電磁表面波とその利用

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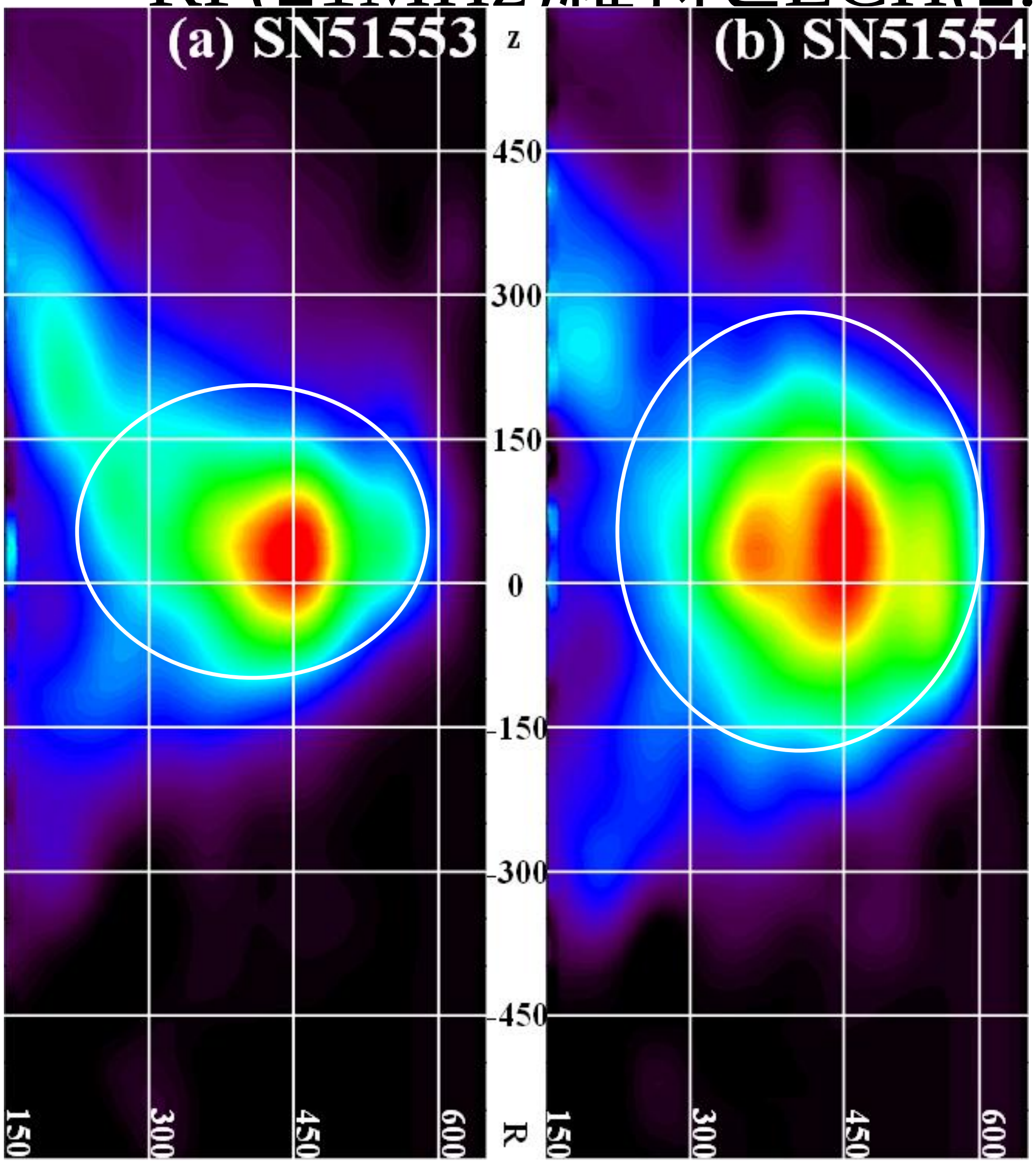
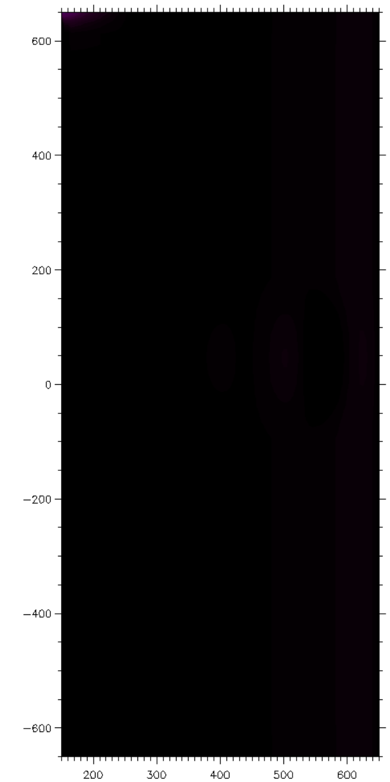
非誘導立ち上げしたトカマクプラズマのRF(21MHz)維持とECH(2.45GHz)維持

トカマク

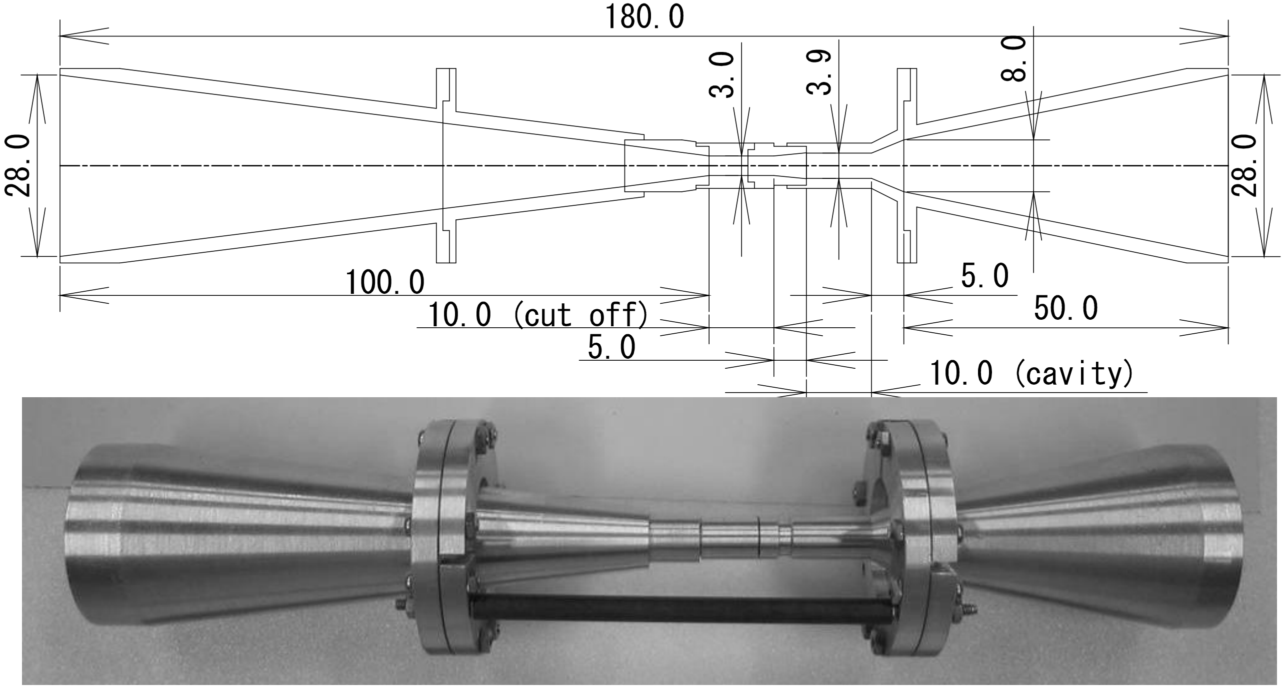


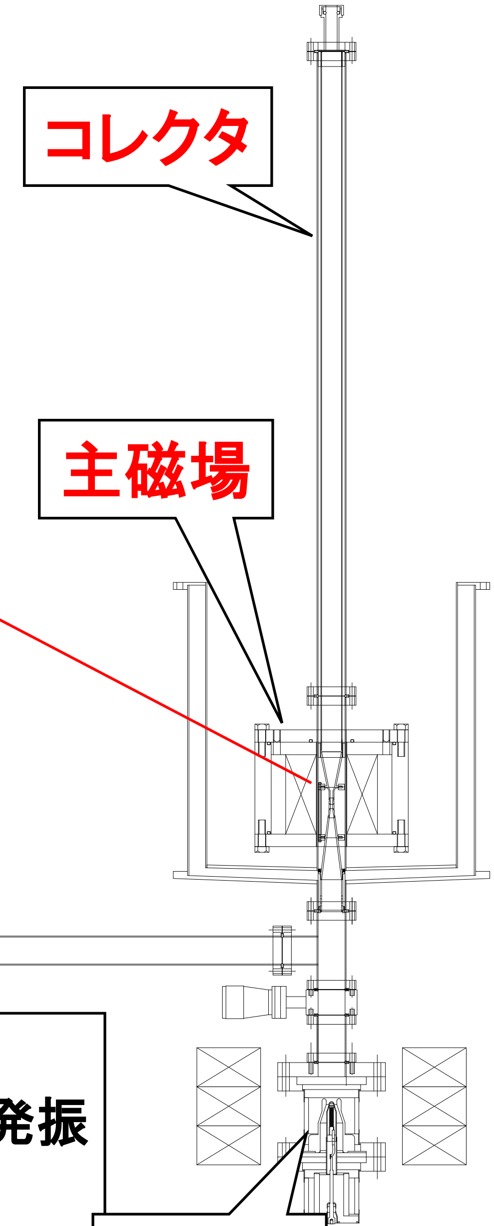


1. Osamu WATANABE, et al., JSPS, Vol. 3, 049 (2008).
2. Osamu Watanabe, et al., JPFR, Vol. 8, pp.1040-1043, (2009).
3. O. Watanabe, et al., submitted to International TOKI Conference

