第１２回 若手研究者によるプラズマ研究会
＠日本原子力開発機構 那珂核融合研究所 ITER会議室
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誤差磁場による磁気島の回転への影響

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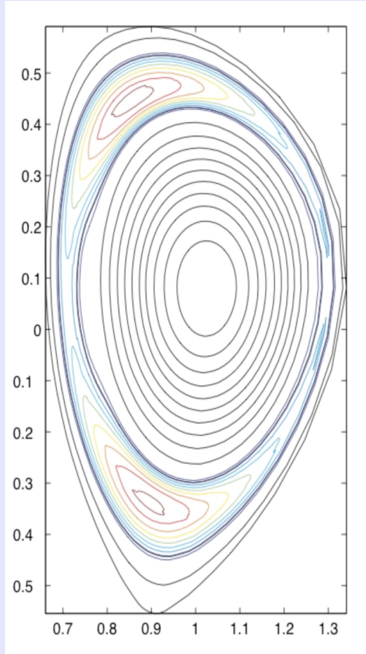
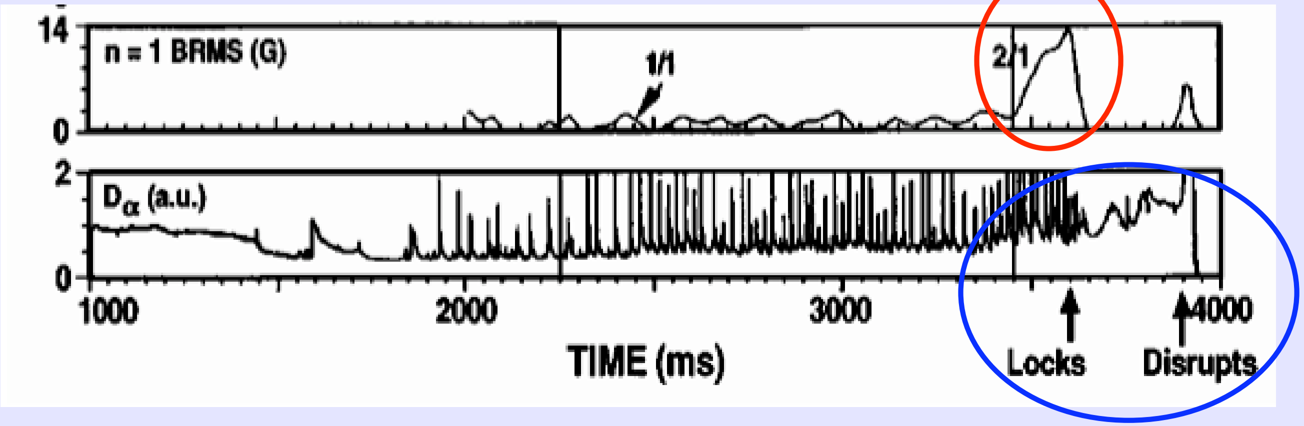
- 解析的モデリング

  0-Dモデルによる磁気島の回転の記述
- Summary

研究背景 1

- トカマクプラズマにおける磁気島は高β(高圧)状態を壊し、さらにはディスラプション(プラズマの瞬間的崩壊)を引き起こす。
- ディスラプションは、磁気島のポロイダル回転が抵抗性壁により減衰し誤差磁場(error-field)によりLock(停止)されて引き起こされることが観測されている。



