Extended MHD simulation of Rayleigh-Taylor/Kelvin-Helmholtz instability

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Background

・Numerical simulations using the single-fluid MHD model for high wave number ballooning instability[1]

・The single-fluid model : ignore Hall effect, Finite Larmor Radius (FLR) effect and so on.

・Since the single-fluid model can be insufficient for high wave number ballooning modes : use extended MHD model[2].

・Though extended MHD model is used as non-ideal MHD code such as M3D[3], NIMROD[4] and so on to explain experimental results, fundamental characteristics are not fully clarified especially in the nonlinear stage.

・Since the ballooning instability is Rayleigh-Taylor(RT) type instability, here we consider a simple RT instability.

Model and Method I

- 2D slab geometry
- Extended MHD equations

(the Hall term and gyro-viscosity are added.)

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}) \,, \tag{1}$$

$$\frac{\partial}{\partial t}(\rho \mathbf{v}) = -\nabla \cdot \left[\rho \mathbf{v}\mathbf{v} + \mathbf{I}\left(p + \frac{B^2}{2} \right) - \mathbf{B}\mathbf{B} \underline{+ \delta \mathbf{\Pi_i}} \right] + \rho \mathbf{g}, \tag{2}$$

$$\frac{\partial e}{\partial t} = -\nabla \cdot \left[\mathbf{v}(e + p) \underline{- \mathbf{v} \cdot \delta \mathbf{\Pi_i}} \right] , \tag{3}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \cdot (\mathbf{v}\mathbf{B} - \mathbf{B}\mathbf{v}) \underline{- \nabla \times \left[\frac{\epsilon}{\rho} (\mathbf{J} \times \mathbf{B} - \nabla p_e) \right]} \,. \tag{4}$$

$$e = \frac{\rho \mathbf{v}^2}{2} + \frac{p}{\gamma - 1}$$

ϵ : Hall parameter

δ : gyro-viscosity

model and method II

2D approximation :

$u_z = 0, \partial/\partial z \to 0$ in eqs.(1)-(4)

$$J_x = \frac{\partial B_z}{\partial y}, J_y = \frac{\partial B_z}{\partial x}, J_z = \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}$$

The gyro-viscosity term is given as

$$(\Pi_i)_{xx} = -(\Pi_i)_{yy} = -p_i \left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y} \right), \tag{5}$$

$$(\Pi_i)_{xy} = (\Pi_i)_{yx} = p_i \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right). \tag{6}$$

g : gravitational acceleration, γ : ratio of the specific heats
total pressure : $\mathrm{p} = \mathrm{p_i} + \mathrm{p_e} = (\alpha + 1)\mathrm{p_e}$, $\alpha = \mathrm{p_i}/\mathrm{p_e}$. $\mathrm{p_i}, \mathrm{p_e}$: ion and electron pressure

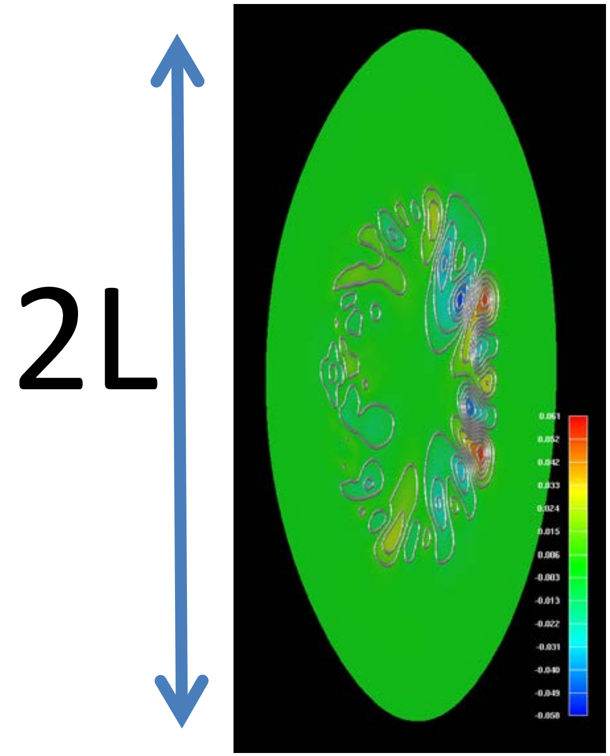
Normalized Hall and gyro-viscosity parameter in Ref.[1]

Wave length : λ=6cm when poloidal mode number m=30

Ion skin depth : $l_i = 5\mathrm{cm}$

(T=0.5keV, B=0.5T, $n = 2 \times 10^{19}[\mathrm{m}^{-3}]$)

Ion Larmor radius : $\rho_i = 6.5\mathrm{mm}$



If $\mathrm{L} = \mathrm{miner\ radius} = 1.0[\mathrm{m}]$

Gyro-viscosity

$$\delta = \frac{\rho_i}{L/2} = \frac{0.65[\mathrm{cm}]}{50[\mathrm{cm}]} \approx O(10^{-2})$$

Hall parameter

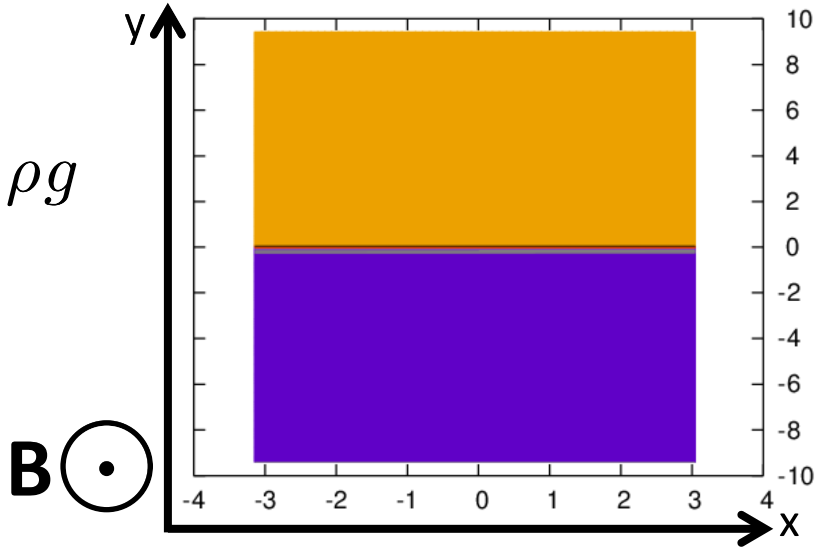
$$\epsilon = \frac{l_i}{L/2} = \frac{5[\mathrm{cm}]}{50[\mathrm{cm}]} \approx O(10^{-1})$$