Gyrotron

シンプルな空洞共振器





- サイクロトロン運動を電磁波に変換
- 導波管モードがサイクロトロン周波数と一致するところで発振

→発振周波数が固定される・出力がリミットされる

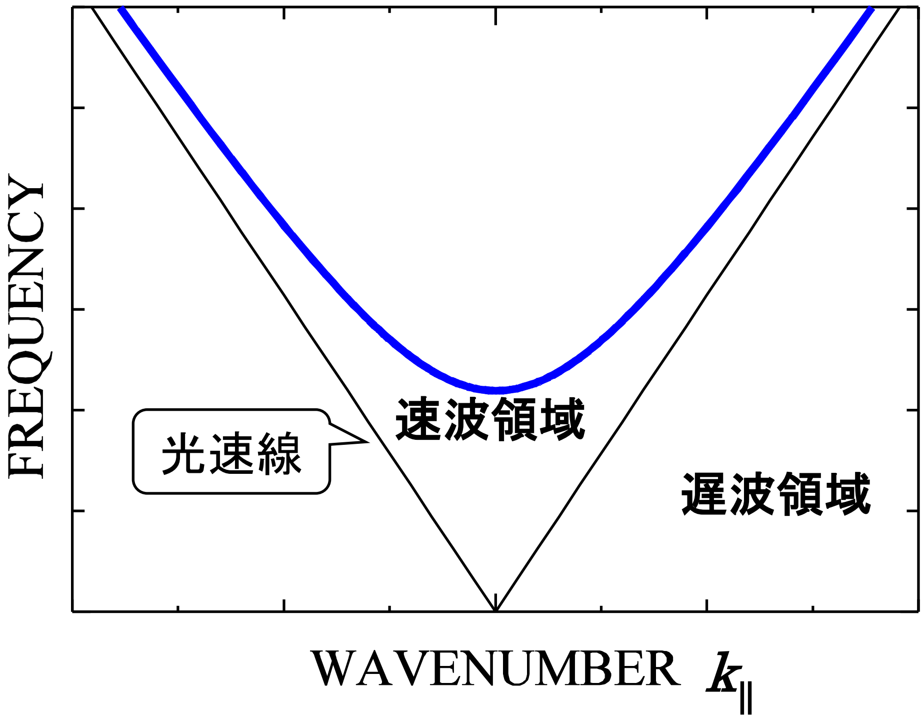
→**発振周波数可変の高出力発振器が欲しい**

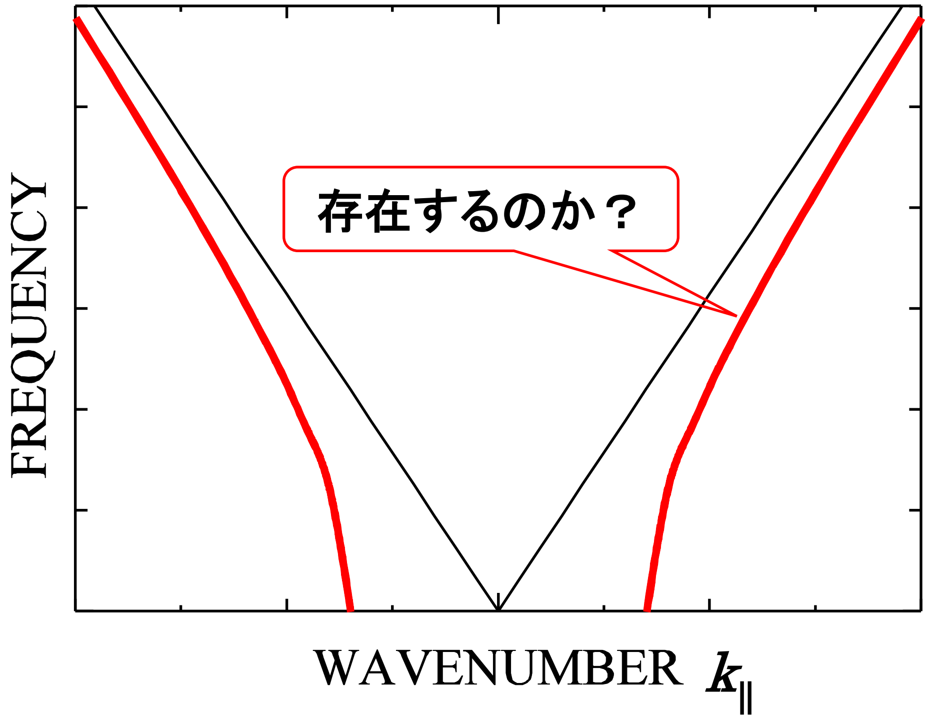
発想の起点

空間を伝わる電磁波: $\omega^2 = c^2(k_\perp{}^2 + k_\parallel{}^2)$

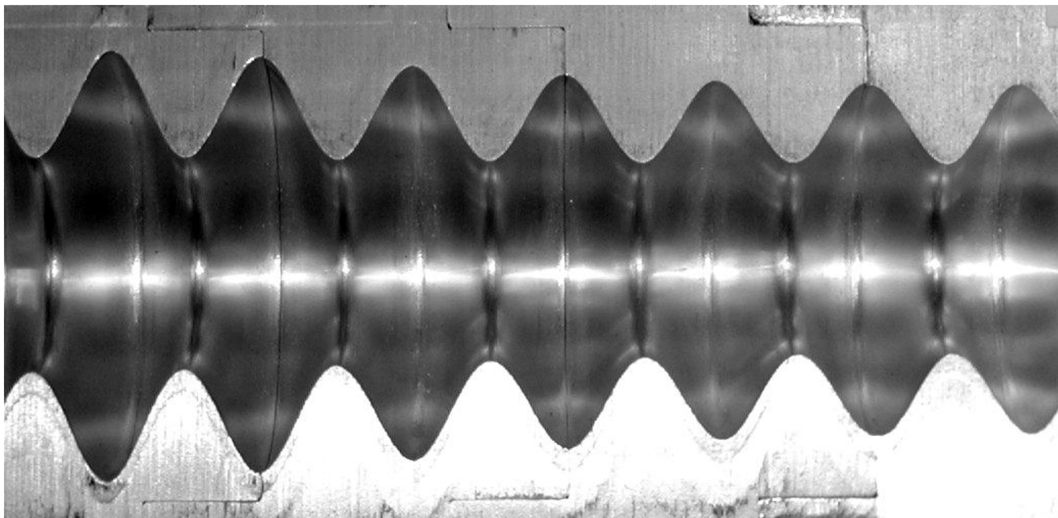
通常は$k_\perp{}^2>0$

数学的には$k_\perp{}^2<0$も可



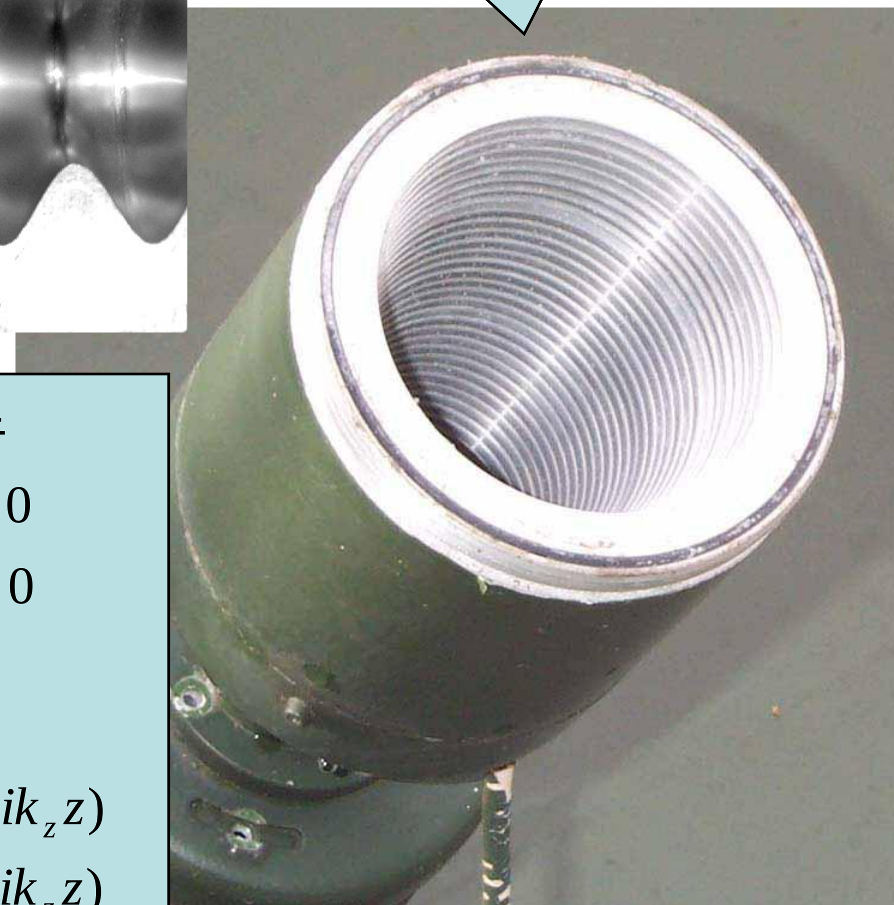


Corrugated waveguide



後進波発振器

伝送管



電磁場成分（未知数6）

$$\boldsymbol{E}=\begin{bmatrix}E_r\\E_\theta\\E_z\end{bmatrix},\boldsymbol{B}=\begin{bmatrix}B_r\\B_\theta\\B_z\end{bmatrix}$$