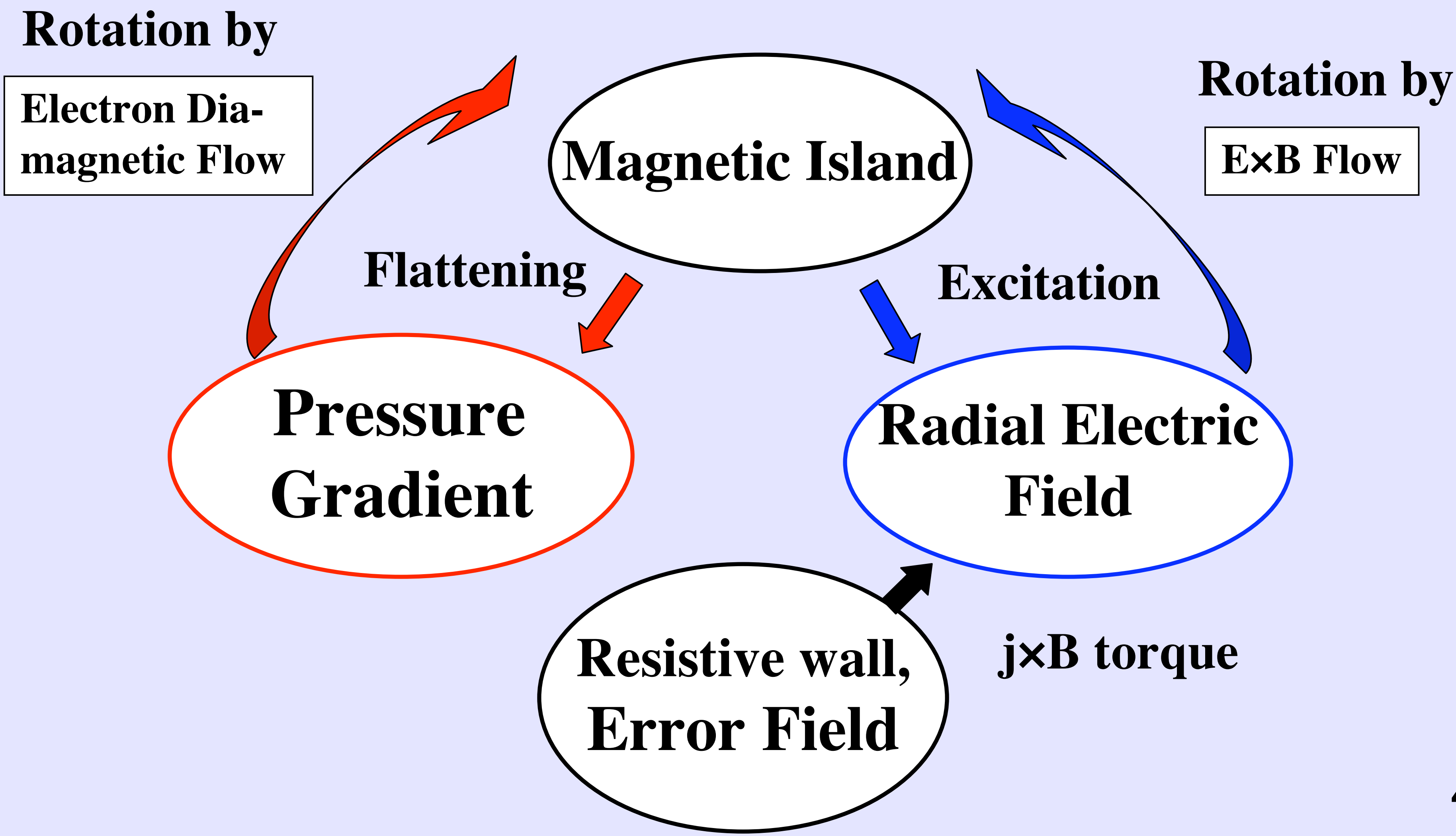


[O. Sauter etal., PoP(1997).]

Lockingの数値シミュレーションに対する解析的モデリング

研究背景 2

Rotation of magnetic island



研究背景 3

- LHDにおける磁気島周りの流れ場(径電場)の観測

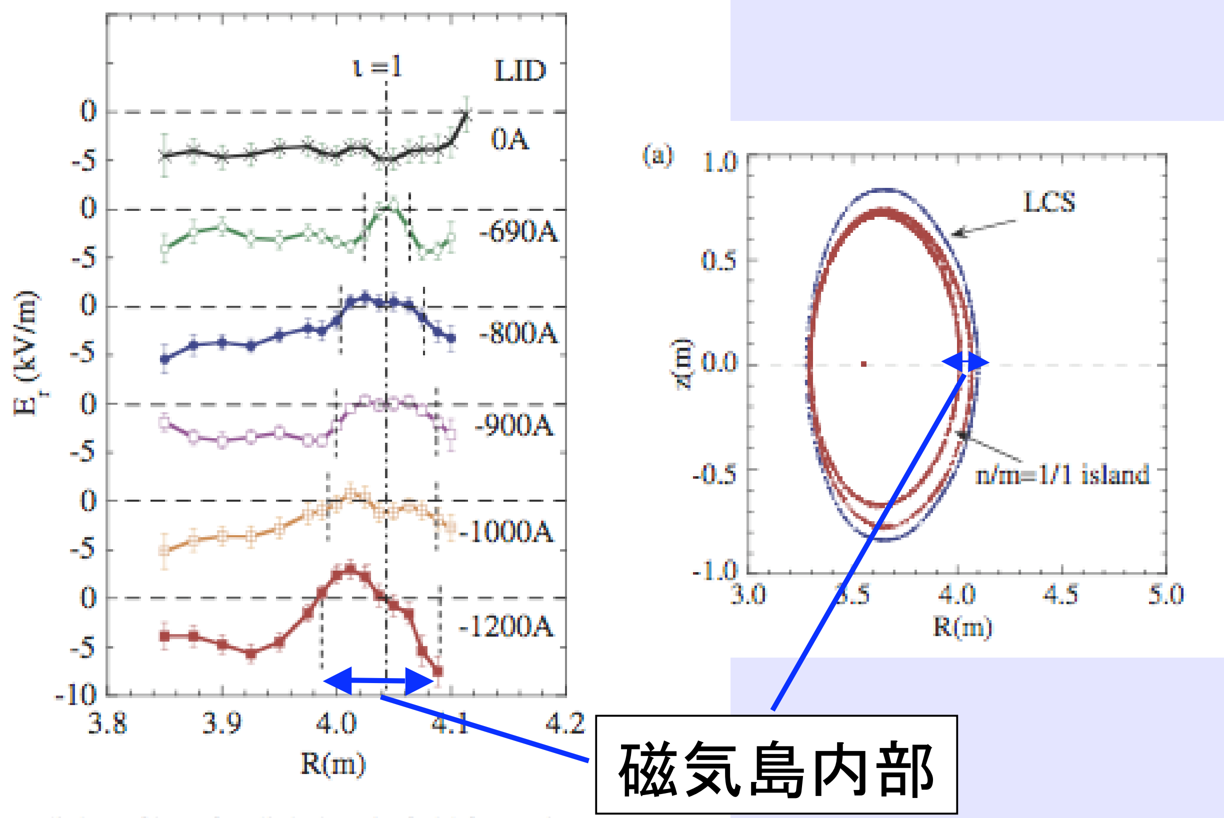
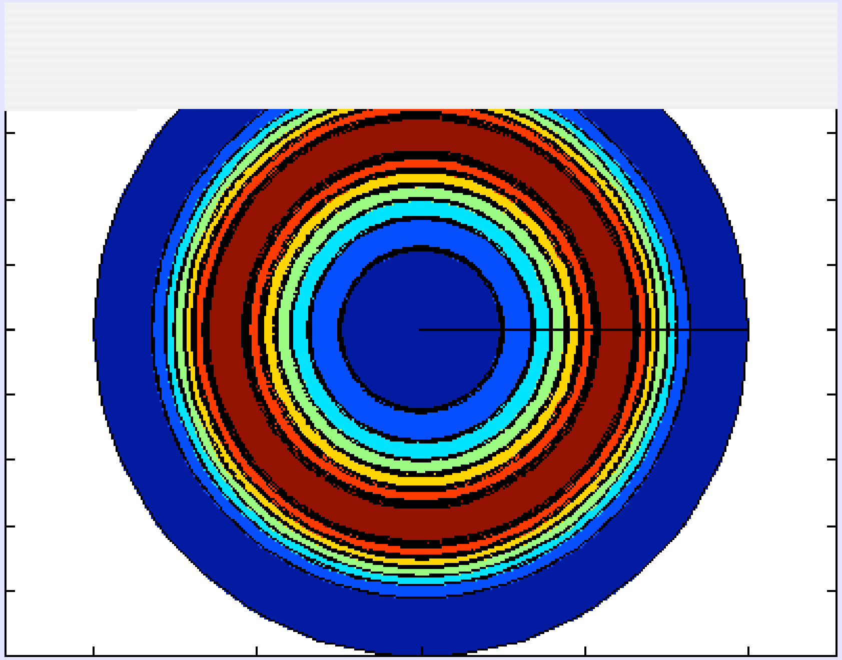