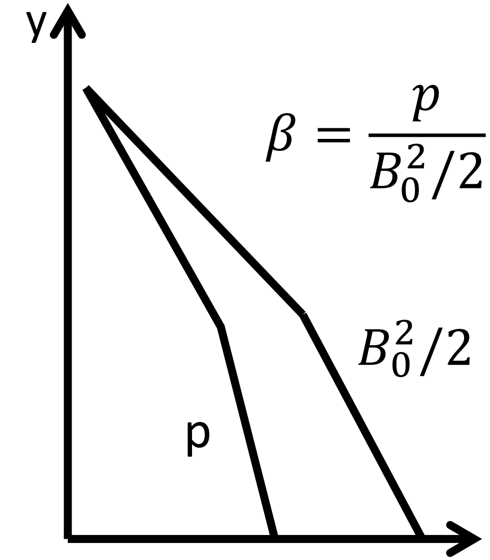
Initial equilibrium in RT instability

- Equilibrium

$$\nabla\left(p_0 + \frac{B_0^2}{2}\right) = -\rho g$$

$$B_z = b_z + B_0(y)$$

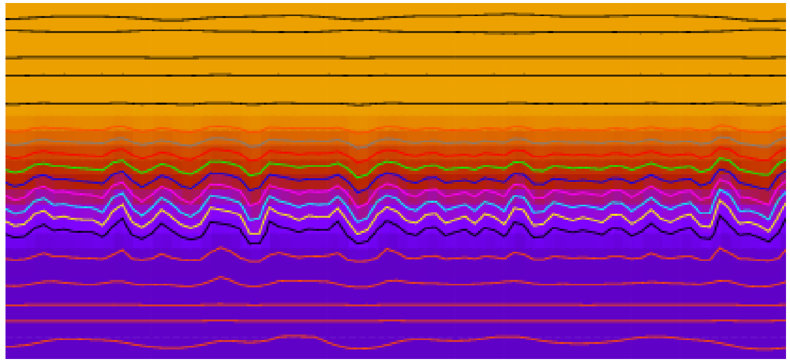




- Numerical method

→ - 4th order central difference
  - Time evolution : 4th order Runge-Kutta-Gill (RKG)
- System size : $-\pi \leq x \leq \pi$, $-3\pi \leq y \leq 3\pi$
- Boundary condition : periodic ($x = \pm\pi$) , $\partial/\partial y \to 0$ ($y = \pm 3\pi$)
- Resolution : $(N_x, N_y) = (1024, 4086)$
- density ratio : $\rho_2/\rho_1 = 2.0$
- $\beta = 10\%$, density jump width : 0.2, $p_i/p_e = 1.0$

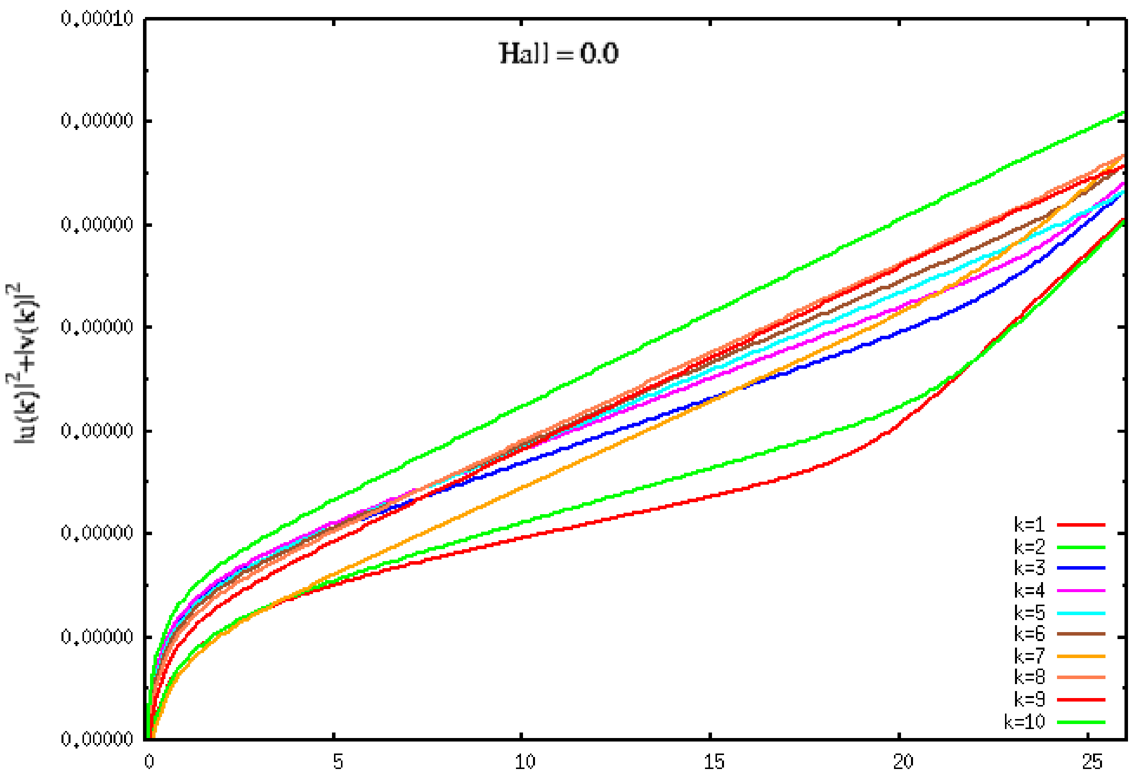
Initial perturbation : random perturbations are added



