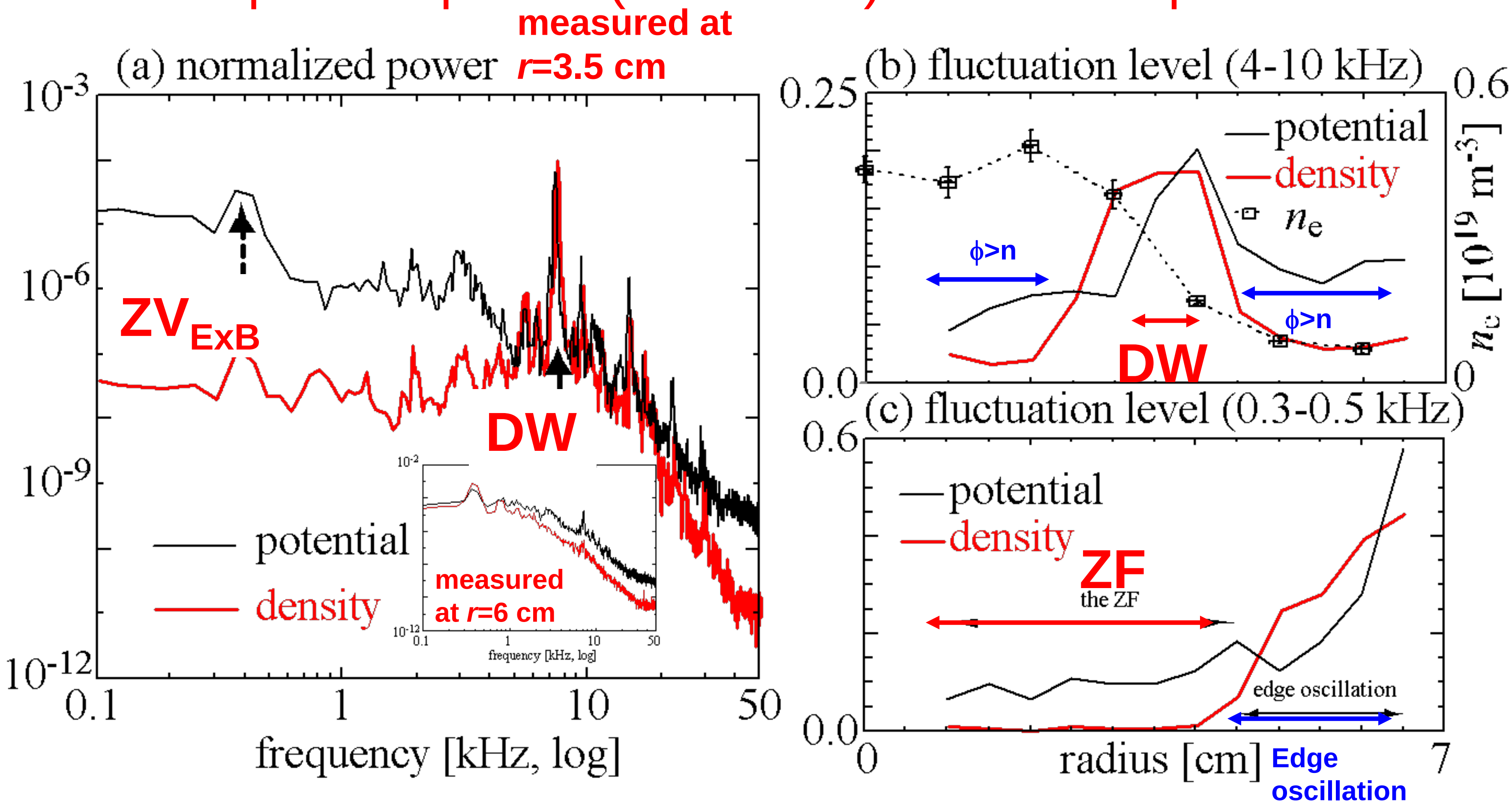


$$\tilde{E}_\theta = \left(\tilde{\Phi}_1 - \tilde{\Phi}_3\right)/d_\theta \ (k_\theta) \qquad \tilde{E}_r = \left\{\tilde{\Phi}_2 - \left(\tilde{\Phi}_1 + \tilde{\Phi}_3\right)/2\right\}/d_r \ (k_r)$$

Reynolds stress per mass density $\left\langle \tilde{E}_r \tilde{E}_\theta \right\rangle / B^2$

Normalized fluctuation spectra

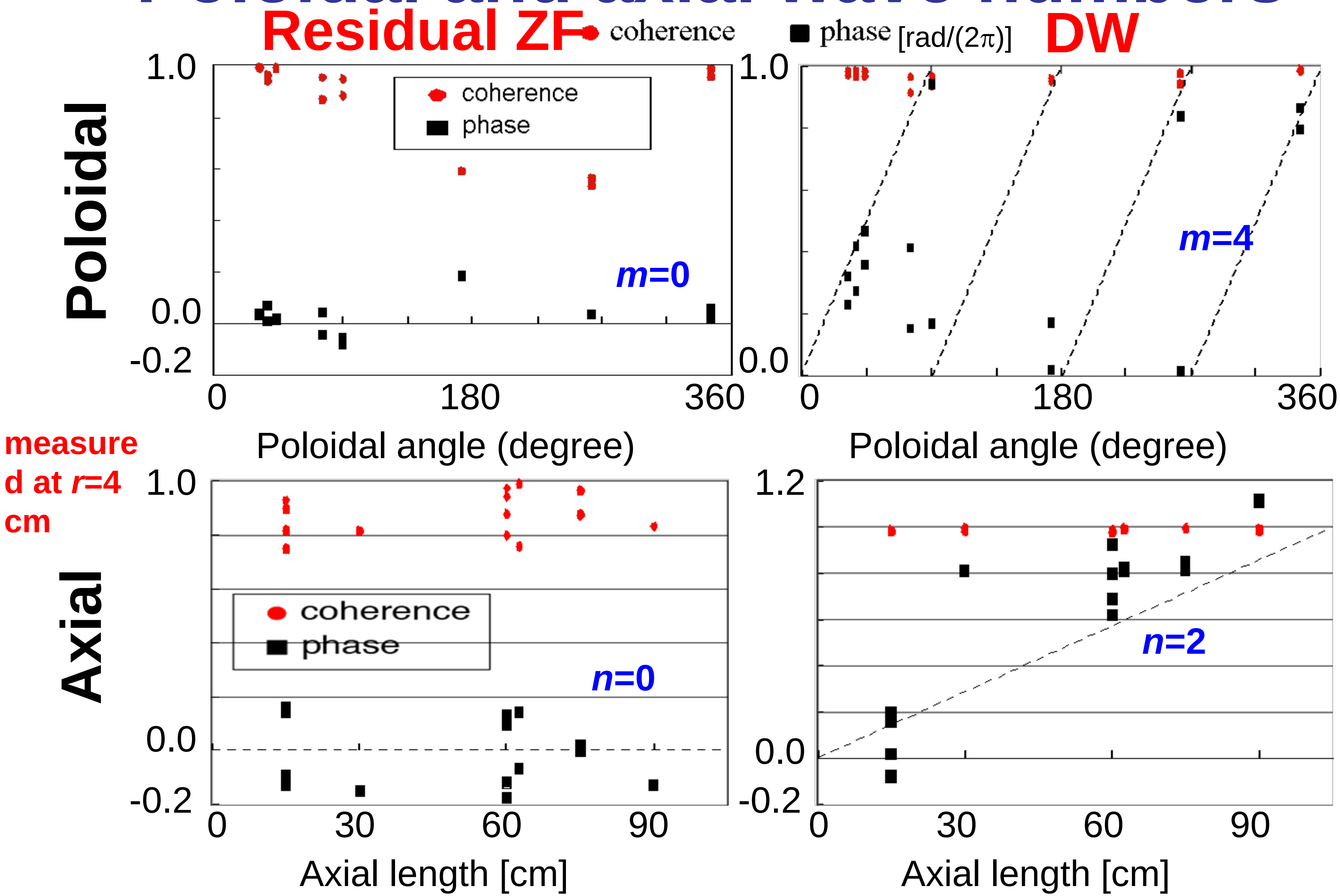
Normalized power spectra (at 0.3 sec) Radial profiles