Figure 3. Radial profiles of radial electric field for various currents of $n/m = 1/1$ perturbation coil of 0, −690, −800, −900, −1000 and −1200 A.

[K.Ida *et al.*, Nucl. Fusion 44 (2004) 209]

Typical nonlinear evolution of magnetic island



Contour of magnetic filed

Reduced two-fluid Equations

Drift-Tearing Mode

Tearing Mode

Electron Dia-magnetic Drift

Vorticity Evolution :

$$\frac{D}{Dt}\nabla_\perp^2\phi = \boxed{\nabla_\parallel j_\parallel} + \mu\nabla_\perp^4\phi,$$

Generalized Ohm's law :

$$\frac{\partial}{\partial t}A = -\nabla_\parallel(\boxed{\phi} - \boxed{\delta p}) \boxed{- \eta_\parallel j_\parallel} + \boxed{\alpha_T\delta\nabla_\parallel T,}$$

Continuity Equation :

$$\frac{D}{Dt}n + \beta\frac{D}{Dt}p = \boxed{\delta\beta\nabla_\parallel j_\parallel} + \eta_\perp\beta\nabla_\perp^2 p,$$

Electron Energy Evolution :

$$\frac{3}{2}\frac{D}{Dt}T - \frac{D}{Dt}n = \boxed{\alpha_T\delta\beta\nabla_\parallel j_\parallel} + \epsilon^2\chi_\parallel\nabla_\parallel^2 T + \chi_\perp\nabla_\perp^2 T,$$

$j_{//} = -\nabla_\perp^2 A$ $\quad$ $p = n + T$ $\quad$ $\alpha_T = 0.71$

β : poloidal β-value $\quad$ $\delta = c/a\omega_{pi}$: ion skin depth

Cylindrical Coordinate: (r, θ, z)

Operator: $\frac{D}{Dt} = \frac{\partial}{\partial t} + [\phi, \ ]$ $\quad \nabla_{//} = \frac{\partial}{\partial z} - [A, \ ]$ $\quad \nabla_{\perp} = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta}$

$$[f, g] = \hat{\mathbf{z}} \cdot \nabla f \times \nabla g$$

Normalization: $r/a \to r, \ \ z/R_0 \to z, \ \ v_A t/R_0 \to t$

Energy Conservation:

$$\begin{aligned}
\frac{d}{dt}H = & -\mu \int \left|\nabla_\perp^2 \phi\right|^2 dV - \eta_\parallel \int \left|j_\parallel\right|^2 dV \\
& - \eta_\perp \int |\nabla_\perp p|^2 \, dV - \frac{\epsilon^2 \chi_\parallel}{\beta} \int \left|\nabla_\parallel T\right|^2 dV \\
& - \frac{\chi_\parallel}{\beta} \int |\nabla_\perp T|^2 \, dV,
\end{aligned}$$

$$\begin{aligned}
H = & \frac{1}{2} \int | \nabla_\perp \phi |^2 \, dV + \frac{1}{2} \int | \nabla_\perp A |^2 \, dV \\
& + \frac{1}{2\beta} \int |n|^2 \, dV + \frac{3}{4\beta} \int |T|^2 \, dV \\
& + \frac{1}{2} \int |p|^2 \, dV.
\end{aligned}$$

Simulation Condition

Expansion:

$$f(\mathbf{r},t) = f_0(r) + \sum_{m,n} \tilde{f}_{m,n}(r,t) e^{i(m\theta - nz)}$$

Numerical Method:

r ← Finite difference

θ, z ← Pseudo Spectrum Method

t ← Predictor-Corrector Scheme

Boundary Condition:

$$\tilde{f}_{m,n}(0) = \tilde{f}_{m,n}(1) = 0 \quad (m,n \neq 0)$$

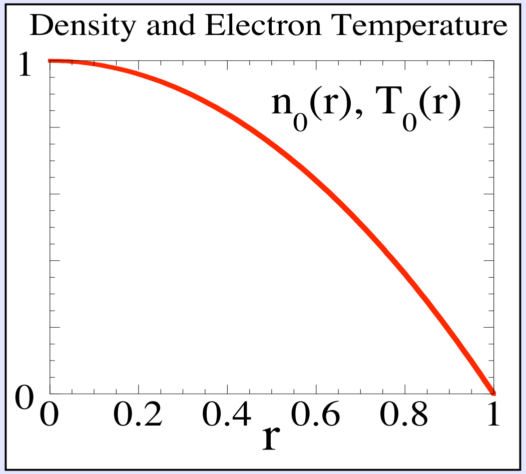
$$\partial \tilde{f}_{0,0} / \partial r \Big|_{r=0} = \tilde{f}_{0,0}(1) = 0$$