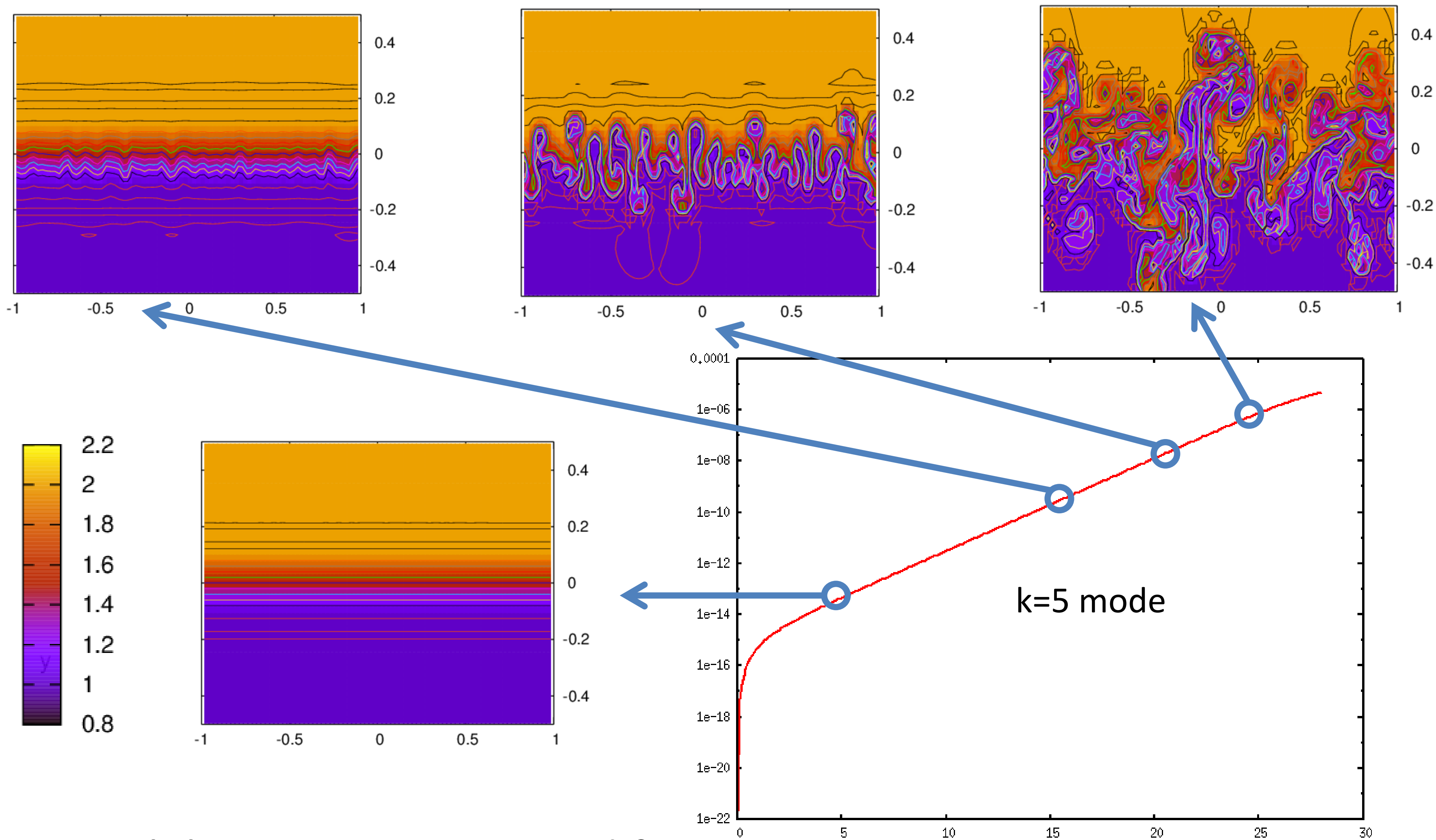
・Giving single-mode perturbation is time consuming

→Add all perturbations in the initial condition.

・Random perturbed wave number : $\mathrm{k} = 0 \sim N_x/4$ are added simultaneously.