DW (density ~ potential) is located at r=3.5-4cm.

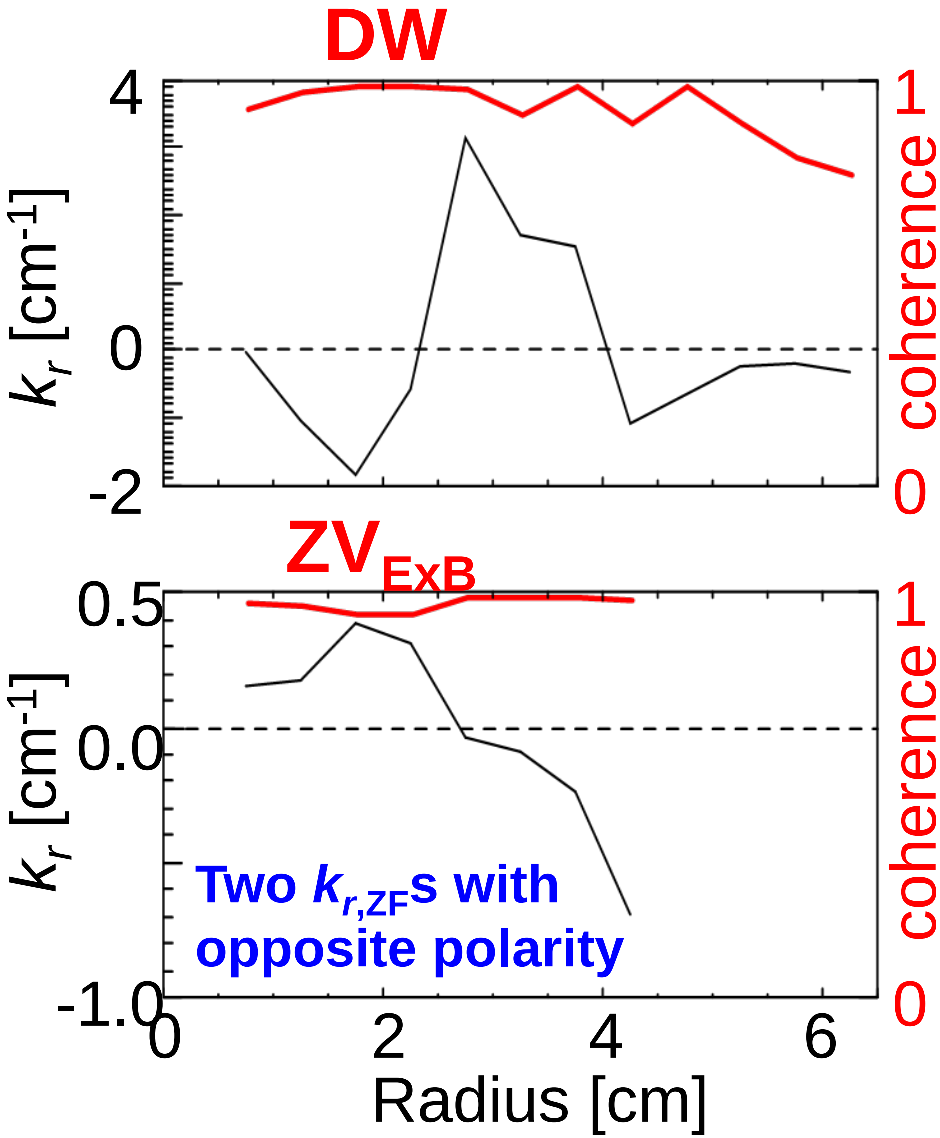
ZV_{ExB} exists at r<~4.5cm (edge oscillations r>~4.5cm).

Poloidal and axial wave numbers

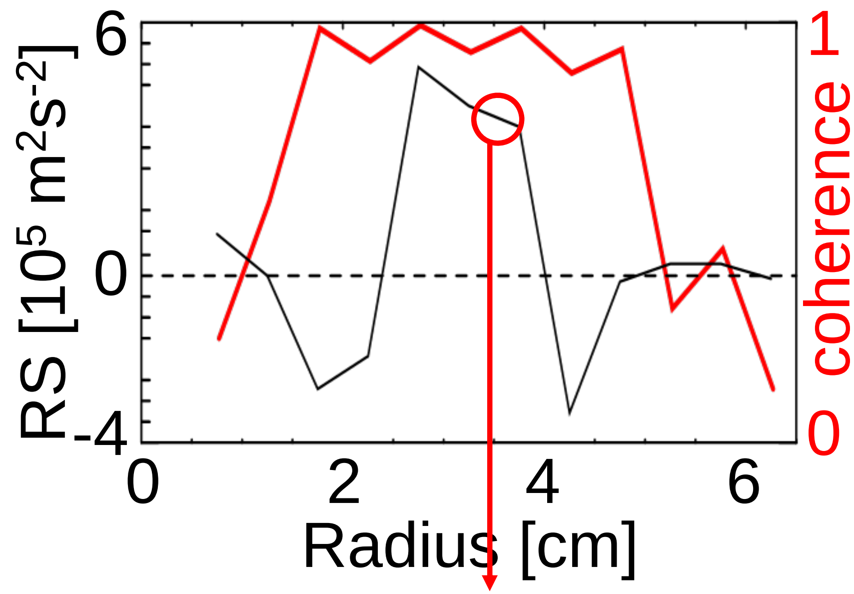


DW has *m*=3-5 and *n*=2-3, while residual ZF potential has *m*,*n*~0.