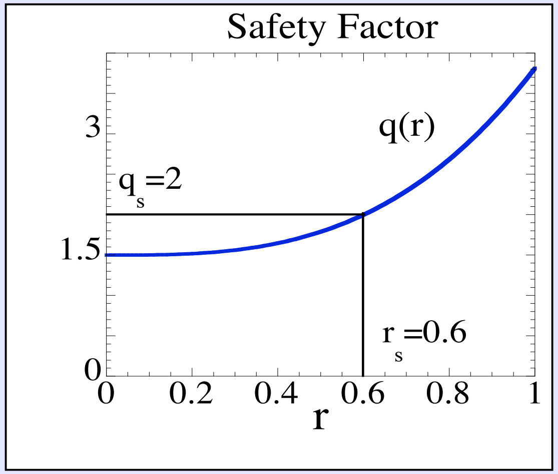
Single helicity m/n = 2/1:
(2,1),(4,2),(6,3) and (8,4)

Default Parameters:

$$\mu = \eta_{//} = \chi_{\perp} = 10^{-5}, \ \ \eta_{\perp} = 2 \times 10^{-5}, \ \ \chi_{//} = 1, \ \ \beta = \delta = 0.01$$

Initial Profile: (Equilibrium)





Only (2,1) is linearly unstable

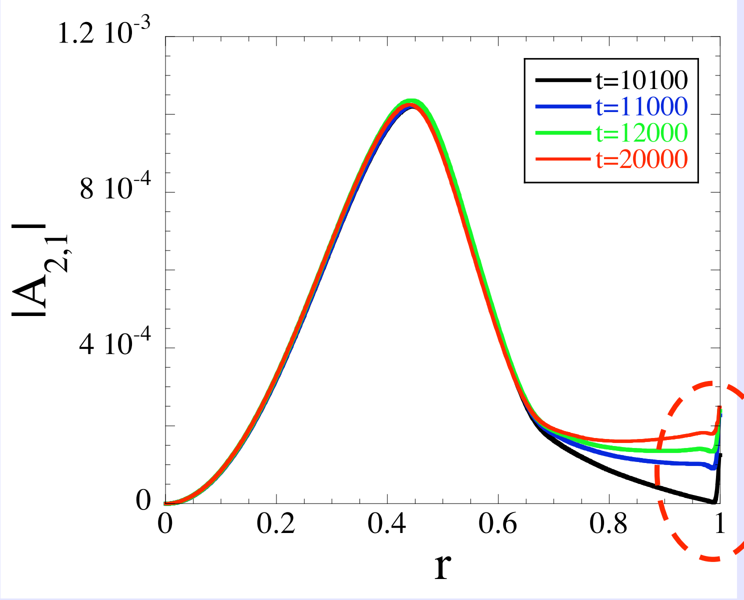
External helical magnetic field

▪ Misalignment of toroidal coils induces radial magnetic field (so-called 'error field'), which affects the island rotation.

Typical value: $\dfrac{B_r}{B_z} \approx 10^{-3} - 10^{-4}$

[ITER Physics Basis Editors et al., Nucl. Fusion 39, 2137 (1999).]

▪ Error field induces finite boundary condition.



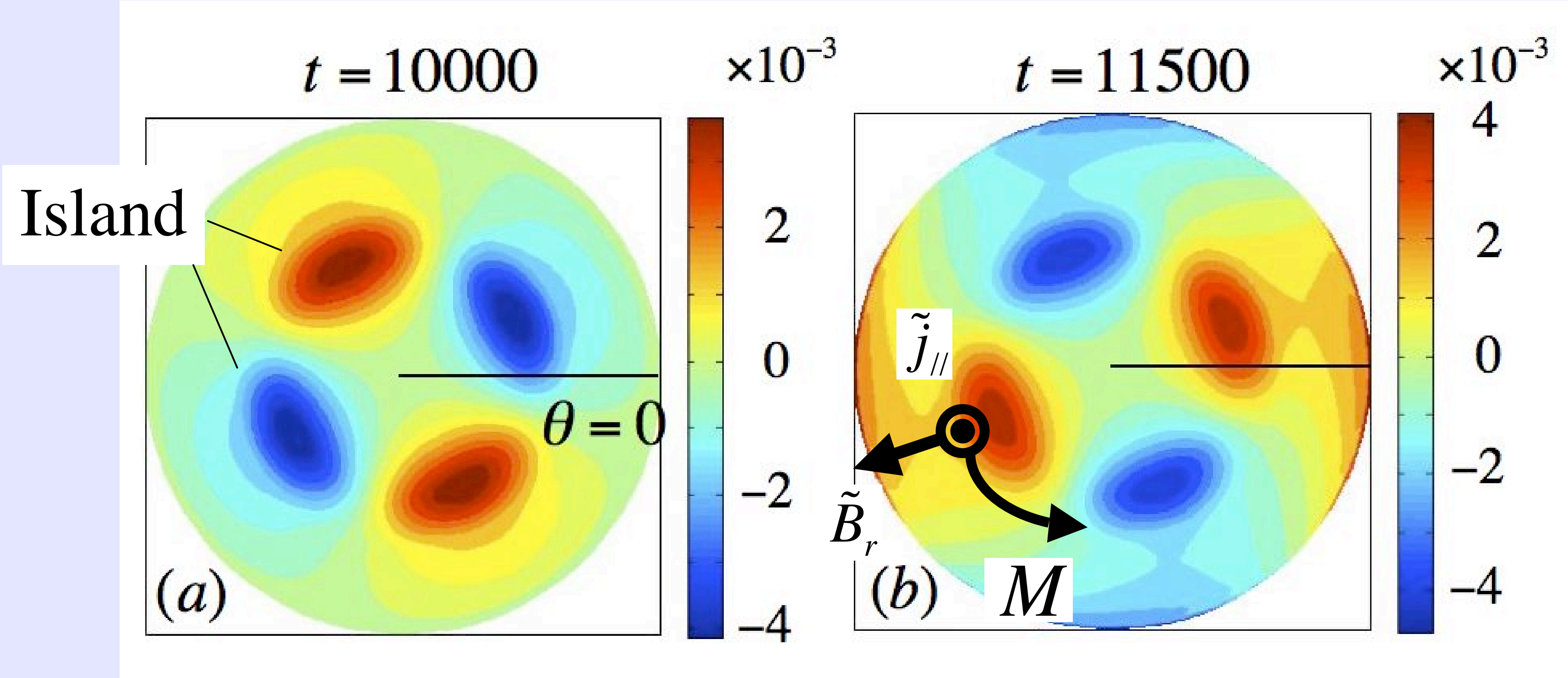
$$\tilde{B}_{r2,1}(1) = ik_\theta(1)\tilde{A}_{2,1}(1)$$