管壁境界条件

$$E_z(r=R_W)=0$$

$$E_\theta(r=R_W)=0$$

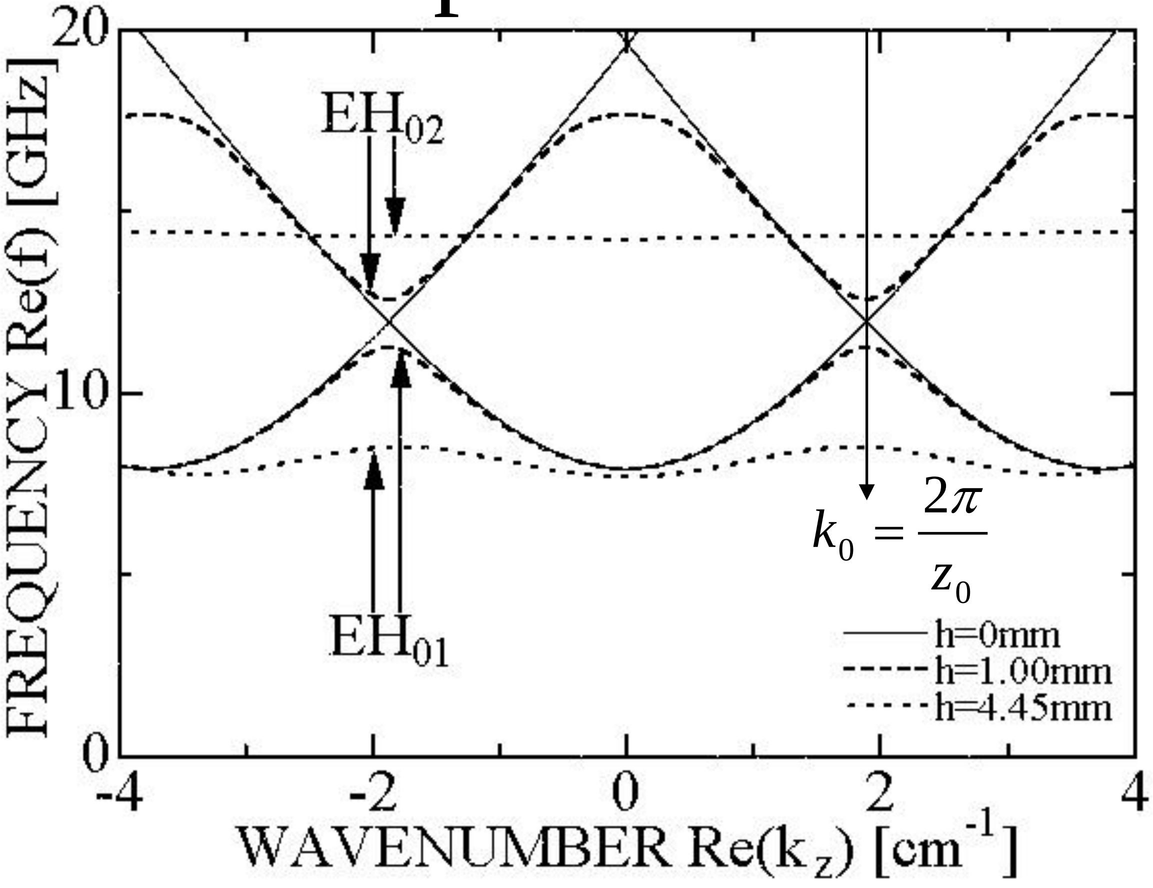
軸方向成分

$$E_z=A_{Ez}\exp(ik_z z)$$

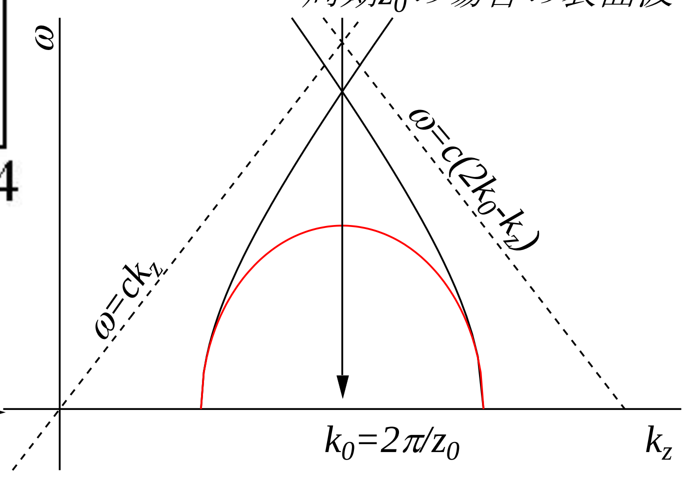
$$B_z=A_{Bz}\exp(ik_z z)$$

Maxwell方程式

Band Structure due to the Connections of Spatial Harmonic Waves

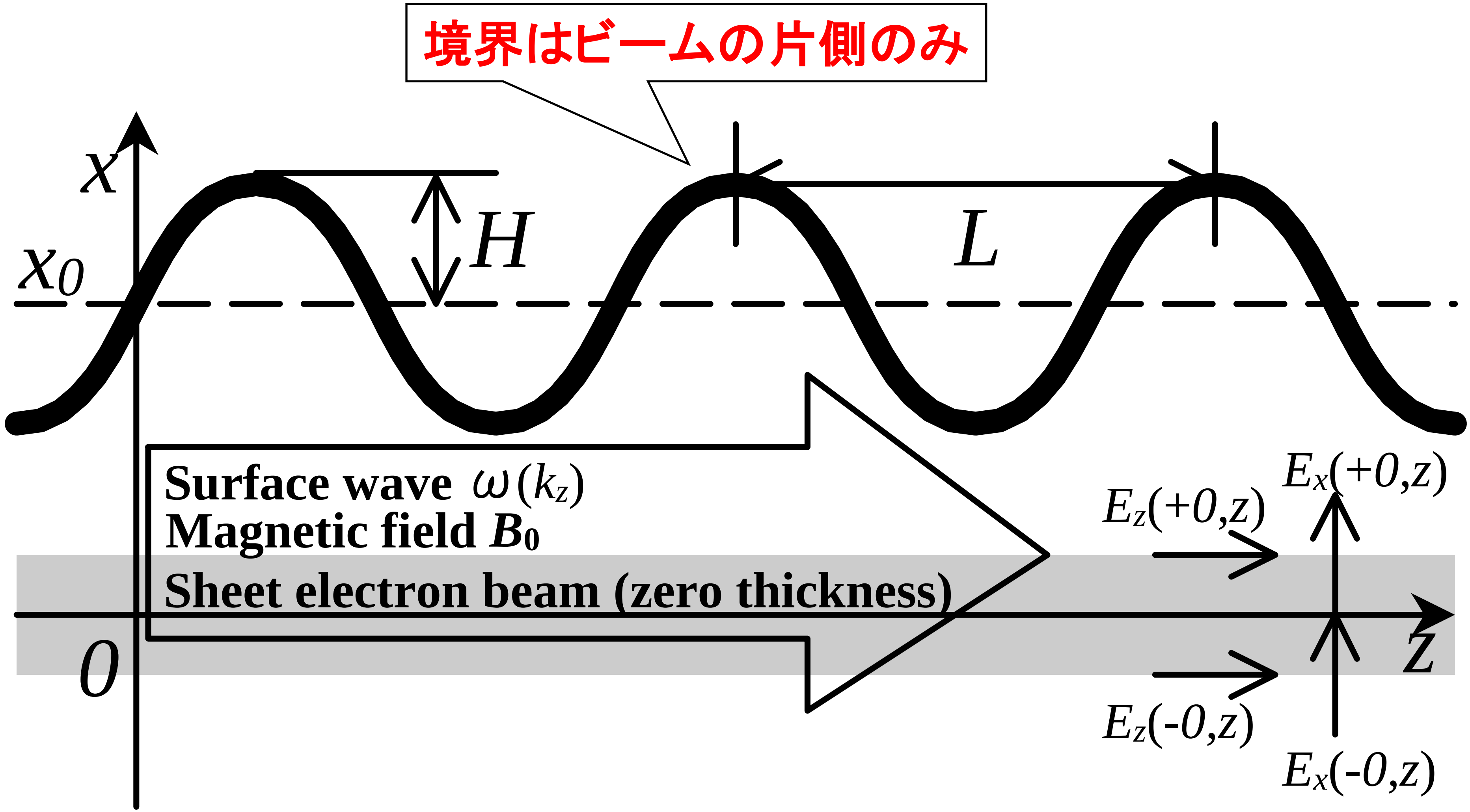


予想



z方向に水平伝搬→電磁表面波

モデル図



境界はビームの片側のみ
x
x0
H
L
Surface wave ω(kz)
Magnetic field B0
Sheet electron beam (zero thickness)
Ex(+0,z)
Ez(+0,z)
Ez(-0,z)
Ex(-0,z)
0
z