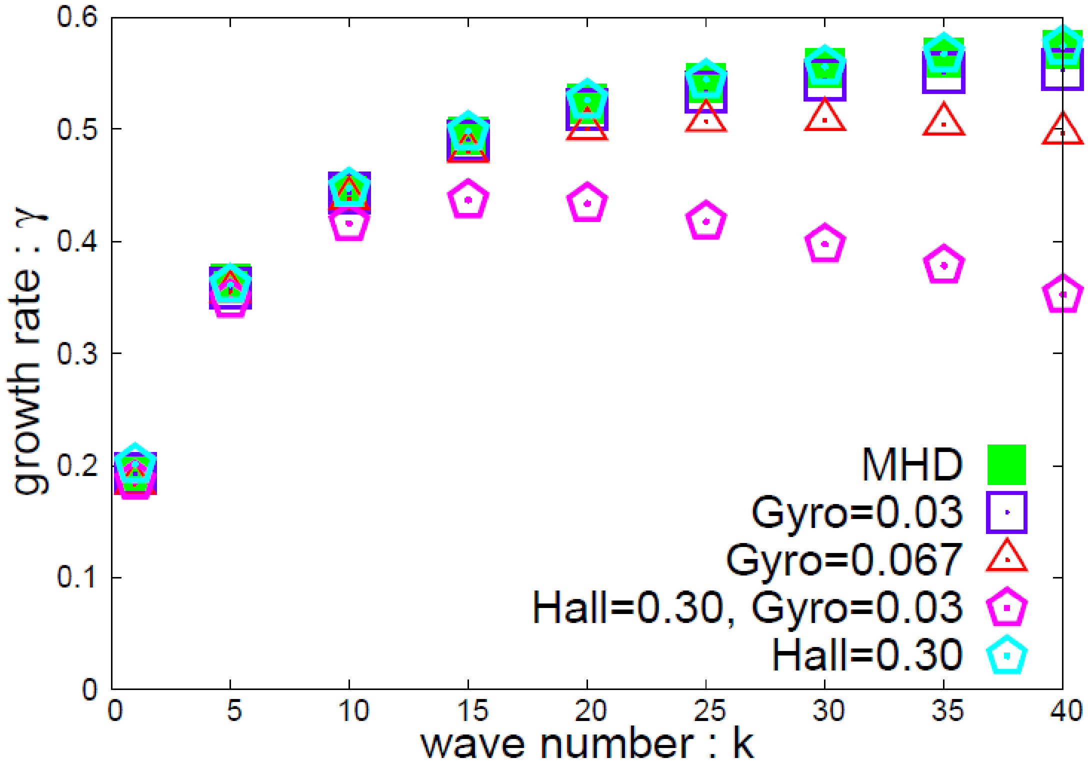


Time evolution of the RT instability : MHD



- RT instability grows in time and forms turbulence-like structure.
- Since each modes are superimposed, mushroom-like structure doesn’t appear even in the linear stage.

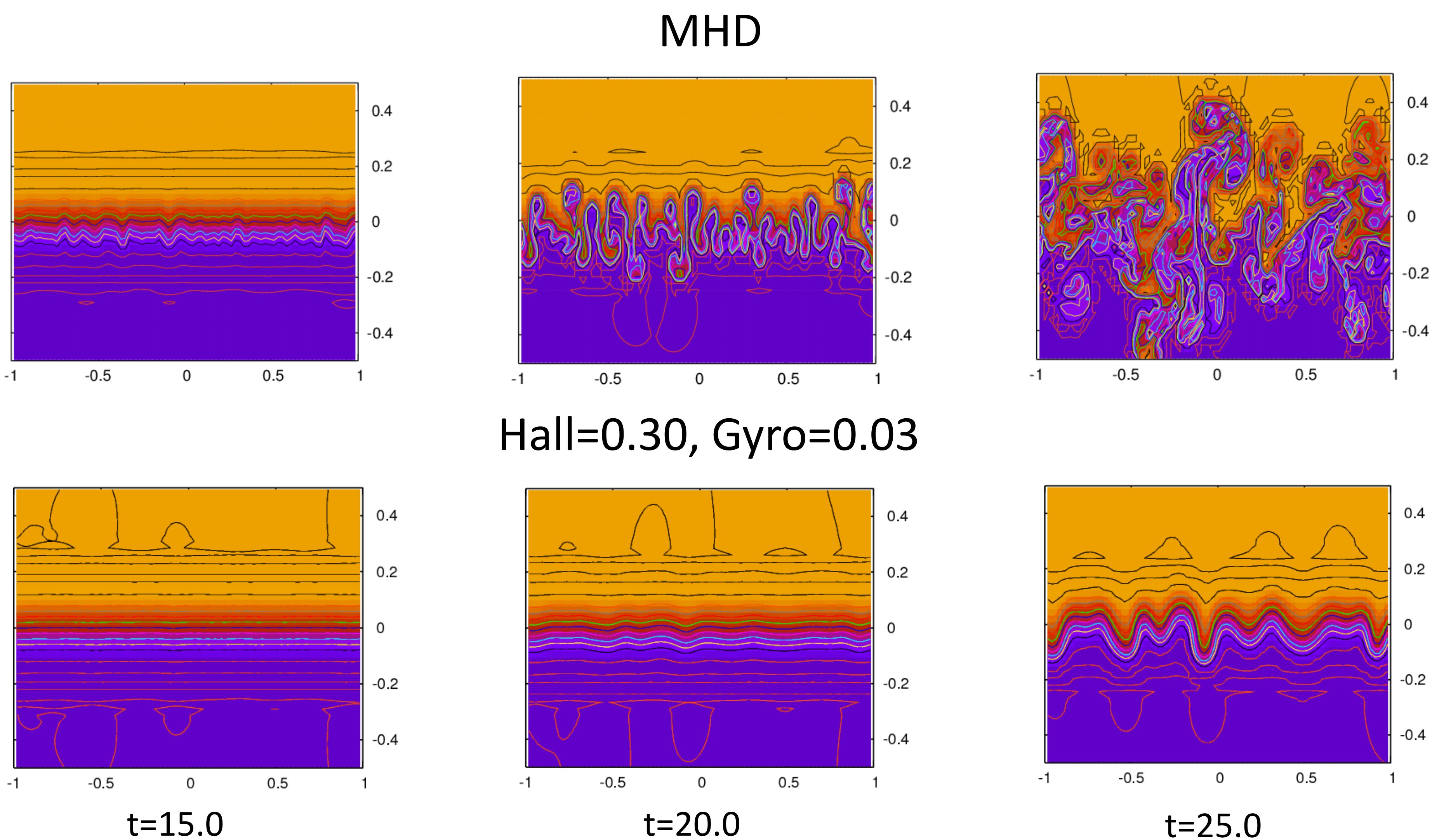
Growth rate : suppression of the higher modes



・Linear growth rate is evaluated by the gradient of the integrated kinetic energy.