$$\tilde{A}_{2,1}(1) \approx 10^{-4}$$

Contour of vector potential

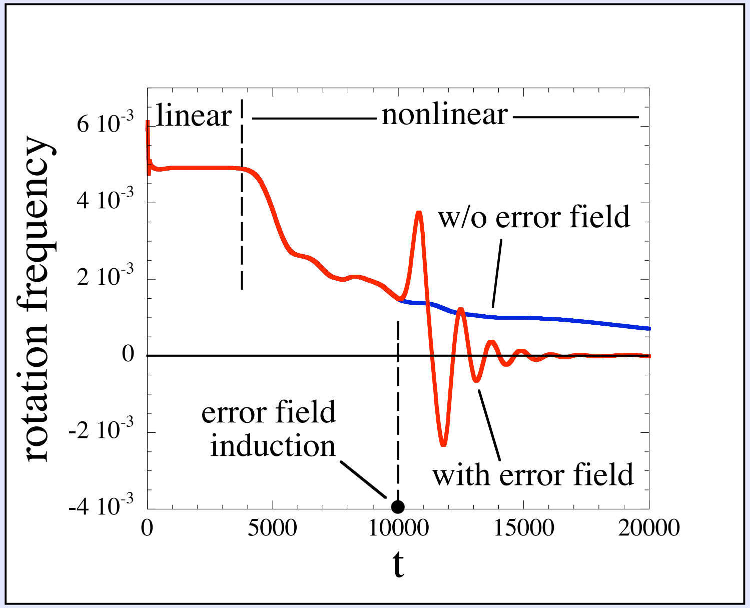
Without error field

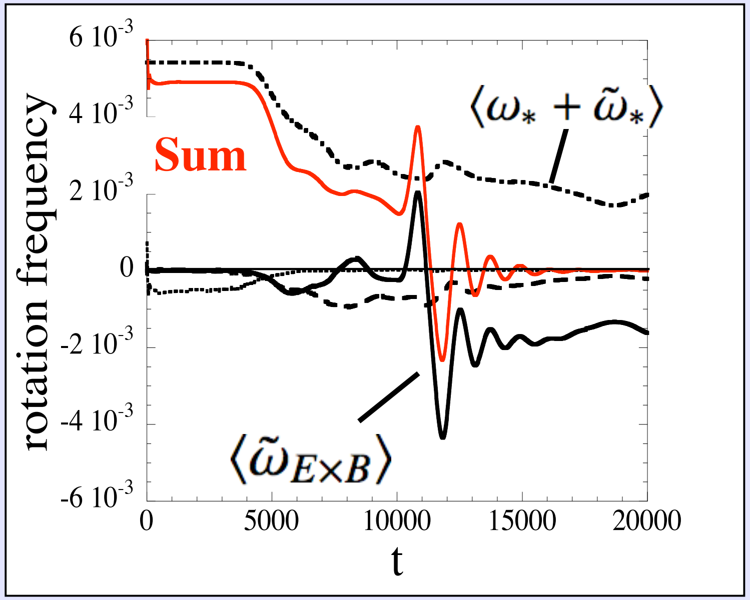
With error field



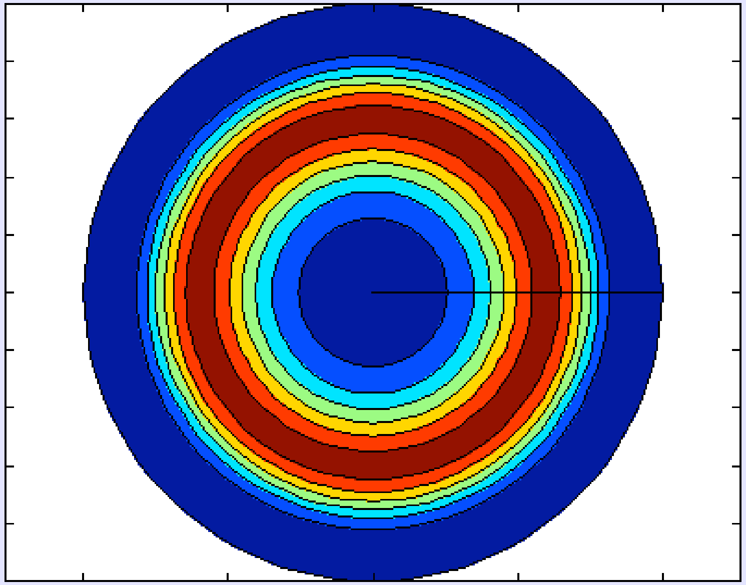
$\mathrm{Re}\left[\tilde{A}_{2,1}(r,t)\exp(im\theta)\right]$ $z = 0$