NORMAL MODE

Electromagnetic field component

$$B(x,z,t)=\nabla\times\begin{bmatrix}\Phi(x,z,t)\\ \Psi(x,z,t)\\ 0\end{bmatrix}=\begin{bmatrix}-\dfrac{\partial\Psi(x,z,t)}{\partial z}\\ \dfrac{\partial\Phi(x,z,t)}{\partial z}\\ \dfrac{\partial\Psi(x,z,t)}{\partial x}\end{bmatrix}$$

$$E(x,z,t)=-\frac{\partial}{\partial t}\begin{bmatrix}\Phi(x,z,t)\\ \Psi(x,z,t)\\ 0\end{bmatrix}-\nabla\lambda(x,z,t)=\begin{bmatrix}-\dfrac{\partial\Phi(x,z,t)}{\partial t}-\dfrac{\partial\lambda(x,z,t)}{\partial x}\\ -\dfrac{\partial\Psi(x,z,t)}{\partial t}\\ -\dfrac{\partial\lambda(x,z,t)}{\partial z}\end{bmatrix}$$

- rot**B**

$$0=\begin{bmatrix}-\dfrac{\partial^2\Phi(x,z,t)}{\partial t^2}-\dfrac{\partial^2\lambda(x,z,t)}{\partial t\partial x}+\dfrac{1}{c^2}\dfrac{\partial^2\Phi(x,z,t)}{\partial z^2}\\ -\dfrac{\partial^2\Psi(x,z,t)}{\partial t^2}+\dfrac{1}{c^2}\left\{\dfrac{\partial^2\Psi(x,z,t)}{\partial z^2}+\dfrac{\partial^2\Psi(x,z,t)}{\partial x^2}\right\}\\ -\dfrac{\partial^2\lambda(x,z,t)}{\partial t\partial z}-\dfrac{1}{c^2}\dfrac{\partial^2\Phi(x,z,t)}{\partial x\partial z}+\dfrac{1}{\varepsilon_0}J_z\end{bmatrix}$$

$$J=(0,0,J_z)$$

- Only TM like mode

$$E=\frac{1}{i\omega c^2}\begin{bmatrix}\dfrac{\partial^2\Phi(x,z,t)}{\partial z^2}\\ 0\\ -\dfrac{\partial^2\Phi(x,z,t)}{\partial x\partial z}\end{bmatrix}$$

$$\Phi(x,z,t)=\sum_{n=-\infty}^{\infty}E_n\exp\{i(k_nz+k_\perp x-\omega t)\}$$

$$k_n=k_z+nk_0$$

$$k_\perp^2=\frac{\omega^2}{c^2}-k_z^2$$

- Wave function

＋:between the beam and the SWS

－:under the electron beam

$$\Phi^+=\sum_{n=-\infty}^{\infty}\{A_n^+\exp(k_\perp x)+B_n^+\exp(-k_\perp x)\}\exp\{i(k_nz-\omega t)\}$$

$$\Phi^-=\sum_{n=-\infty}^{\infty}A_n^-\exp(k_\perp x)\exp\{i(k_nz-\omega t)\}$$

分散式の導出

Equation of Motion

- Equation of fluid

$$m\left(\frac{\partial}{\partial t}+v_0\frac{\partial}{\partial z}\right)\frac{\partial v_z}{\partial z}=-e\frac{\partial E_z}{\partial z}$$

- Equation of continuity

$$\nabla\cdot J+\frac{\partial\rho}{\partial t}=0$$

$$\therefore 0=N_0\boxed{\frac{\partial v_z}{\partial z}}+v_0\frac{\partial N_1}{\partial z}+\frac{\partial N_1}{\partial t}$$

- Thus

$$0=\omega^2 N_1-2i\omega v_0\frac{\partial N_1}{\partial z}-v_0^2\frac{\partial^2 N_1}{\partial z^2}+\frac{eN_0}{m}\frac{\partial^2\Phi}{\partial x\partial z}$$

1

Boundary Condition

- Beam surface

$$\boxed{E_x^+ - E_x^- = -\frac{eN_1}{\varepsilon_0}}$$

2

$$E_y^+ - E_y^- = 0$$

$$\boxed{E_z^+ - E_z^- = 0}$$

3

$$B_x^+ - B_x^- = 0$$

$$B_y^+ - B_y^- = \mu_0 \kappa_z$$

$$B_z^+ - B_z^- = 0$$

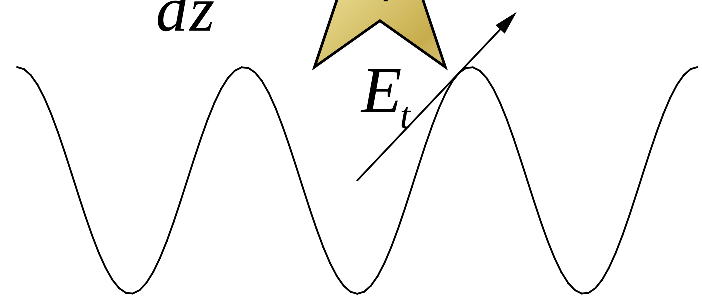
- SWS surface

$$x_w = x_0 + h\cos(k_0 z)$$

$$0 = E_t = E_z \cos(k_0 z) + E_x \sin(k_0 z)$$

$$\propto E_z + E_x \frac{dx}{dz}$$

4



$$0 = \left(\omega^2 - 2\omega v_0 k_n + k_n^2 v_0^2\right)\left\{\left(A_n^+ - A_n^-\right)\exp(k_\perp x) + B_n^+ \exp(-k_\perp x)\right\} + \frac{e^2 N_0 k_\perp}{m\varepsilon_0} A_n^- \exp(k_\perp x)$$

$$0 = A_n^+ - B_n^+ - A_n^-$$

分散式の導出

Dispersion Relation

$$0=\sum_{n=-\infty}^{\infty}B_n^{+}\left\{\frac{e^2N_0k_\perp}{2m\varepsilon_0}-\left(\omega^2-2\omega v_0k_n+k_n^2v_0^2\right)\right\}k_n\exp\left(k_\perp x_0\right)\int_{-\pi/k_0}^{\pi/k_0}\exp\left\{k_\perp h\cos\left(k_0z\right)\right\}\cos\left\{\left(n-m\right)k_0z\right\}dz$$

$$-\frac{e^2N_0}{2m\varepsilon_0}\sum_{n=-\infty}^{\infty}B_n^{+}k_\perp k_n\exp\left(-k_\perp x_0\right)\int_{-\pi/k_0}^{\pi/k_0}\exp\left\{-k_\perp h\cos\left(k_0z\right)\right\}\cos\left\{\left(n-m\right)k_0z\right\}dz$$