Radial wave numbers (radial profile)



Time averaged Reynolds stress per mass density



$\omega_{de,th}$ = 6.22-8.26 kHz at minimum RS gradient

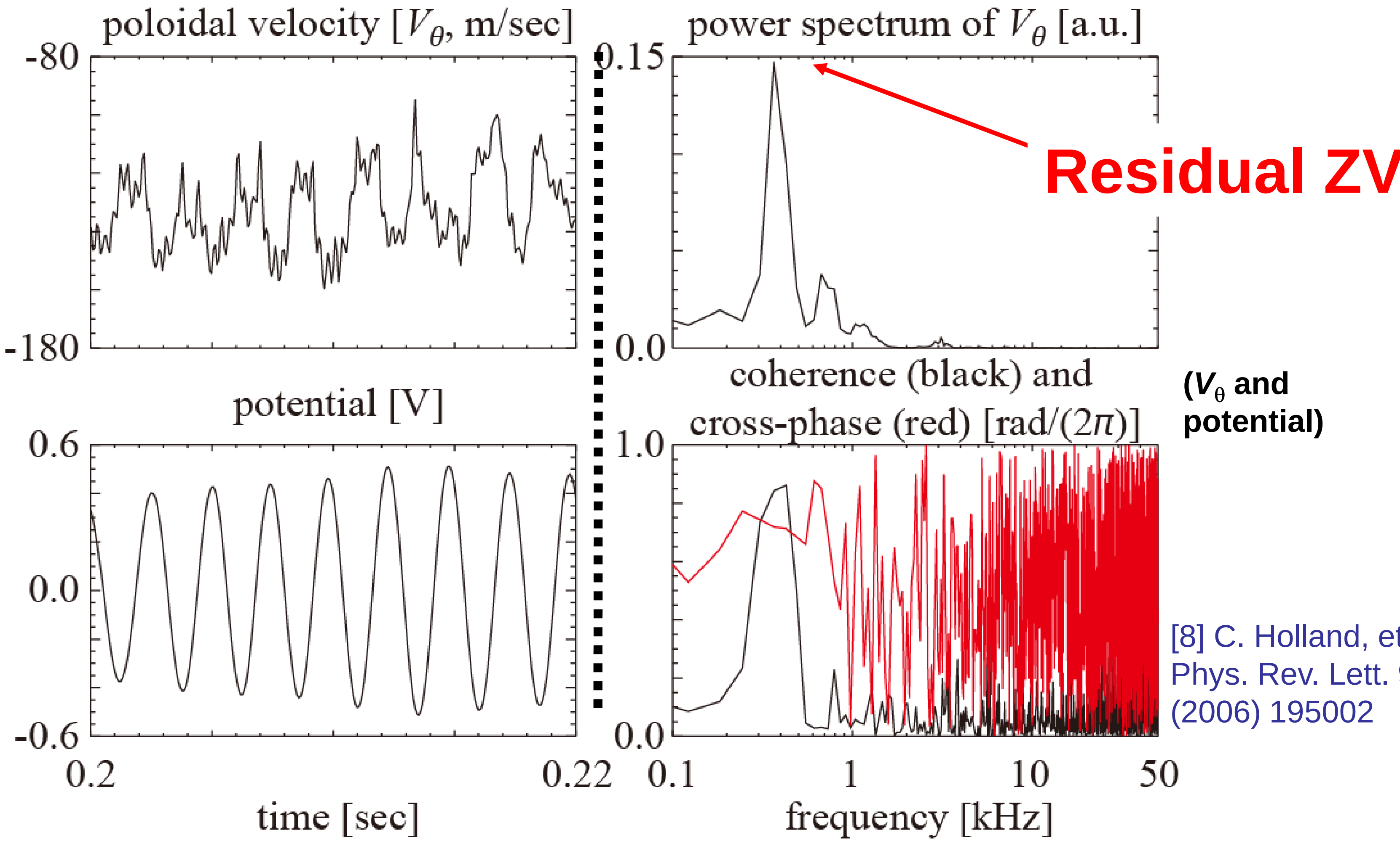
$\omega_{de,exp}$ = 7-8 kHz

$$\omega_{\mathrm{de,th}} = \frac{\omega_{*,\mathrm{e}}}{1 + \mathrm{k}^2_{\perp}\rho^2_{\mathrm{s}}}$$

ZV$_{ExB}$ has two radial wave numbers with opposite polarity.

Poloidal velocity fluctuation

measured at r=2.5 cm where E_r of the ZV is large.



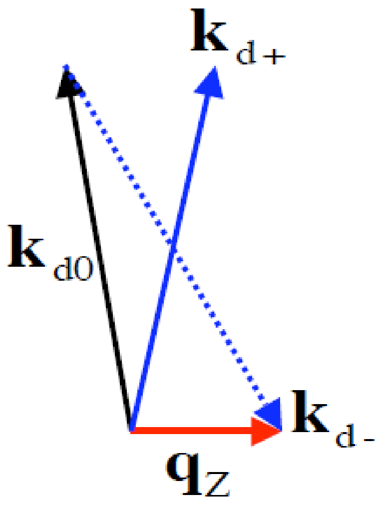
(V_θ and potential)

[8] C. Holland, et al., Phys. Rev. Lett. **96** (2006) 195002

ZV is associated with the poloidal velocity fluctuation derived from the Time Delay Estimation[8].

Y. Nagashima, et al., JPSJ 77 114501 (2008)

Four wave process(parametric-modulational instability)



$$\mathbf{k}_{d+} = \mathbf{k}_{d0} + \mathbf{q}_Z$$

$$\mathbf{k}_{d-} = \mathbf{q}_Z - \mathbf{k}_{d0}$$

Quoted from P.H. Diamond, et al., PPCF **47** R35 (2005)

k_{d0} : Primary drift wave wavenumber

k_{d+}, k_{d-} : Secondary drift wave wavenumbers

q_z : GAM radial wavenumber

Modulation of k, f, Φ_{ZV} and $(v_r v_\theta)$

Notice: coordinate is corrected from left-handed (APS2007) to right-handed…

k_r, k_θ (two point cross-phase):

k_r modulation is significant.

k_θ or $V_{\theta,ph}$ is modulating at r=4.25cm.