Locking of island rotation





with error field



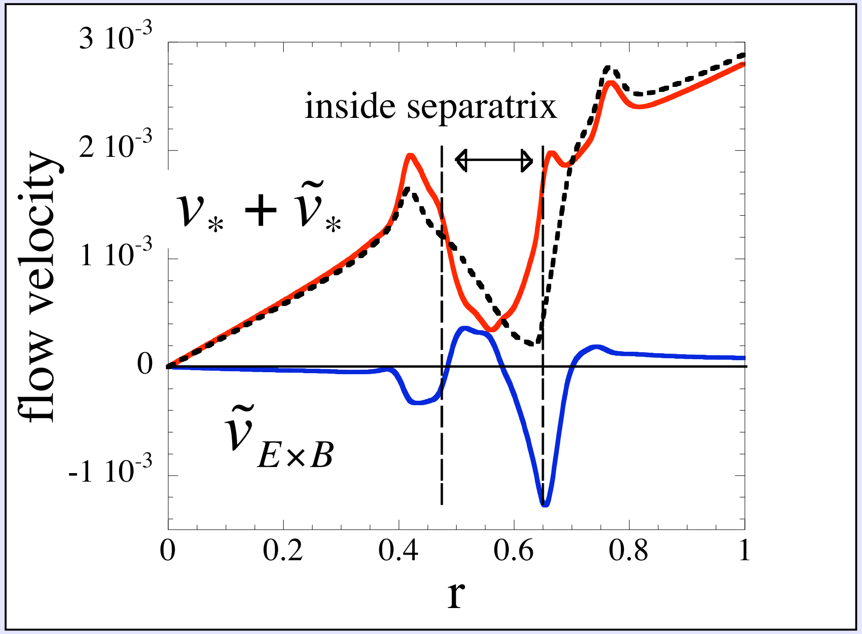
$$\omega_r \approx \left\langle \omega_* + \tilde{\omega}_* \right\rangle + \left\langle \tilde{\omega}_{E\times B} \right\rangle$$

Electron diamagnetic flow

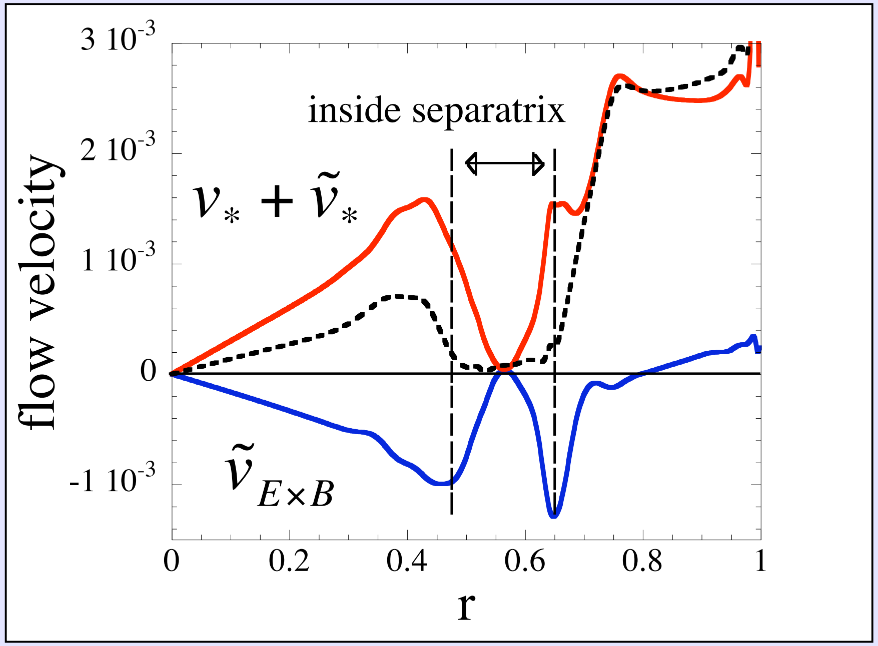
E×B flow

Radial profile of flows

$t = 10000$ **(Before locking)**

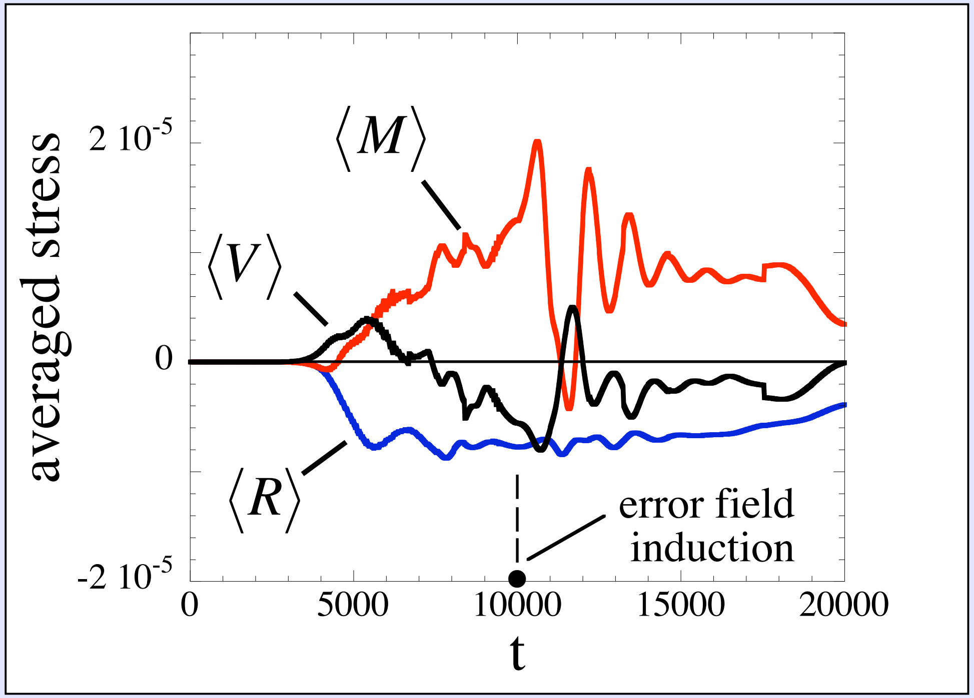


$t = 20000$ **(After locking)**



Time evolution of stresses

with error field



$$\frac{\partial}{\partial t}\omega_r \approx \frac{\partial}{\partial t}\left\langle \tilde{\omega}_{E\times B} \right\rangle \approx \underbrace{\langle M \rangle}_{\text{Maxwell stress}} + \underbrace{\langle V \rangle}_{\text{Ion viscosity}}$$