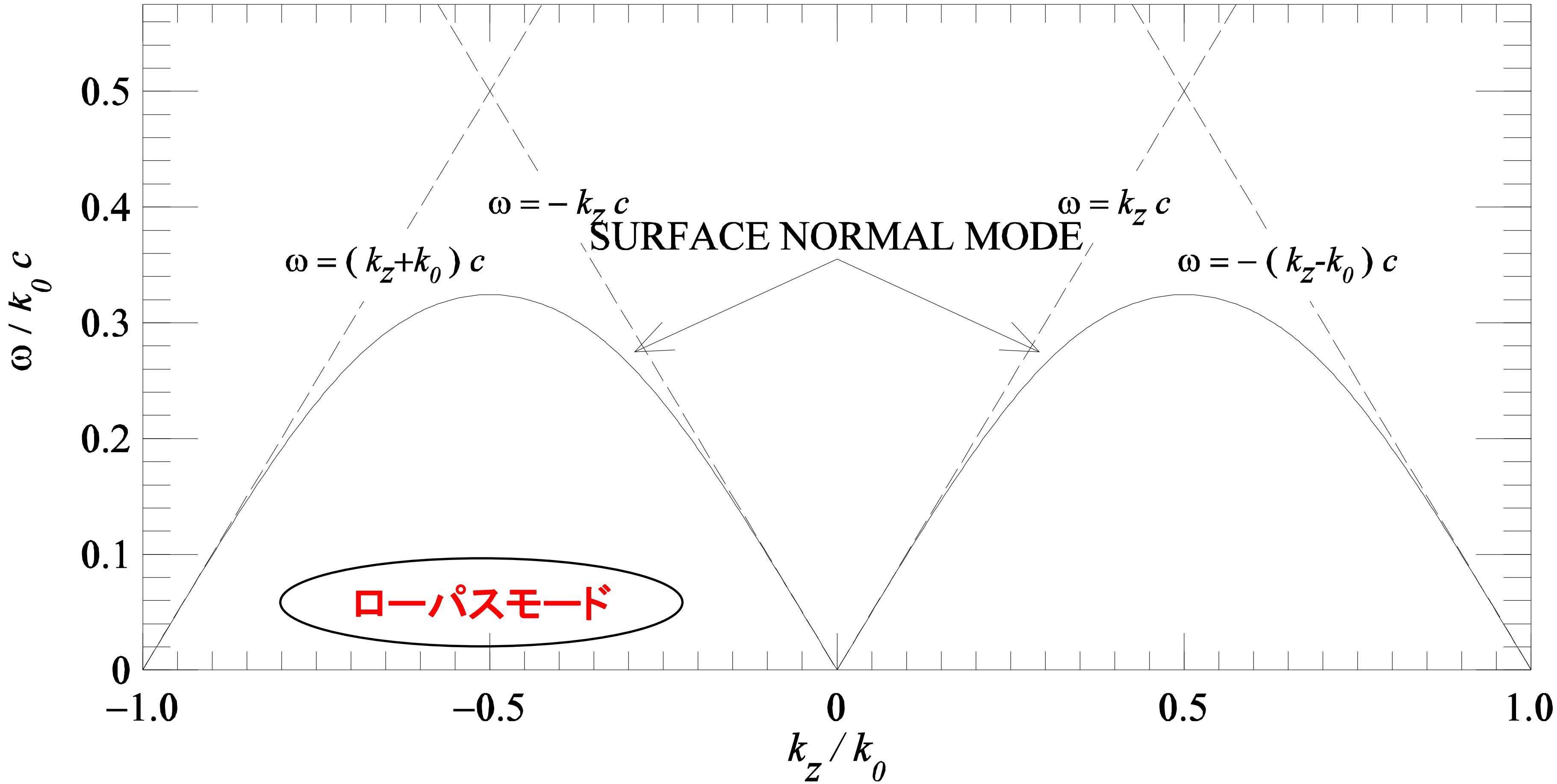
$$-hk_0\sum_{n=-\infty}^{\infty}B_n^{+}\left\{\frac{e^2N_0k_\perp}{2m\varepsilon_0}-\left(\omega^2-2\omega v_0k_n+k_n^2v_0^2\right)\right\}\frac{k_n^2}{k_\perp}\exp\left(k_\perp x_0\right)\int_{-\pi/k_0}^{\pi/k_0}\sin\left(k_0z\right)\exp\left\{k_\perp h\cos\left(k_0z\right)\right\}\sin\left\{\left(n-m\right)k_0z\right\}dz$$

$$-\frac{e^2N_0k_\perp}{2m\varepsilon_0}hk_0\sum_{n=-\infty}^{\infty}B_n^{+}k_n^2\exp\left(-k_\perp x_0\right)\int_{-\pi/k_0}^{\pi/k_0}\sin\left(k_0z\right)\exp\left\{-k_\perp h\cos\left(k_0z\right)\right\}\sin\left\{\left(n-m\right)k_0z\right\}dz$$

$$D_{mn}\cdot B_n=0$$

$$\therefore\left|D_{mn}\right|=0$$

表面波モード(H/L=0.35)の分散関係



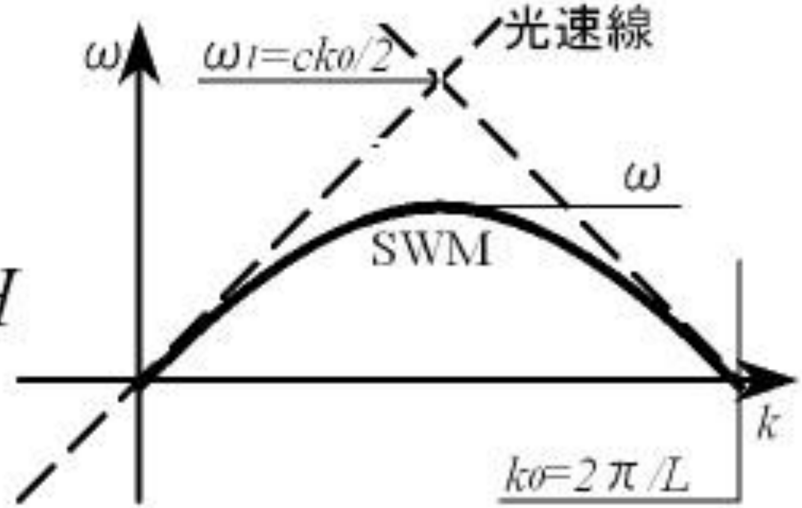
Watanabe Osamu, *et al*., “*Numerical Study of Microwave Generation by Electromagnetic Surface Wave on Deeply Corrugated Metal Plate*”, The Japan Society of Plasma Science and Nuclear Fusion Research, Rapid Communications, Vol. 1, 025 (2006).

高次モードの発生条件

$2H > \lambda(2n+1)/4$

$\omega/2\pi c = 1/\lambda > (2n+1)/8H$

$k_0 = 2\pi/\lambda$



$\omega_l = c\pi/L > \omega$

$\omega_l/2\pi c > \omega/2\pi c > (2n+1)/8H$

$H/L > (2n+1)/4$

$n < 2H/L - 1/2$

類似研究1

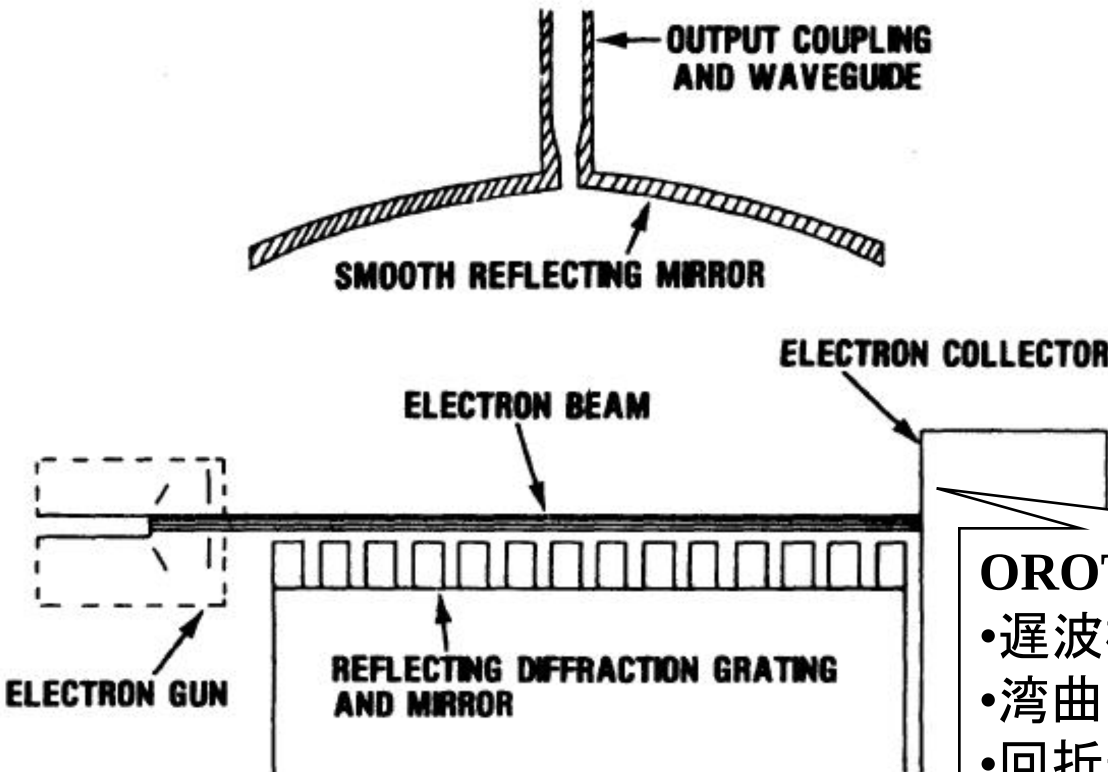


FIG. 1. Schematic diagram of the orotron.