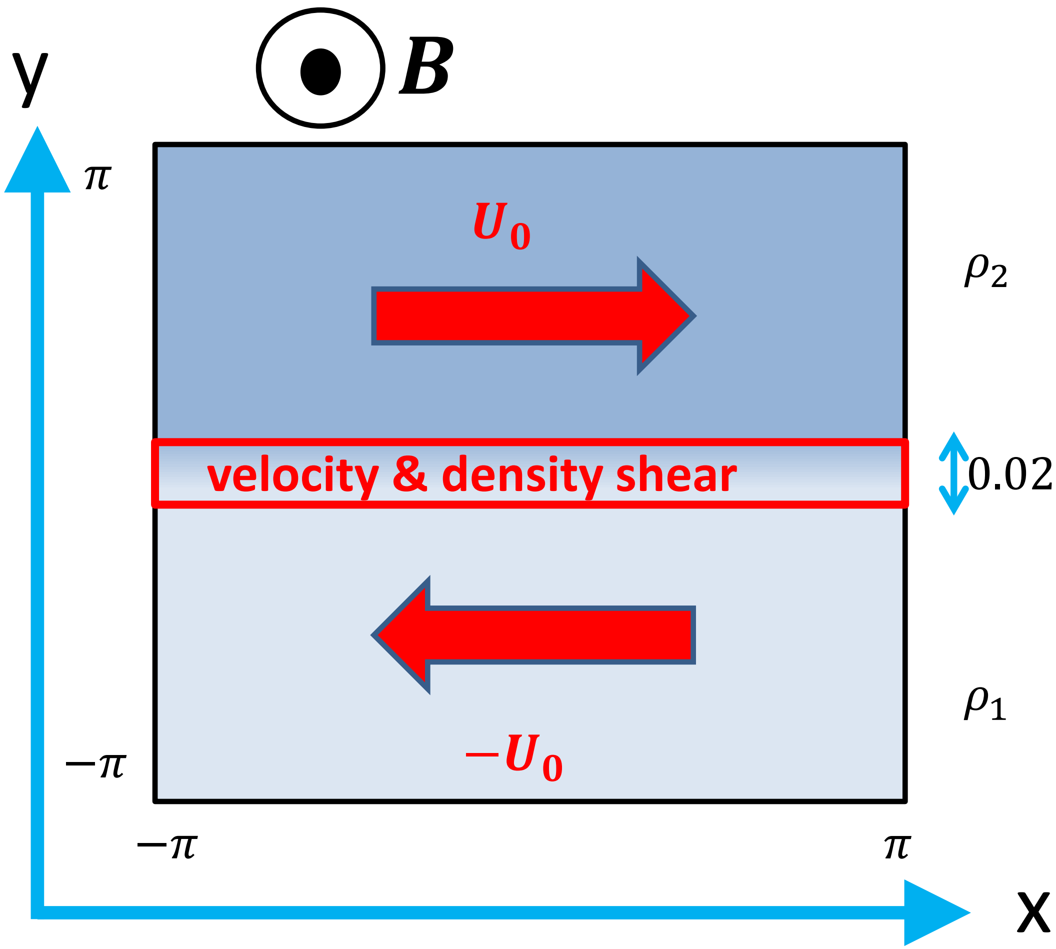
・In this parameter, combination of the Hall term and the gyro-viscosity term stabilize the growth rate of the higher wave number modes.

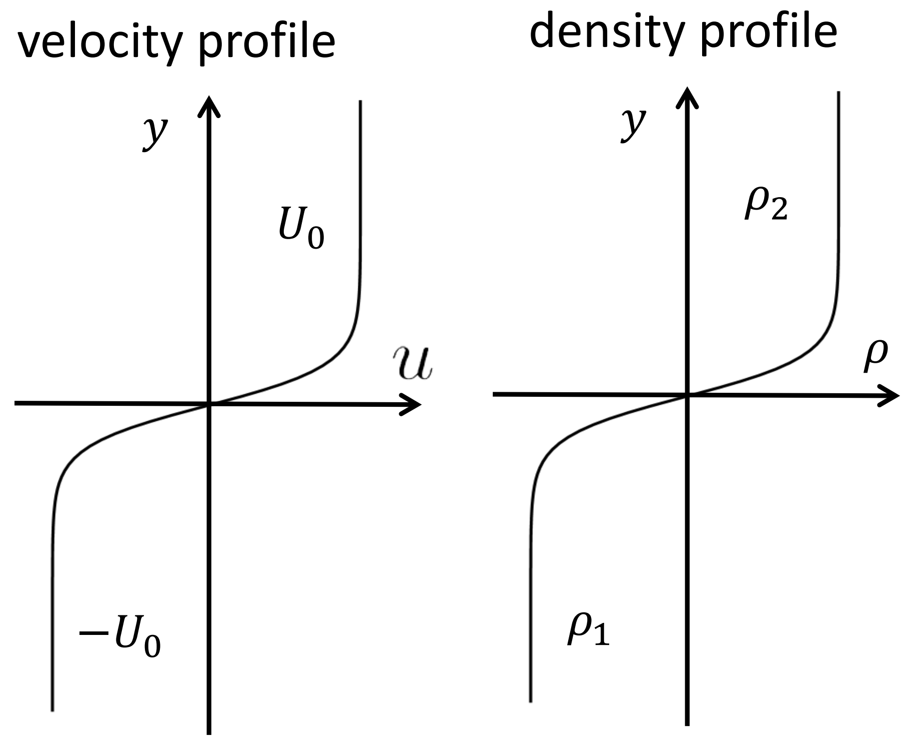
Density plot : low wave number modes grow



▪High wave number modes are suppressed and only low wave number modes grow.