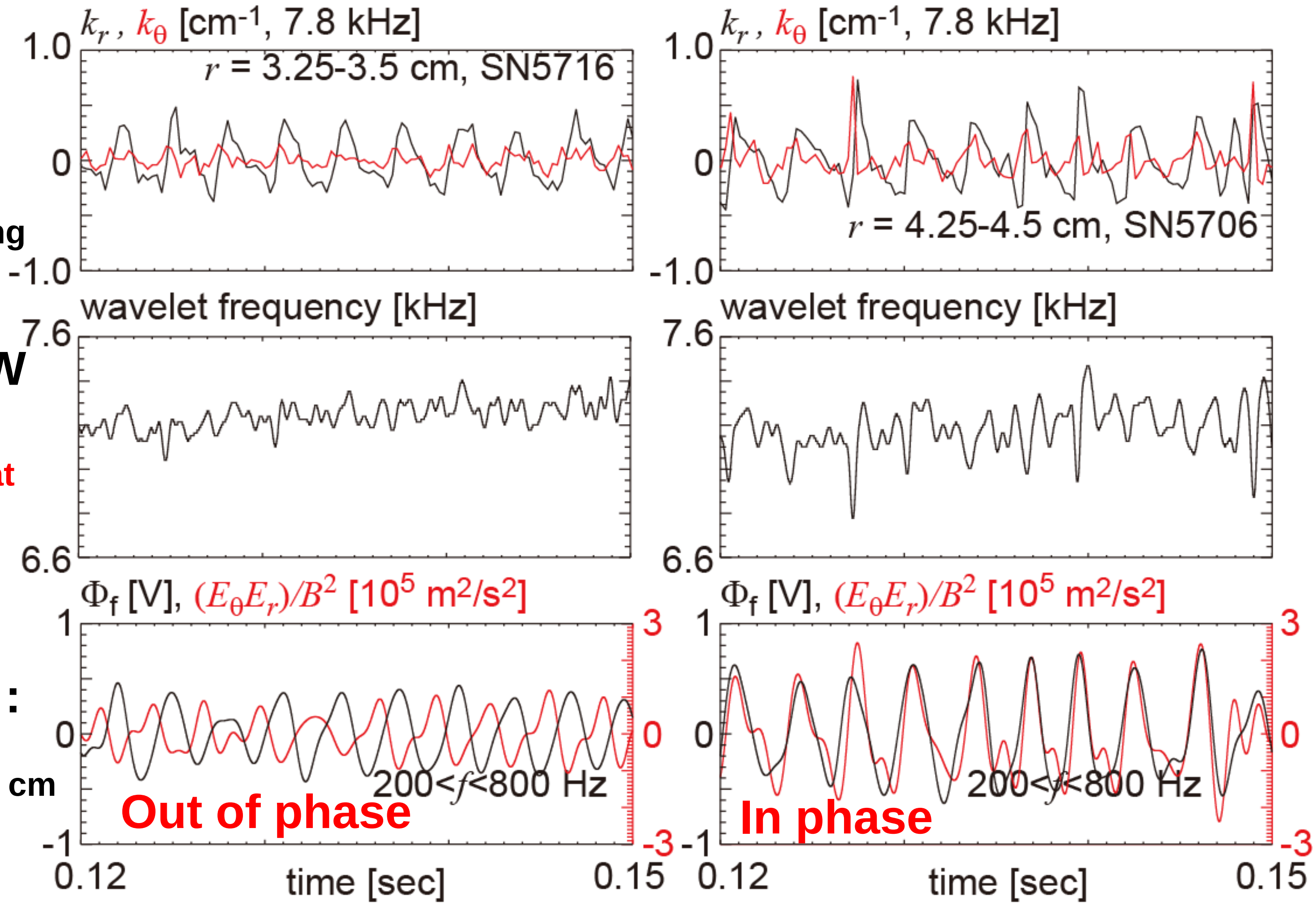
Peak f of the DW (wavelet) :

***f* modulation is large at large $E_{r,ZV}$ location.**

Φ_{ZV} , $(E_\theta E_r)_{DW}/B$:

In phase at r=4.25 cm.