OROTRON
- 遅波構造に対し垂直に放射
- 湾曲したリフレクター
- 回折光

類似研究2

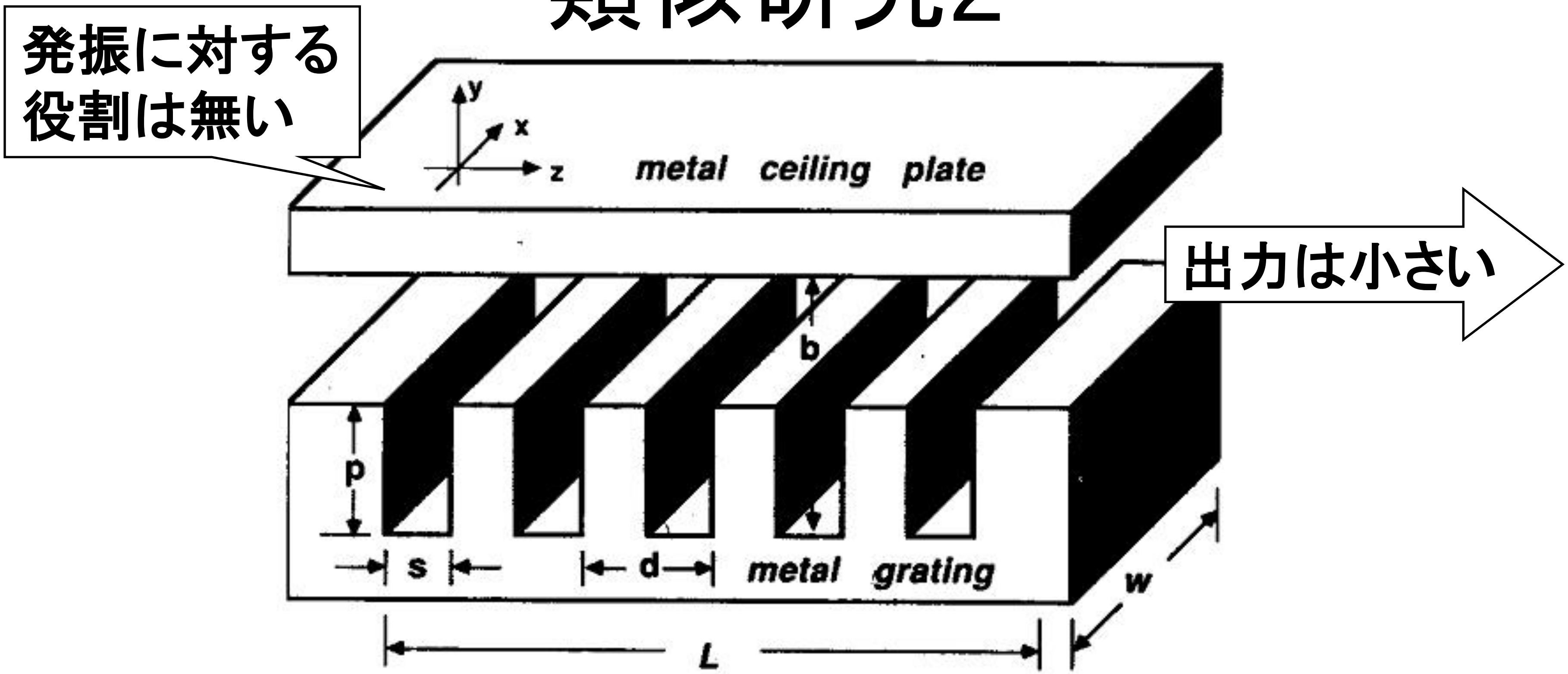


Fig. 1. The resonator for a planar orotron, consisting of a metal grating and ceiling plate.

"*Planar orotron experiments in the millimeter-wavelength band*",
IEEE Trans. Plasma Sci., (1988) 199,
E. M. Marshall, P. M. Phillips, and J. E. Walsh

類似研究3

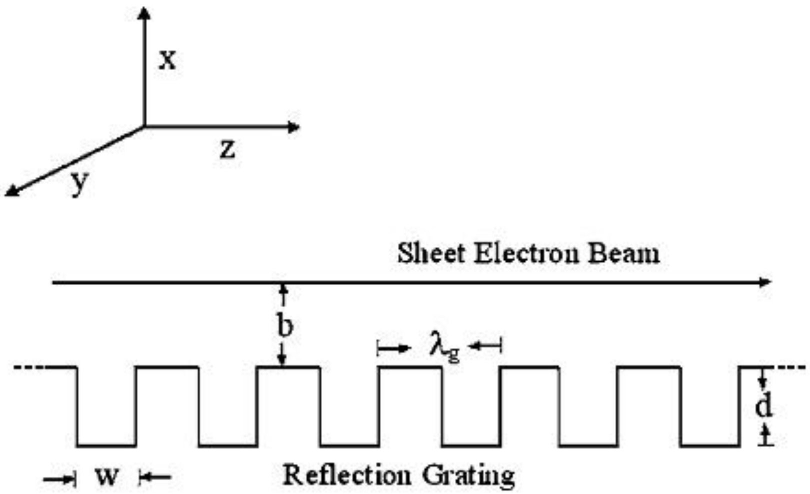


FIG. 1. Schematic of an SP-FEL using a sheet electron beam. The sheet electron beam is in the plane $x=0$.

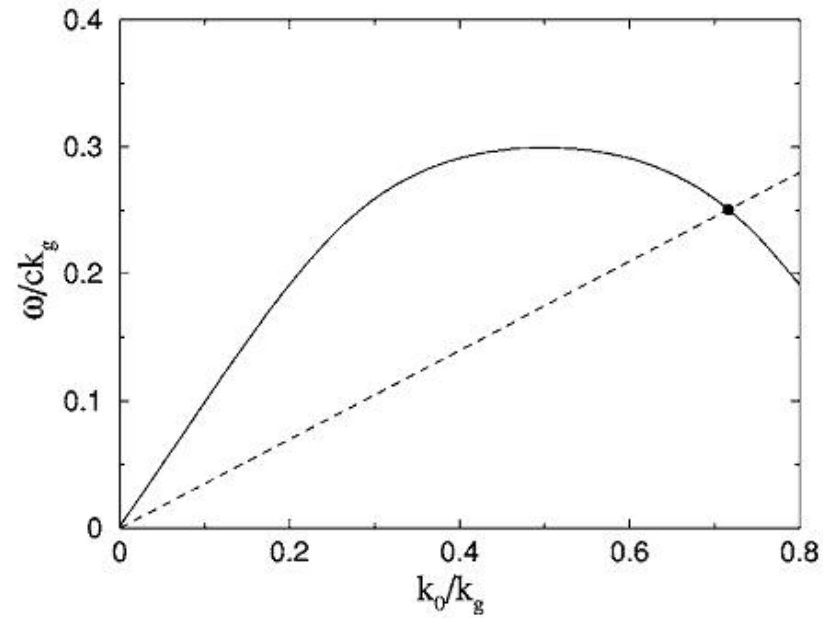
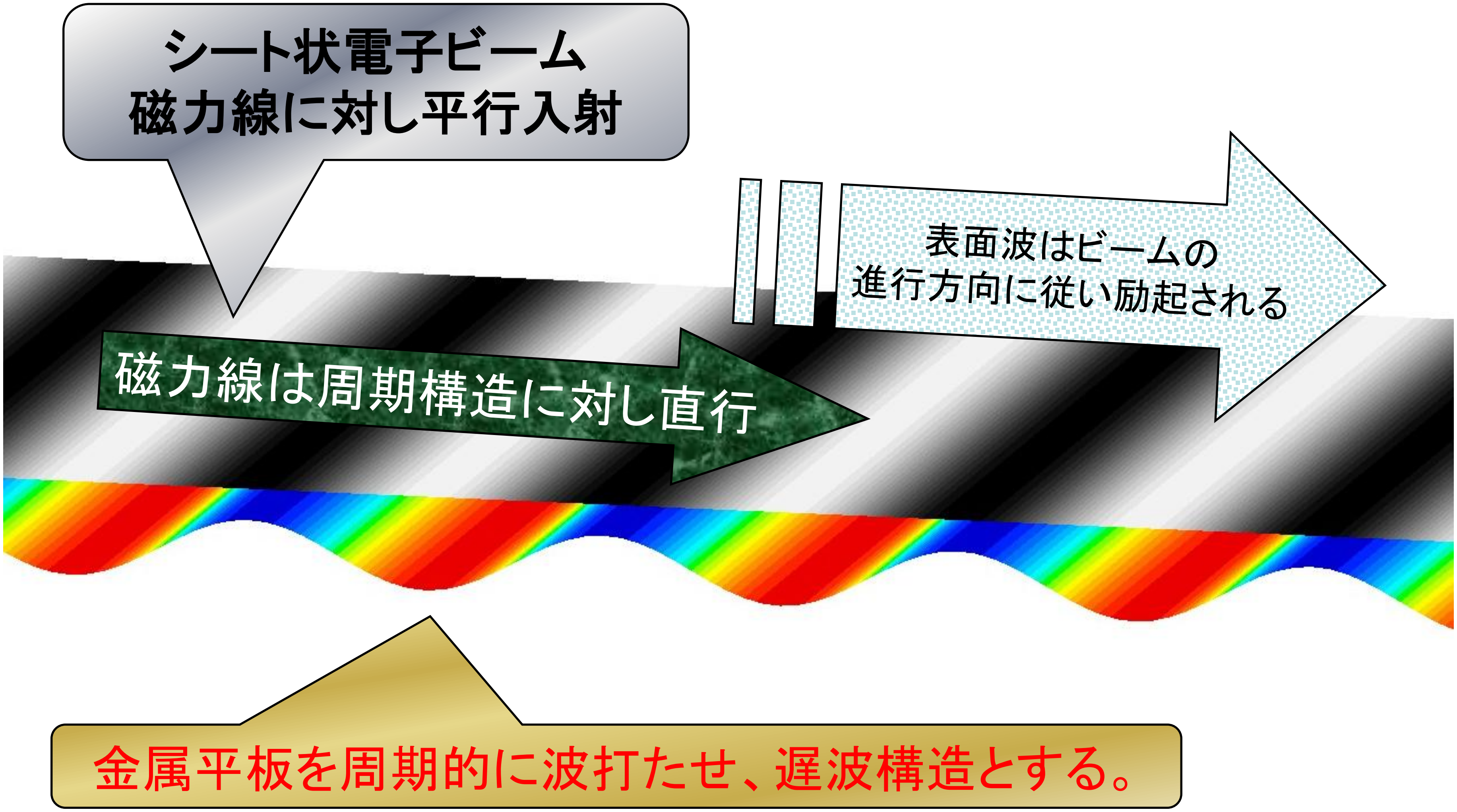