Initial Condition in KH instability





$$u = U_0 \tanh(y/d)$$

$$\rho = \frac{\rho_1 + \rho_2}{2} + \frac{\rho_2 - \rho_1}{2} \tanh(y/d)$$

$d = 0.01\pi$

$\rho_2 = 4.0$

$U_0 = \begin{cases} 0.1 \\ -0.1 \end{cases}$

$\rho_1 = 1.0$

$(N_x, N_y) = (256, 1024)$

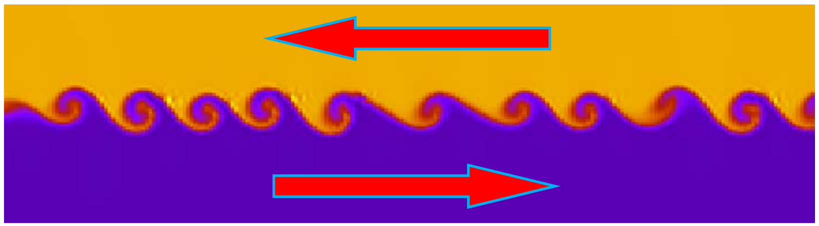
Equilibrium

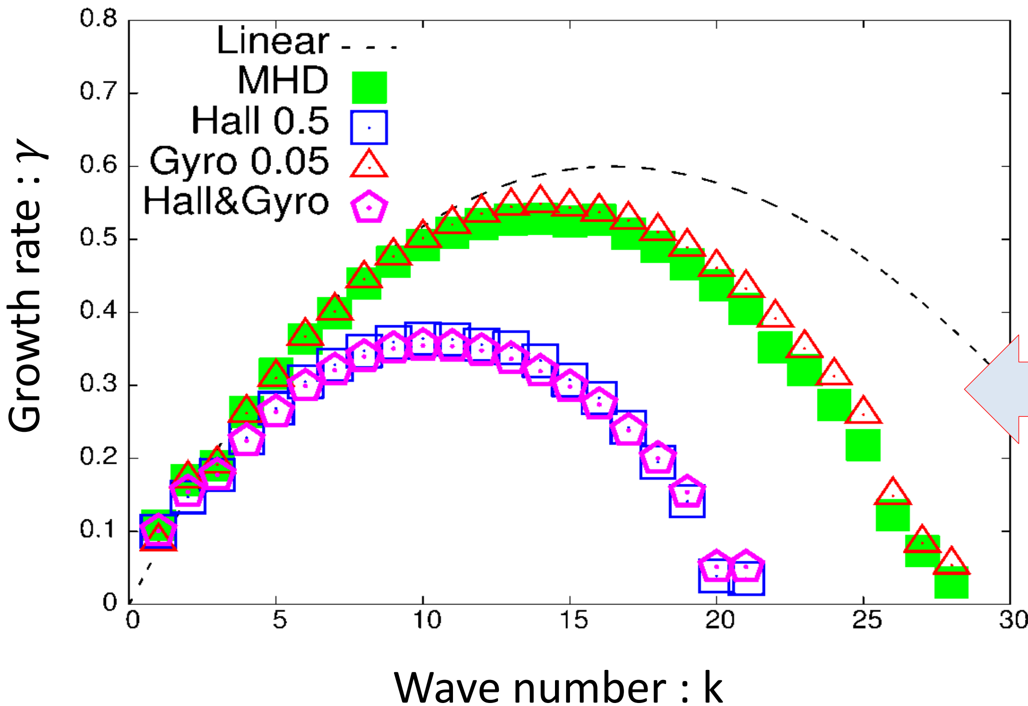
$$\frac{\partial}{\partial y}\left(p + \frac{B_0^2}{2} + \delta P_i \frac{\partial u}{\partial y}\right) = 0$$