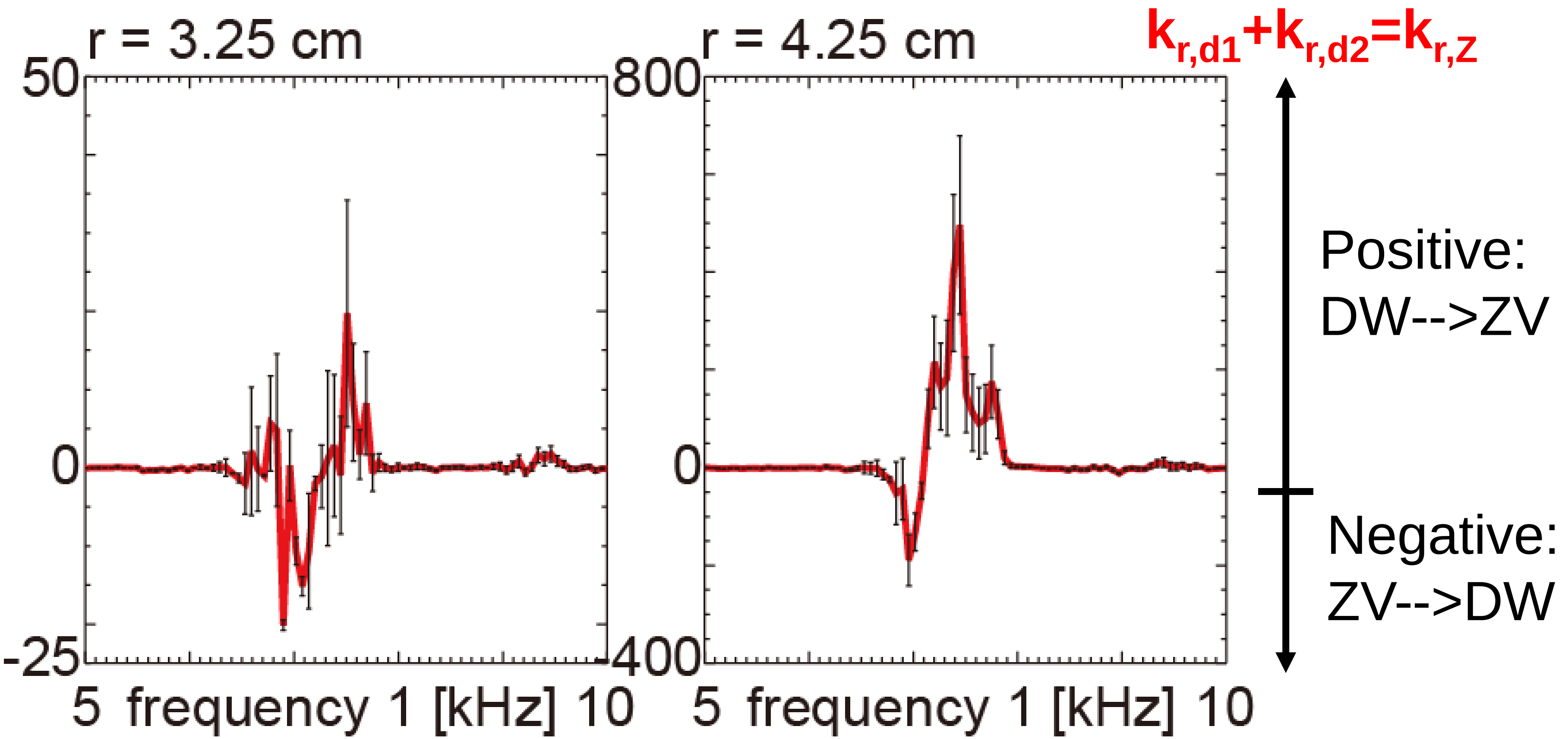
Out of phase at r=3.25 cm



Measured at r=3.25 cm **Measured at r=4.25 cm**

Modulational relationship is different in radius.

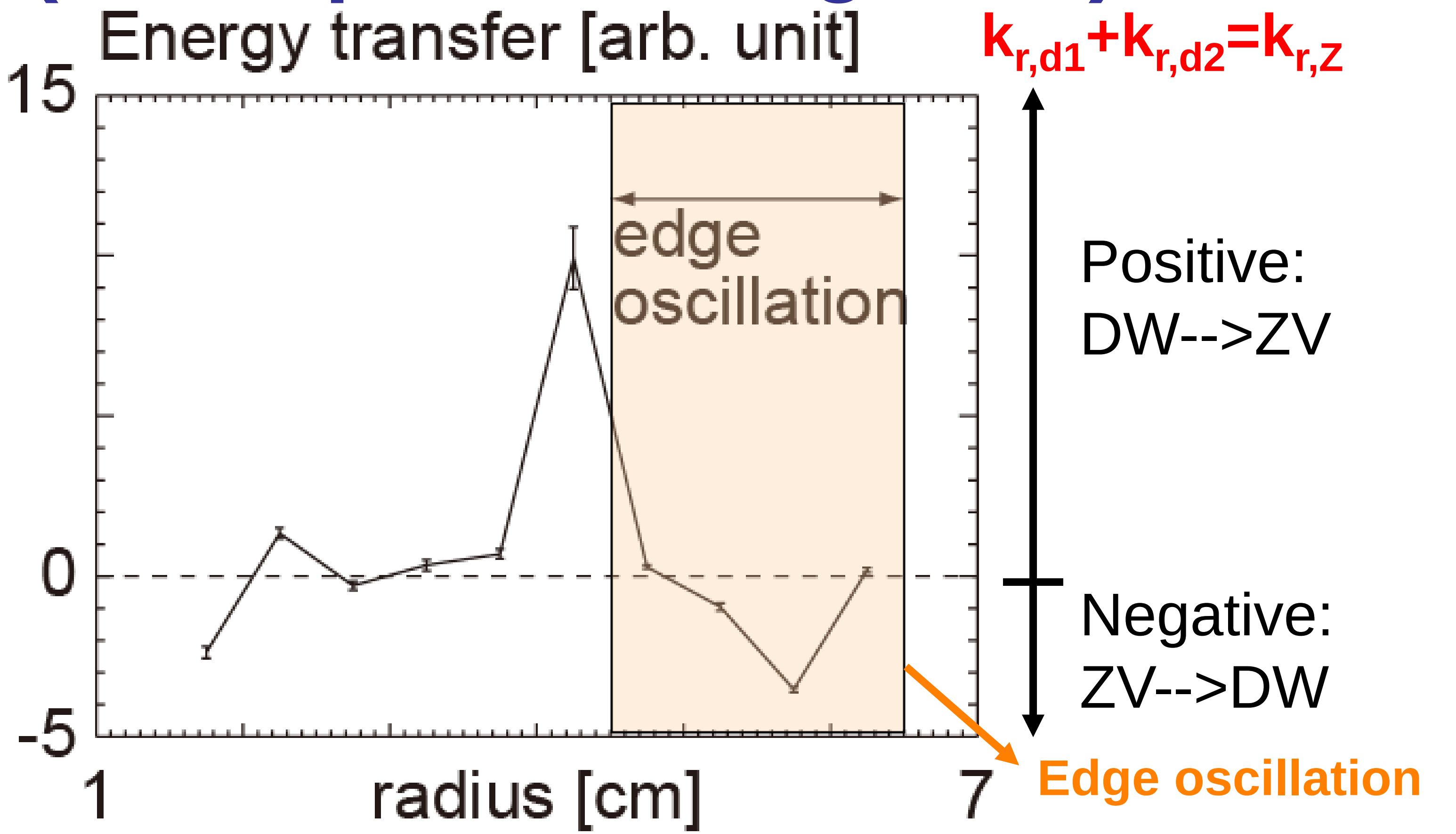
$(k_{r,d1}+k_{r,d2})$ Im$<E_{\theta,d}E_{r,d}E_{r,Z}^{*}>$ estimation (frequency space)



Transfer direction is different in frequency.

Momentum transfer from the DW to the ZV_{ExB} occurs totally at r=4.25 cm.