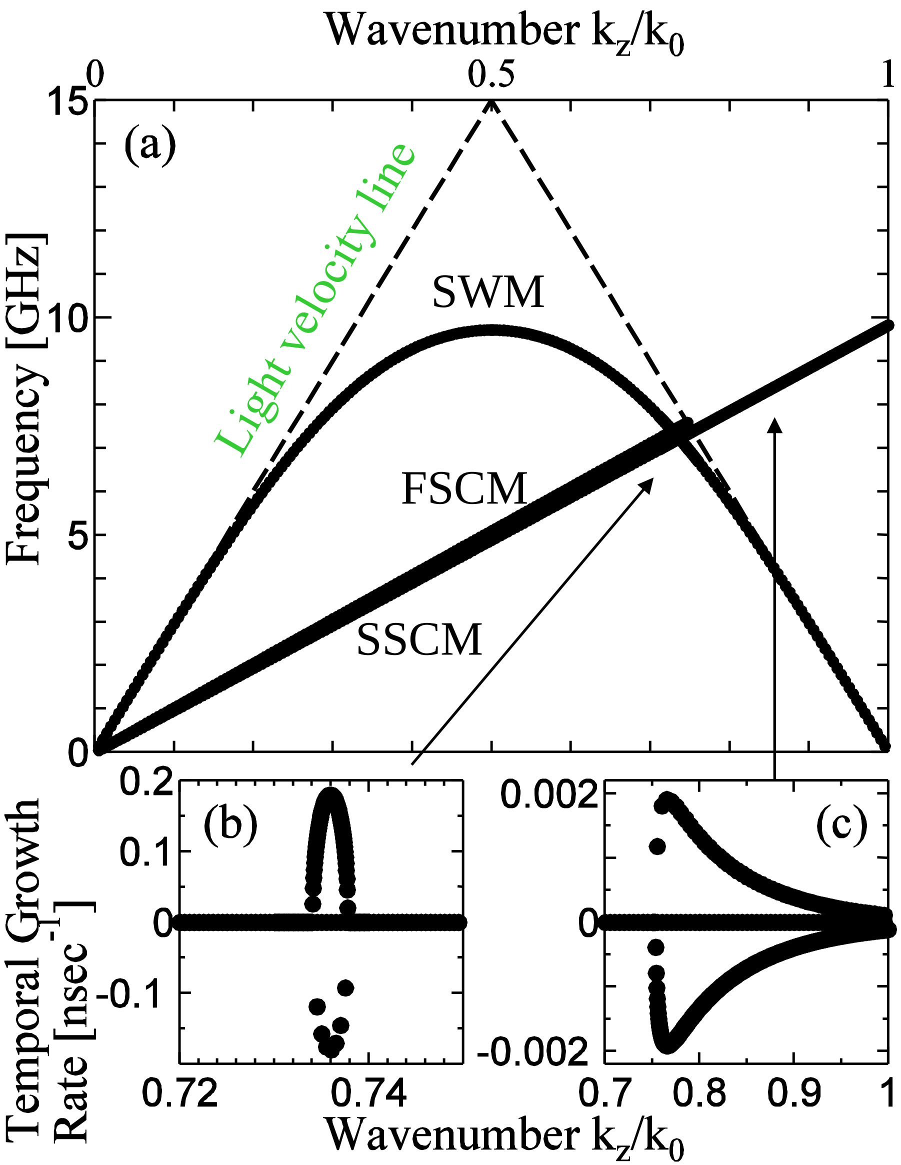
FIG. 6. The solid line shows the dispersion curve of the surface mode supported by the grating for the parameters mentioned earlier. The dashed line is the Doppler line for the beam. At the intersection point, the group velocity v_g is negative and its value is $0.54c$, as obtained from this plot.

"*Analysis of Smith-Purcell free-electron lasers*", Phys. Rev. E **73,** 026501 (2006) *(15 pages)*, Vinit Kumar and Kwang-Je Kim

表面波発振器構造図


シート状電子ビーム
磁力線に対し平行入射
表面波はビームの
進行方向に従い励起される
磁力線は周期構造に対し直行
金属平板を周期的に波打たせ、遅波構造とする。

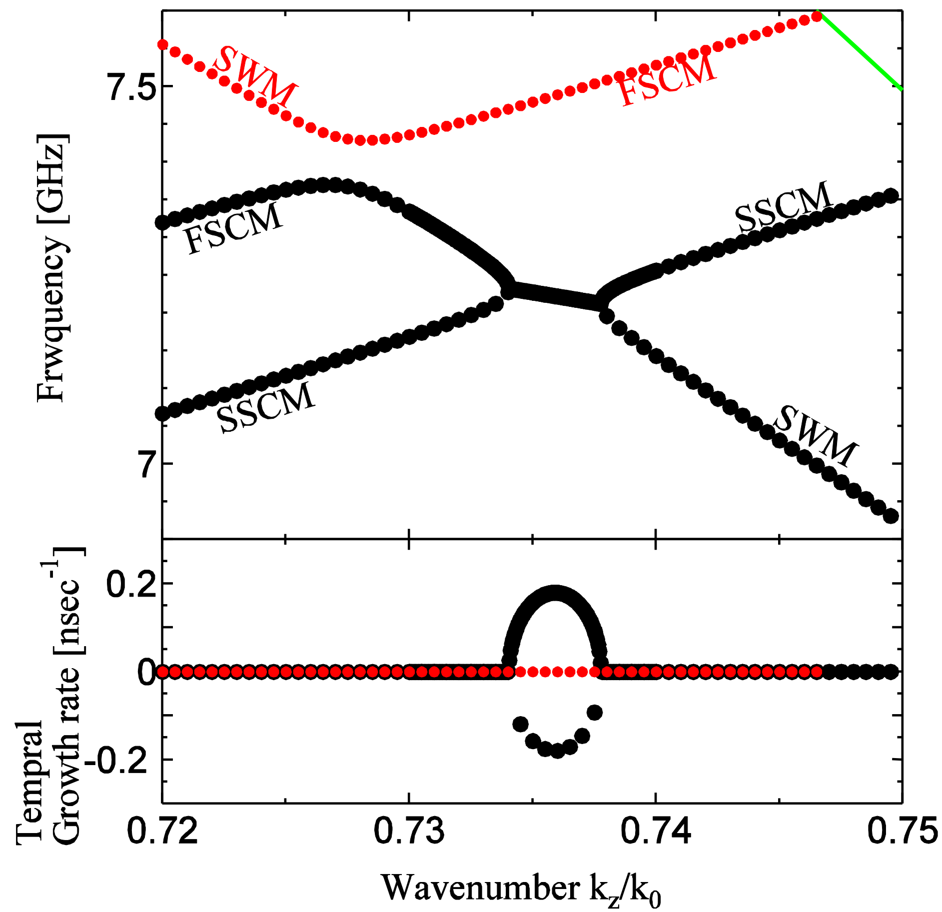
分散関係



SWM:電磁表面波モード

FSCM:速波空間電荷波

SSCM:遅波空間電荷波



Smith-Purcell radiation

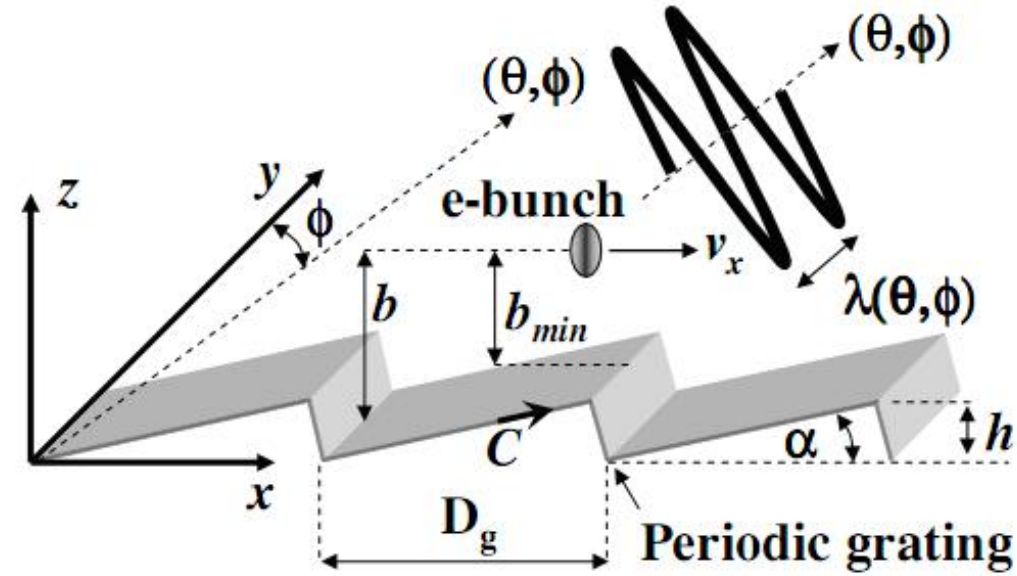


FIG. 1. The SPR scheme. An electron bunch is traveling at an axial velocity v_x above an echelle grating of period D_g.