Result

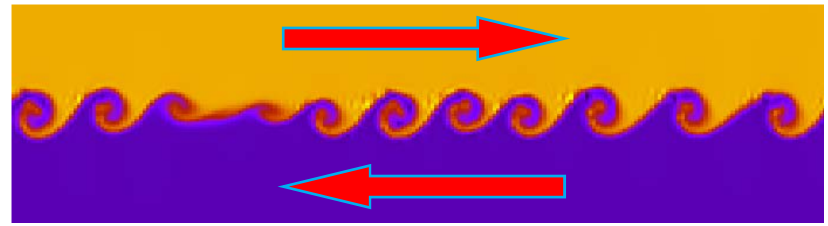
$U_0 = -0.1$ | $U_0 = 0.1$

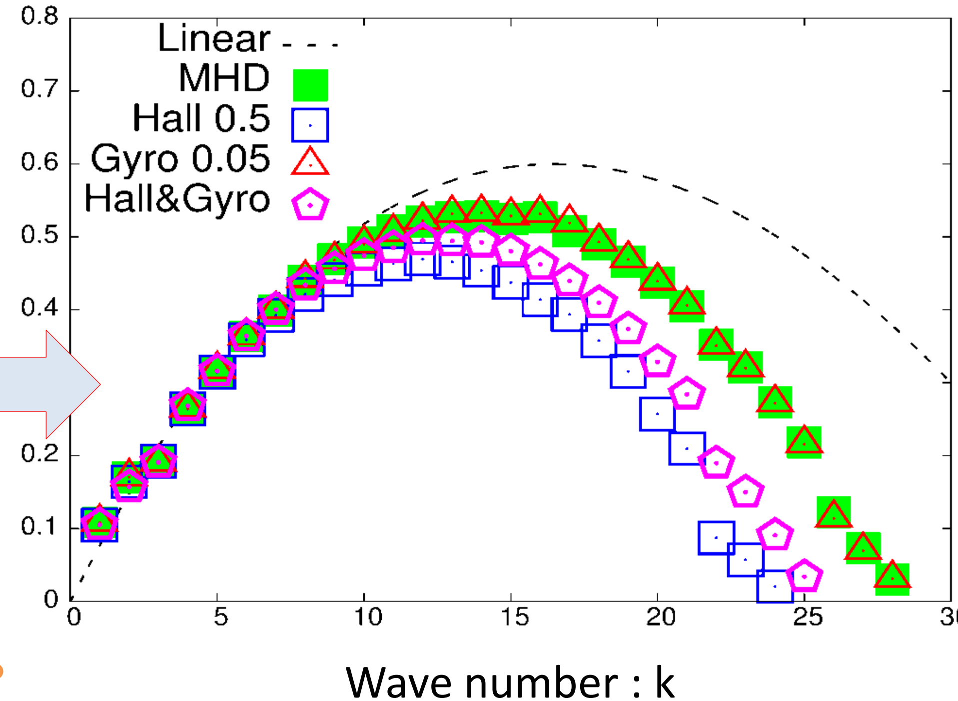
density contour of (1-fluid) MHD





density contour of (1-fluid) MHD



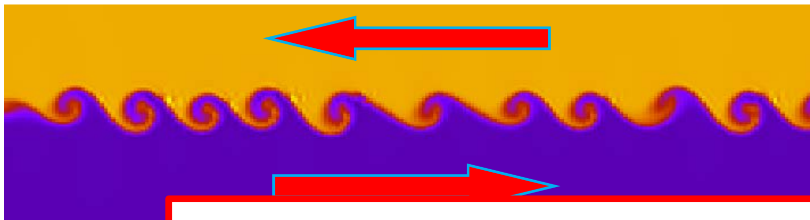


Result

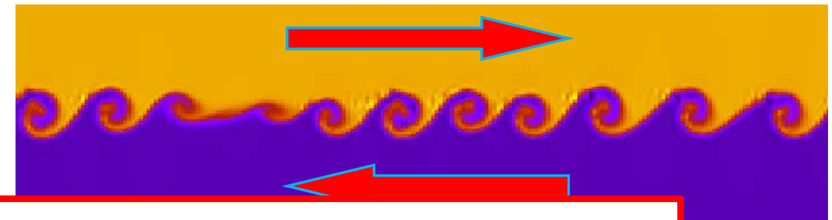
$U_0 = -0.1$

$U_0 = 0.1$

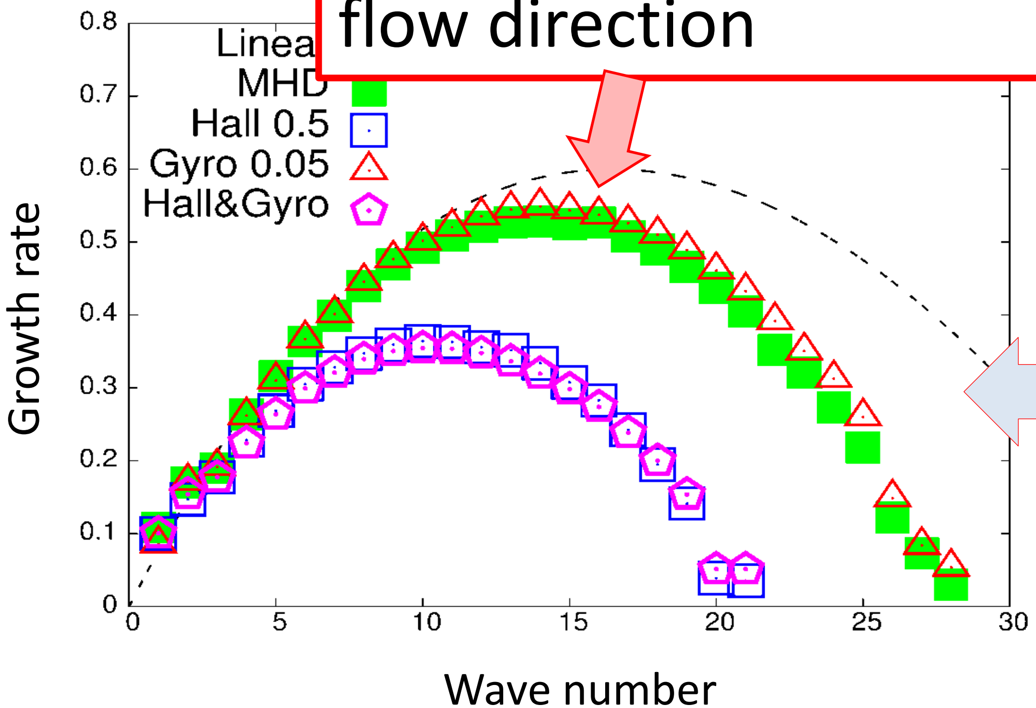
density contour of (1-fluid) MHD

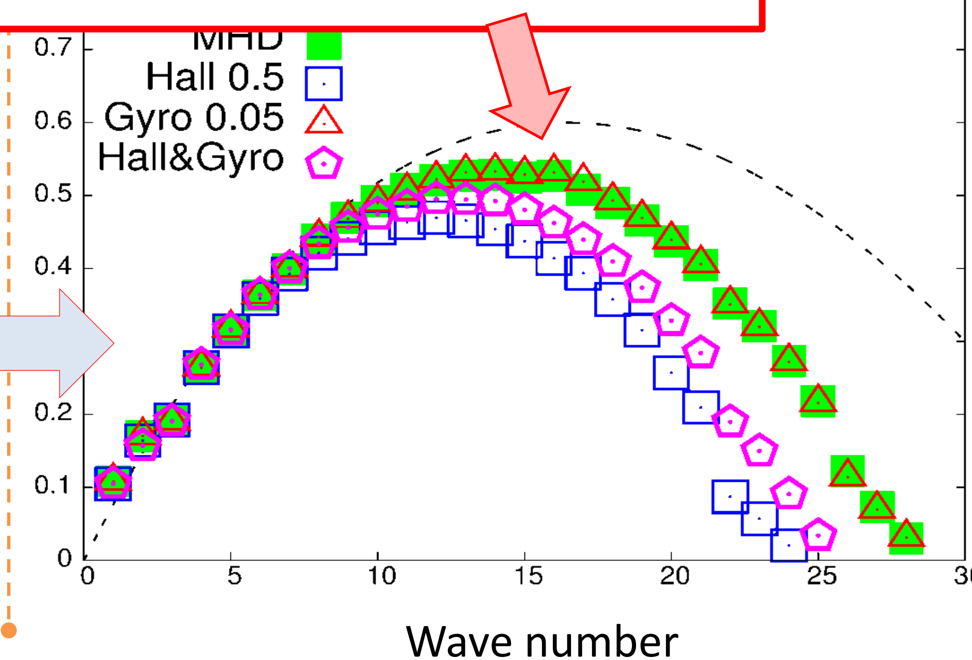


density contour of (1-fluid) MHD



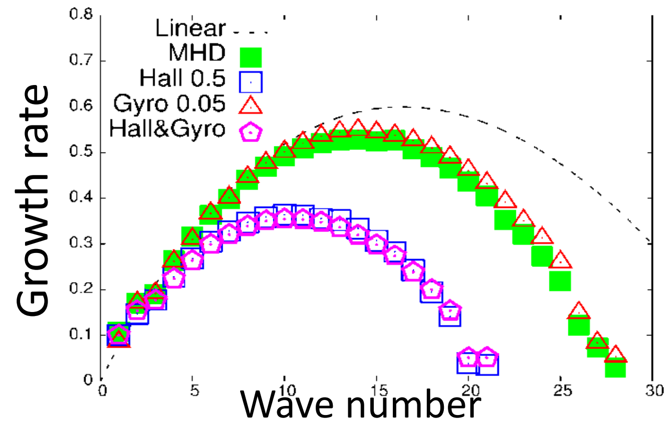
the Hall and FLR effects differ depending on the flow direction