$(k_{r,d1}+k_{r,d2})$ Im$<E_{\theta,d}E_{r,d}E_{r,Z}^*>$ (radial profile, integrated)



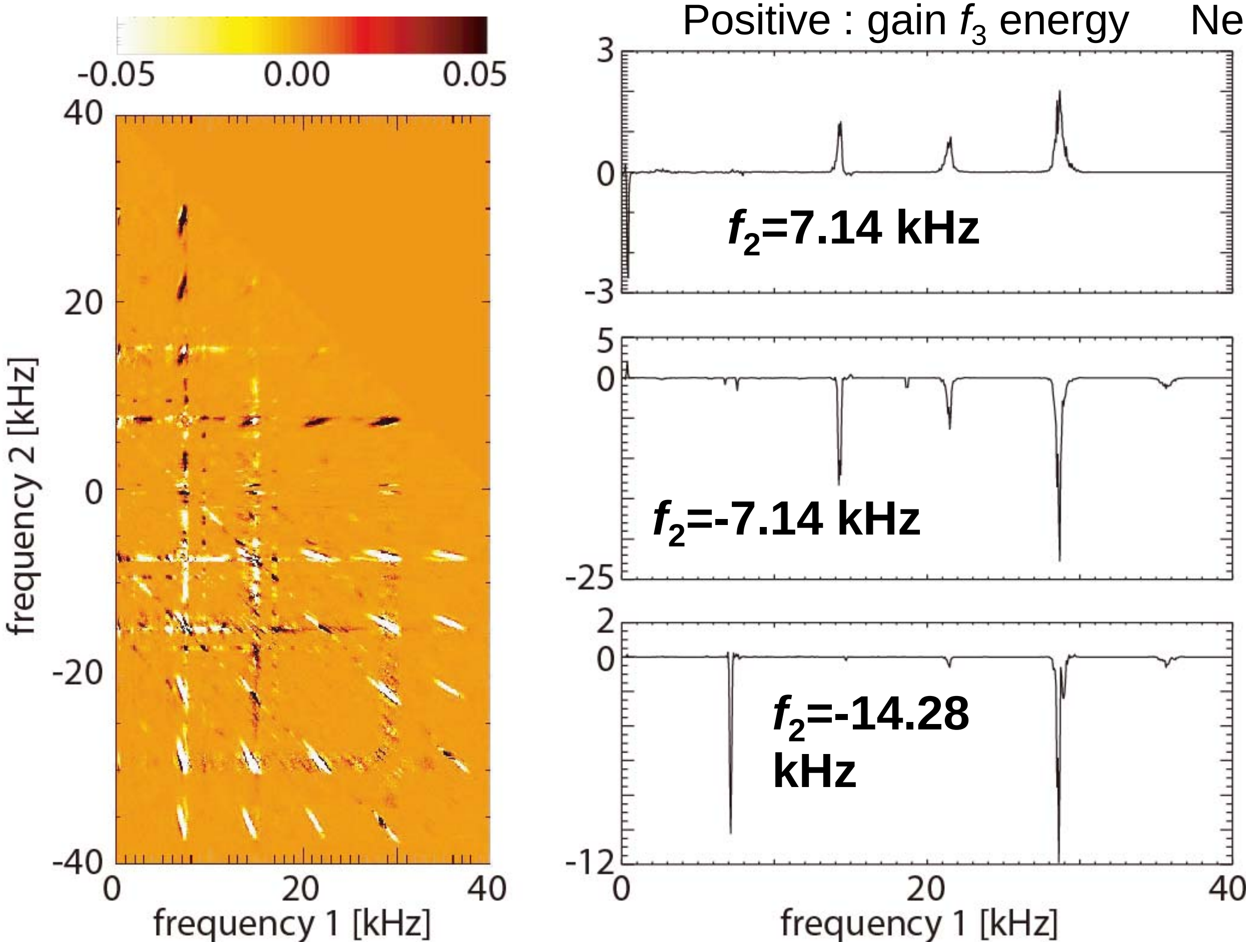
Transfer direction is different in radius.

Strong momentum transfer from the DW to the ZV_{ExB} occurs at r=4.25 and 2.25 cm.

Y. Nagashima, et al., POP 16 020706(2009)

Estimation of Hasegawa-Mima type equation $\Sigma k_1 x k_2 .e_z .(k_2^2-k_1^2)Re<\Phi_1\Phi_2\Phi_3^*>$

Positive : gain f_3 energy Negative : release f_3 energy



1: fundamental mode,
2: 2nd harmonic,…

1 + 2 --> 3,
1 + 3 --> 4,…

2 - 1 <-- 1,
4 - 1 <-- 3,…

4 - 2 <-- 2,…

r=3.25 cm

Forward transfer of the DW energy to higher harmonics occurs in frequency space

Y. Nagashima, et al., PFR 3 056 (2008)

Conclusion

1. In the JFT-2M tokamak, bispectral analysis of the drift-wave-zonal flow turbulence succeeded in clarifying nonlinear process of turbulence.

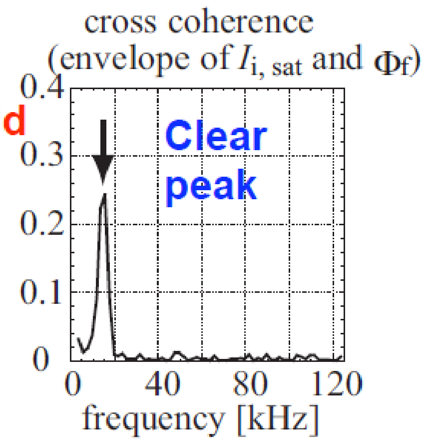
2. The combination of the bispecral and envelope analysis is proposed to detect GAMs by density fluctuation measurement and "GAM spectroscopy".

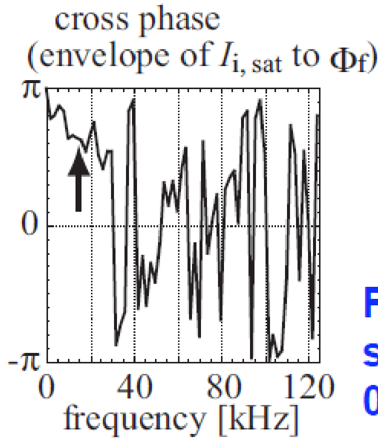
3. In the Large Mirror device (linear plasma device), coexistence of the low-frequency Er oscillation and drift-wave fluctuation is observed.

4. Nonlinear /nonlocal energy transfers between drift wave and the low-frequency Er are identified in real and spectral spaces.