$$\lambda = \frac{D_g}{n}(\beta^{-1} - \sin\theta \sin\phi), \qquad (1)$$

一方、分散関係より

$$\begin{cases} \omega = -ck_z \\ \omega = v_x(k_z + k_0) \end{cases}$$

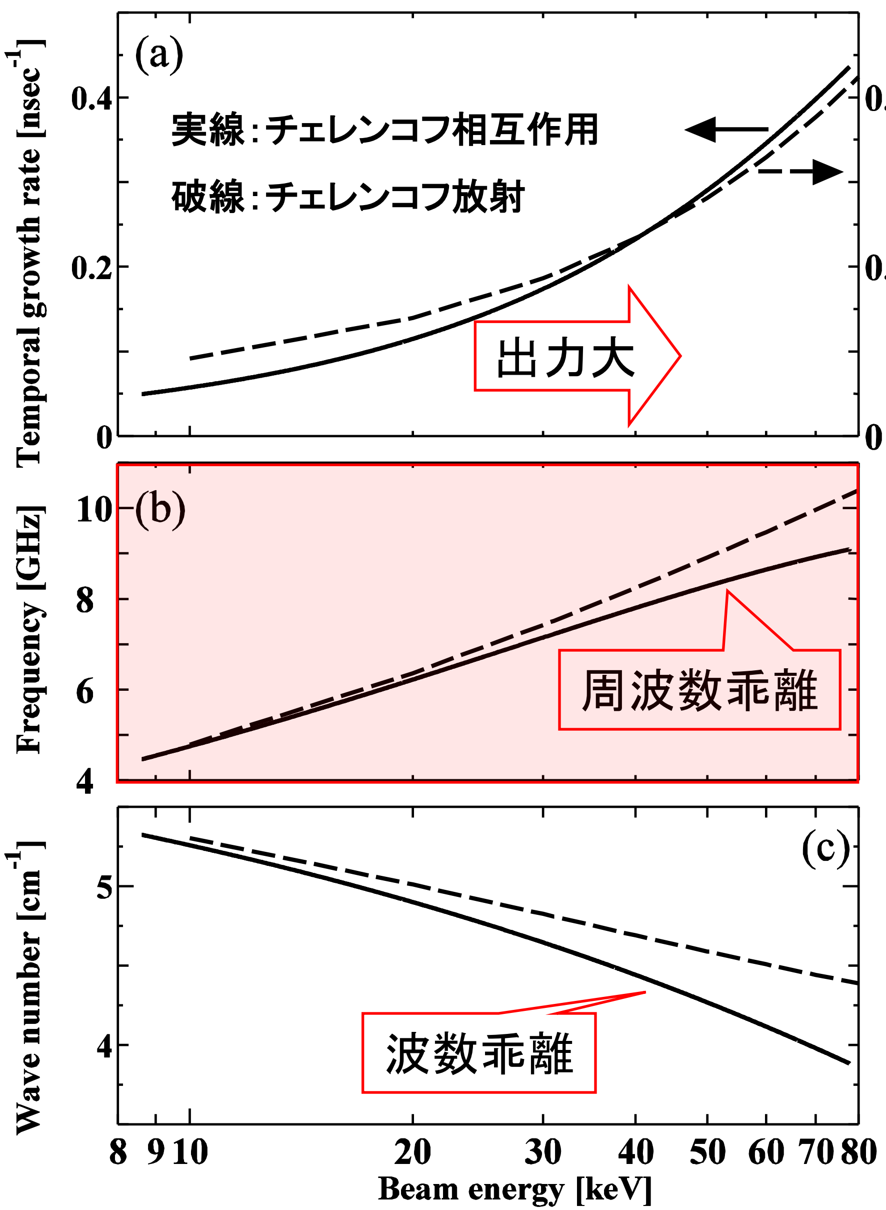
$$-ck_z = v_x(k_z + k_0)$$

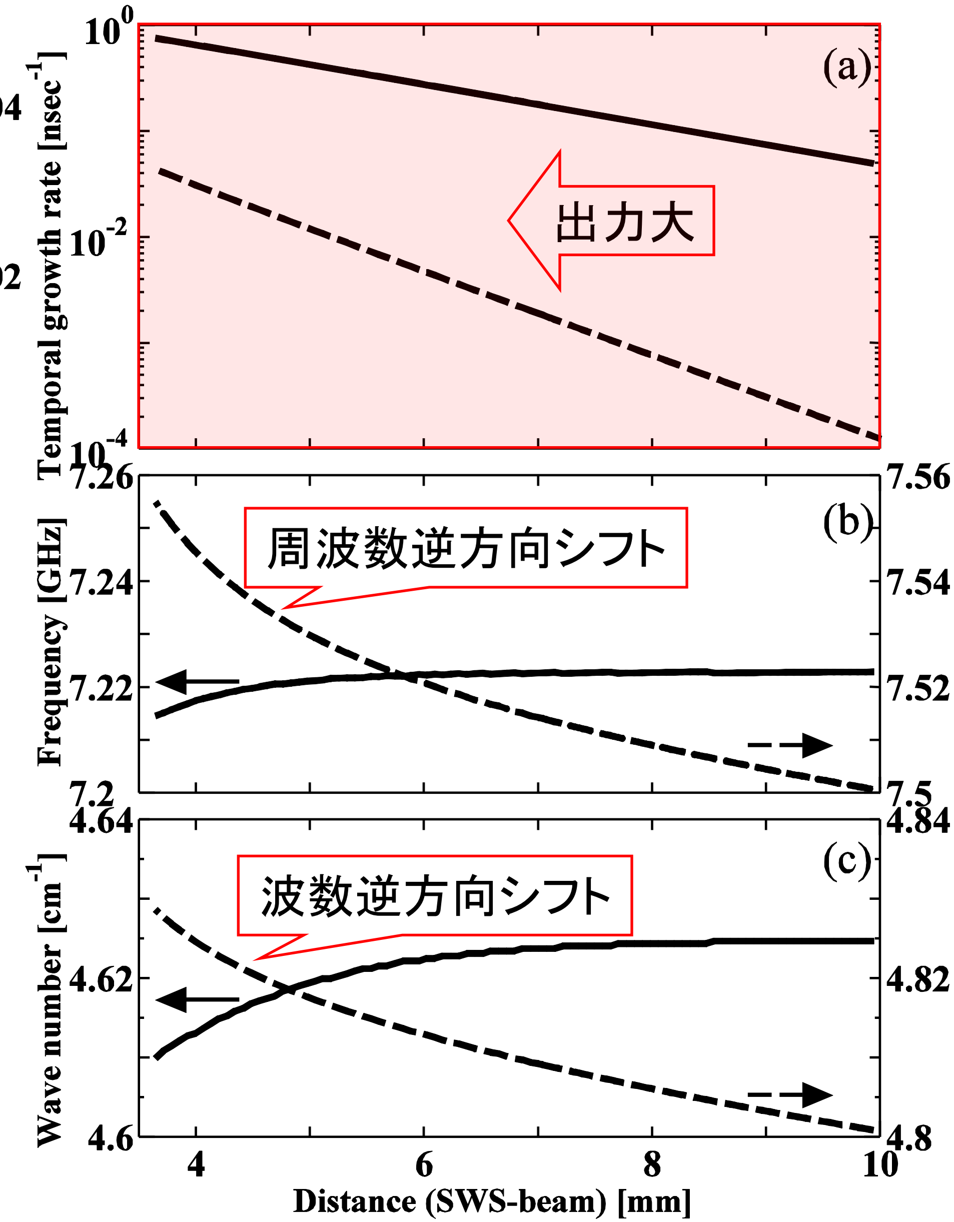
$$(\beta^{-1} + 1)k_z = k_0$$

$$\therefore (\beta^{-1} + 1)D = \lambda$$

“Smith-Purcell radiation from a charge moving above a finite-length grating”, Amit S. Kesar, Phys. Rev. ST-AB, **8**, 072801 (2005)

発振特性(周波数/電圧、出力/距離)





まとめ

- コルゲート金属表面近傍に、金属に沿って伝播する伝送モードが存在する。→電磁表面波。
- $H/L>1/4$を満たすとき表面波は存在する。→高次モード数は、$n<2H/L-1/2$。
- 表面波モードのカットオフ周波数はゼロ。→ローパスフィルタとして使える。
- シート電子ビームとチェレンコフ相互作用を起こす。
- スミスパーセル放射と同じ周波数特性の不安定性を得た。
- 電子ビームと共振器の面積を増加させると、表面波発振器の出力は増大する。
- 発振周波数は、ビーム電圧で制御される。
- 遅波構造とビームの距離を狭めると、相互作用強度は指数関数的に増大する。