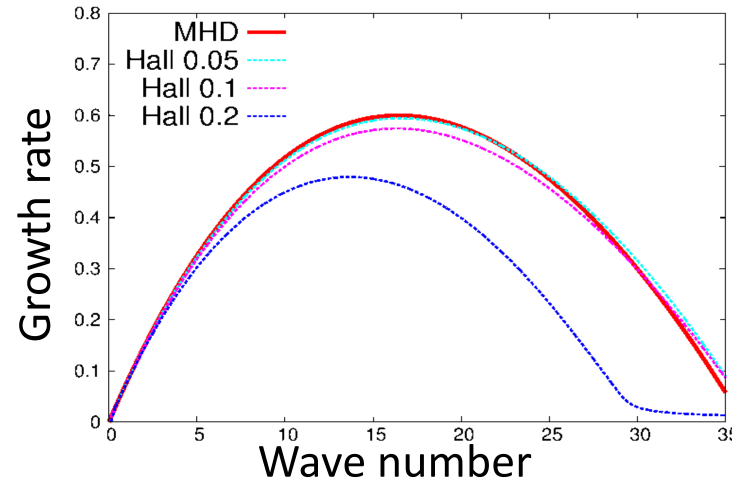


On the Hall effect, the relation is qualitatively consistent with the result of linear analysis.

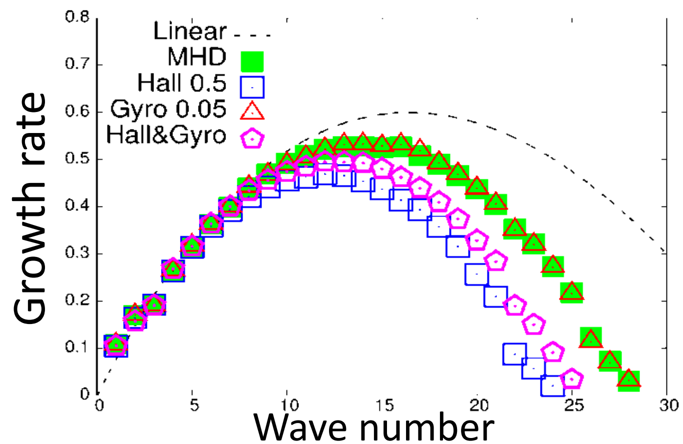
$U_0 = -0.1$

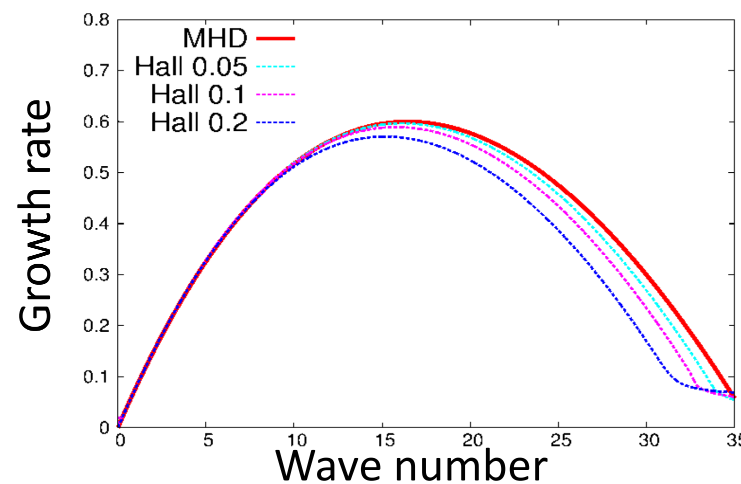


Result of linear analysis by A. Ito (ITC22,P4-3)



$U_0 = 0.1$





Summary I (RT instability)

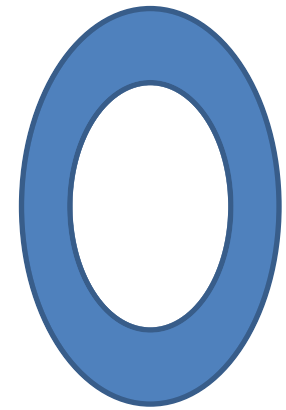
・The effects of the Hall term and the gyro-viscosity to the RT and KH instabilities are studied by the nonlinear extended MHD simulations.
・Linear region has the same time scale when random perturbed modes are added simultaneously.
・In RT simulation, the Hall term slightly increase and the gyro-viscosity decrease the growth rate in our parameter.
This result is consistent with the result by Winske[5].
・Combination of the Hall term and the gyro-viscosity highly stabilize the linear growth rate of the high wave number modes and only low wave number modes grow.

Summary II (KH instability)

- The Hall term can stabilize KH modes, especially in high wave number.
- The dispersion relation obtained from this simulation is qualitatively consistent with the result of linear analysis on the Hall effect.
- The gyro-viscosity can destabilize KH modes.

Future plan

・Simulate both the RT instability and the KH instability simultaneously.
・Extend the geometry to a 3D torus system to analyze the evolution of the ELMs.



2D hollow cylinder → 3D hollow cylinder → 3D torus

Reference

[1] H. Miura and N. Nakajima: Nucl. Fusion **50** (2010) 054006.
[2] P. Zhu et al: Phys. Rev. Lett. **101** (2008) 085005.
[3] W. Park et al: Phys. Plasmas **6** (1999) 1796.
[4] C. R. Sovinec et al: J. Comp. Phys. **195** (2004) 355.
[5] D. Winske : Phys. Plasmas **3** (1996) 11.