Phase difference (Φ_f,and $Env(I_{is})$)

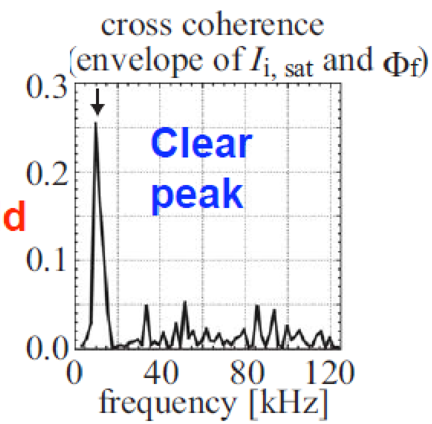
B field CCW

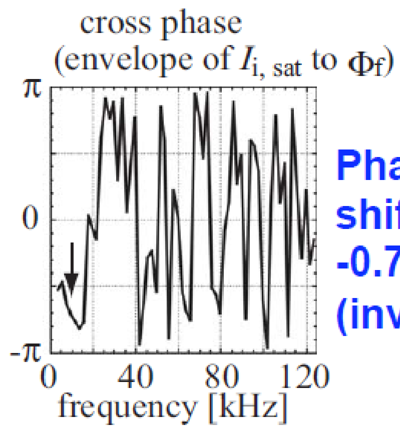




Phase shift 0.6~0.7π

B field CW





Phase shift -0.7~-0.8π (invert)

Theoretical prediction of phase shift between potential and the envelope

K. Itoh, et al., PPCF **47** (2005) 451

$$\frac{\delta\left|\phi_{\mathrm{micro}}\right|^2}{\left|\phi_{\mathrm{micro}}\right|^2} \cong \frac{4\rho_s^2 k_\theta^2}{\omega_{\mathrm{GAM}}\left(1+\rho_s^2 k_{\perp,0}^2\right)}\left(-i\theta s q_r^2 \phi \frac{B_z}{\left|B_z\right|^2}\right)$$

$$\propto -i\theta s q_r^2 \frac{B_z}{\left|B_z\right|}$$

θ: poloidal angle
s: shear parameter
q_r: zonal flow radial wavenumber
B_z: magnetic field

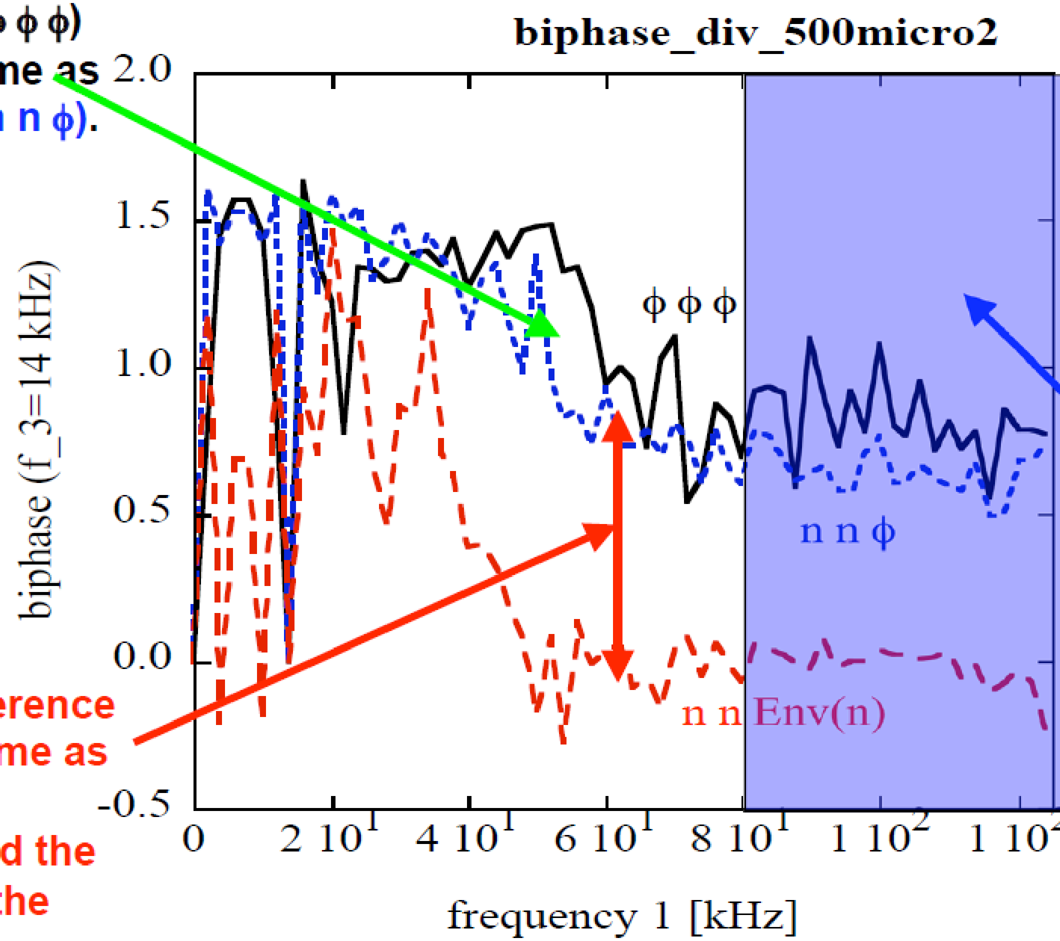
Phase shift depends upon
- Poloidal angle,
- shear parameter,
- complex radial wavenumber of zonal flow and
- direction of magnetic field

Inversion of phase shift is observed in opposite B direction

Biphase (Φ_f, I_{is}, $Env(I_{is})$)

Biphase of (ϕ ϕ ϕ) is almost same as biphase of (n n ϕ).