Maxwell stress $\langle M \rangle$ ➡ **Direct influence of error field via J×B**

Ion viscosity $\langle V \rangle$ ➡ **Response of E×B flow**

Low dimensional model

$$\frac{\partial}{\partial t}\omega_r \approx \langle M \rangle + \langle V \rangle$$

$\Delta w << w_0$

$$\frac{\partial}{\partial t}\omega_r = \underbrace{-C_M \sin 2\theta}_{\text{Maxwell stress}} \underbrace{- C_V \omega_r}_{\text{Ion viscosity}}$$

with

$$C_M = \frac{m k_{\theta 2,1}\big|_{rs} s_s^2 \Delta_0' w_0^2}{256 q_s^2} |\Delta w|$$

$$C_V = \frac{8\mu}{w_0^2 k_{\theta 2,1}\big|_{rs}}$$

$C_V \to 0$

$$\omega_r = \pm 2\sqrt{\frac{C_M}{k^2}} \sqrt{1 - k^2 \mathrm{sn}^2\left(\sqrt{\frac{C_M}{k^2}} t\right)}$$

$$\frac{1}{k^2} = \sin^2\frac{\theta_0}{2} + \frac{\omega_{r0}^2}{4C_M}$$

$0 < k^2 < 1$ ：非捕捉

$k^2 > 1$ ：捕捉

$C_V \neq 0$

$$\frac{\partial}{\partial t}E = -C_V \omega_r^2$$

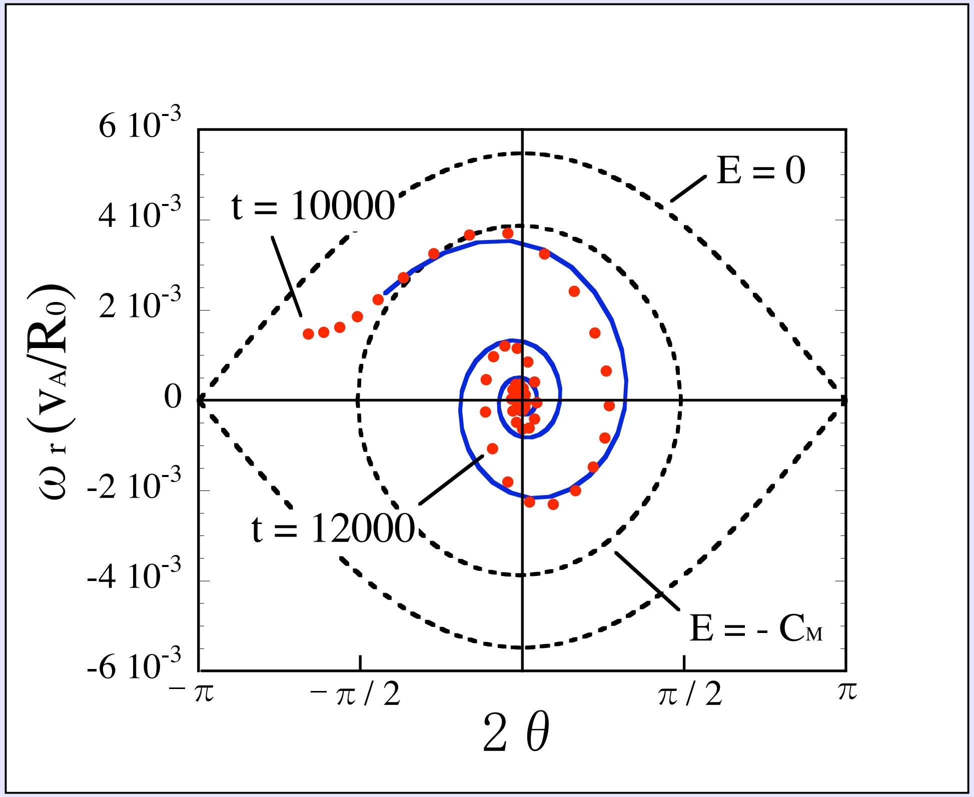
$$E = \frac{1}{2}\omega_r^2 - 2C_M \cos^2\frac{\theta}{2}$$

臨界： $k^2 = 1 \leftrightarrow E = 0$

Comparison with low dimensional model and nonlinear simulation

$C_M = 7.5 \times 10^{-6}$

$C_V = 8.1 \times 10^{-4}$



Locking mechanism

Trapping by error field

Damping by ion viscosity

Summary

- **Locking of rotation of drift-tearing mode by error field is investigated.**
- **Analytical decomposition model for rotation frequency of magnetic island is introduced, and importance of contribution of E×B flow is found.**
- **Detailed analysis shows that coupling of contributions from Maxwell stress and ion viscosity plays an essential role.**
- **A low dimensional mode is derived to understand the physical mechanism of locking event.**
- **It is concluded that the locking is caused by rotation trapping by error field and damping by ion viscosity.**

Future work:

- Collisionless case
- Forced reconnection (initially stable tearing mode).
- Influence on turbulent transport (RMP).

Collisionless limit

If $\Delta w << w_0$ is broken.