High-pass filter 80 kHz <*f* for envelope



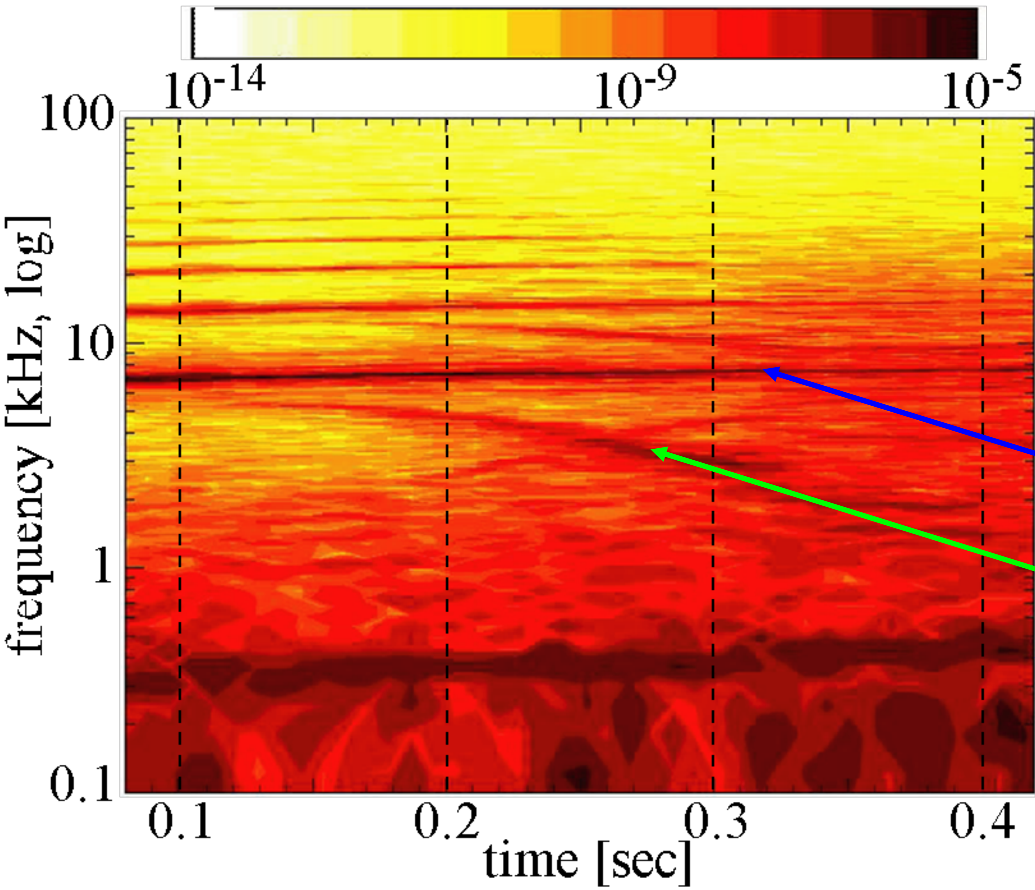
Polluted area

Biphase difference ~0.7-0.8π, same as phase shift between f and the envelope of the GAM

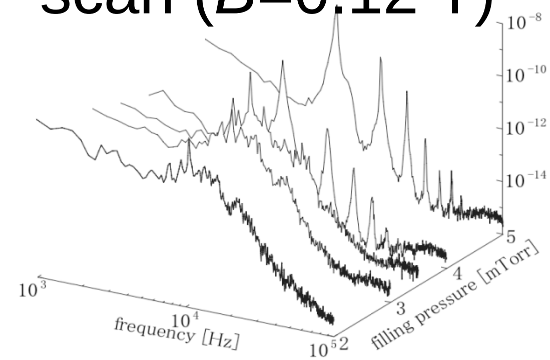
Biphase analysis is applicable considering phase between ϕ and the envelope

The drift wave and the zonal ExB flow

Time evolution of the potential spectrum during a single discharge



Filling pressure scan (B=0.12 T)



Marginal transition to strong turbulence at 3.5 mTorr

the Drift Wave (DW)

quasi-modes (future scope)

the residual Zonal ExB Velocity (ZV_{ExB})

Measured at r=3.5 cm (plasma radius a=~5 cm)

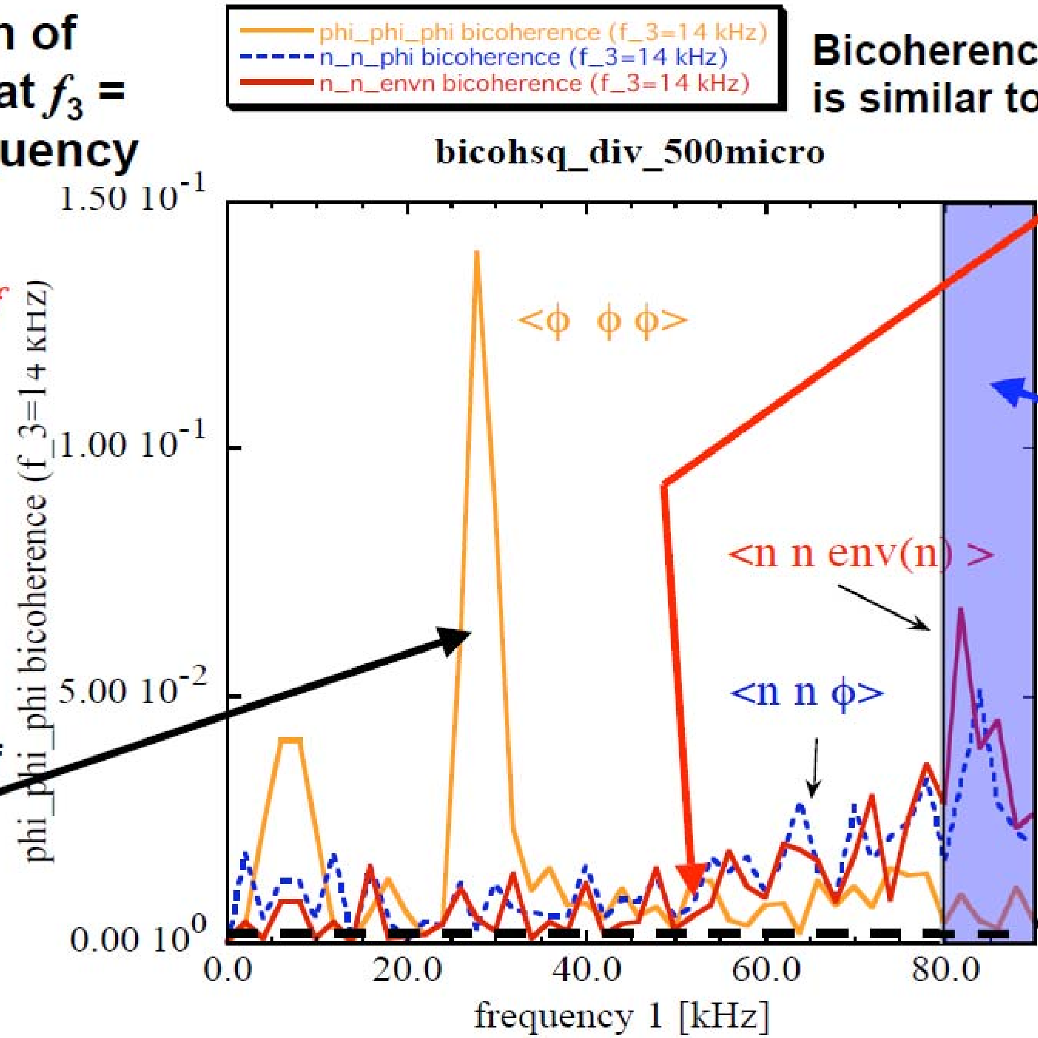
DW (7-8 kHz) and residual ZV_{ExB} (~400 Hz) are observed.

Comparison of bicoherence

Cross-section of bicoherence at f_3 = the GAM frequency

High-pass filter 80 kHz < f for envelope

Bicoherence of (n n env(n)) is similar to that of (n n ϕ)



Self coupling of GAM in (φ φ φ), which is not observed in (n n φ)

Bicoherence of <*nn*Env(*n*)> is same as that of <*nn*Φ>