Collisionless plasmas

$$\frac{\partial}{\partial t}\omega_r = -C_M \sin 2\theta - (\cancel{C_V} + \underbrace{C_w}_{})\omega_r \qquad C_w = \frac{1}{w}\frac{\partial \Delta w}{\partial t}$$

Effect of island growth by error field

Locking Threshold

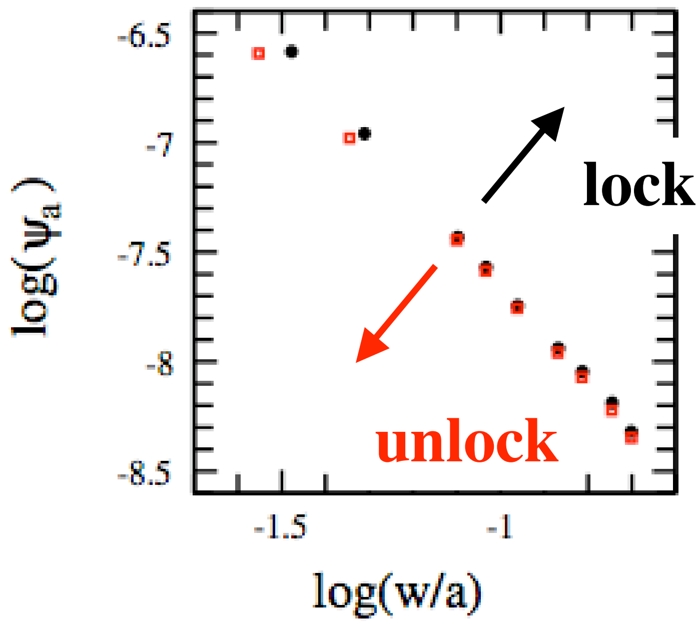


Fig.1 Mode locking threshold versus normalized island width w/a.

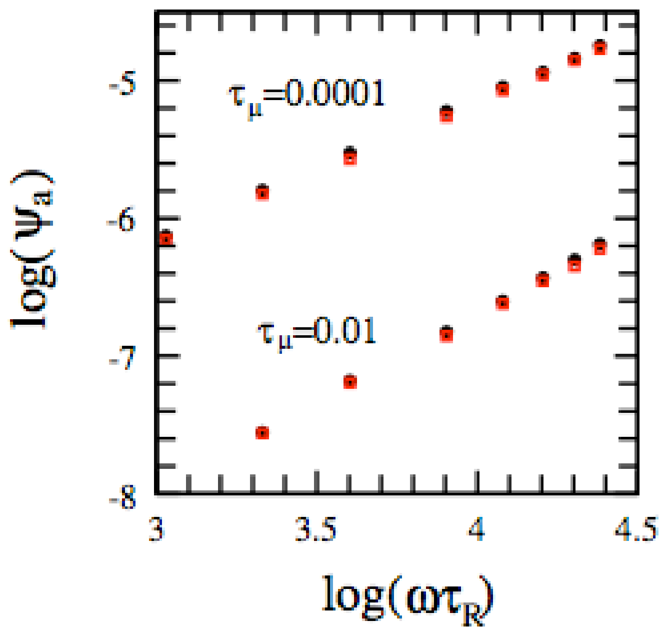


Fig.2 Mode locking threshold versus normalized mode frequency.

★Threshold value depends on magnetic island 'width' and 'rotation frequency'.

[Yu *et al*., Proc. EPS (2008)]

▪ Analytic formula of rotation frequency of islands

$$\omega_r \approx \langle \omega_* + \tilde{\omega}_* \rangle + \langle \tilde{\omega}_{E\times B} \rangle + \mathrm{Re}\langle L_{k//} \rangle + \mathrm{Re}\langle L_{\eta //} \rangle$$

Electron diamagnetic flow

E×B flow

Self-generated perturbed field

$$\omega_* = k_{\theta_{2,1}} \left(v_* + \tilde{v}_* \right)$$
$$v_* = -\delta \left[n_0' + \left(1+\alpha_T\right) T_0' \right]$$
$$\tilde{v}_* = -\delta \left[\tilde{n}_0' + \left(1+\alpha_T\right) \tilde{T}_0' \right]$$

$$\tilde{\omega}_{E\times B} = k_{\theta_{2,1}} \tilde{v}_{E\times B}$$

$$L_{\eta_\parallel} = i\eta_\parallel \frac{\nabla_\perp^2 \tilde{A}_{2,1}}{\tilde{A}_{2,1}}$$

$$L_{k_\parallel} = k_{\parallel 2,1} \frac{\tilde{\phi}_{2,1} - \delta \left\{ \tilde{n}_{2,1} + (1+\alpha_T) T_{2,1} \right\}}{\tilde{A}_{2,1}}$$

Radial Average inside island:

$$\langle\ \rangle = \frac{1}{W} \int_{r_{\mathrm{in}}}^{r_{\mathrm{out}}} dr$$

$$W = r_{\mathrm{out}} - r_{\mathrm{in}}$$

▪ Ion E×B flow generation

$$\langle \tilde{\omega}_{\mathrm{E\times B}} \rangle = \left\langle \int^t R dt' \right\rangle + \left\langle \int^t M dt' \right\rangle + \left\langle \int^t V dt' \right\rangle$$

Reynolds stress

Maxwell stress

Ion viscosity

$$\mathbf{v} \times \left(\nabla \times \mathbf{v} \right)$$

$$\mathbf{j} \times \mathbf{B}$$

$$R = -\frac{k_{\theta_{2,1}}}{r} \int^r dr\ r \left[\tilde{\phi}, \nabla_\perp^2 \tilde{\phi} \right]_{0,0},$$
$$M = \frac{k_{\theta_{2,1}}}{r} \int^r dr\ r \left[\tilde{A}, \nabla_\perp^2 \tilde{A} \right]_{0,0},$$
$$V = \mu k_{\theta_{2,1}} \frac{\partial}{\partial r} \left\{ \frac{1}{r} \frac{\partial}{\partial r} (r v_\theta) \right\}.$$

[S.Nishimura etal., Phys.Plasmas 15 093506 (2008)]